

Brinell hardness tester

steel ball of diameter D pressed into flat plate with constant force F

measure size of circular indentation

steel F = 3000 kgf D = 10 mm

aluminum F = 500 kgf D = 10 mm

calculate subsurface stress for testing aluminum

steel ball on flat aluminum plate

$$F = 500 \text{ kgf} = 4905 \text{ N}$$

$$\text{steel ball } R_1 = 5 \text{ mm } E_1 = 206.8 \text{ GPa } v_1 = 0.28 \quad \text{Norton Table A-1}$$

$$\text{aluminum plate } R_2 = \infty \quad E_2 = 71.7 \text{ GPa} \quad v_2 = 0.34 \quad \text{Norton Table A-1}$$

$$m_1 = \frac{1-v_1^2}{E_1} = \frac{1-(0.28)^2}{206.8 \text{ GPa}} \left(\frac{\text{GPa}}{10^3 \text{ MPa}} \right) \left(\frac{\text{MPa} \cdot \text{mm}^2}{\text{N}} \right) = 4.4565 \times 10^{-6} \frac{\text{mm}^2}{\text{N}} \quad \text{Norton Eq. 7.9a}$$

$$m_2 = \frac{1-v_2^2}{E_2} = \frac{1-(0.34)^2}{71.7 \text{ GPa}} \left(\frac{\text{GPa}}{10^3 \text{ MPa}} \right) \left(\frac{\text{MPa} \cdot \text{mm}^2}{\text{N}} \right) = 12.335 \times 10^{-6} \frac{\text{mm}^2}{\text{N}} \quad \text{Norton Eq. 7.9a}$$

$$B = \frac{1}{2} \left(\frac{1}{R_1} + \frac{1}{R_2} \right) = \frac{1}{2} \left(\frac{1}{5 \text{ mm}} + \frac{1}{\infty} \right) = 0.1 \text{ mm}^{-1} \quad \text{Norton Eq. 7.9b}$$

$$a^3 = 0.375 \frac{m_1 + m_2}{B} F = 0.375 \frac{(4.4565 + 12.335)}{0.1 \text{ mm}^{-1}} \left(\frac{10^{-6} \text{ mm}^2}{\text{N}} \right) 4905 \text{ N} = 0.3089 \text{ mm}^3 \quad \text{Eq. 7.9d}$$

$$a = 0.6760 \text{ mm}$$

$$p_{\text{MAX}} = \frac{3}{2} \frac{F}{\pi a^2} = \frac{3}{2} \frac{(4905 \text{ N})}{\pi (0.6760 \text{ mm})^2} = 5126 \text{ MPa} \quad \text{Norton Eq. 7.8b}$$

$$\tau_{\text{MAX}} = \frac{p_{\text{MAX}}}{2} \left[\frac{(1-2v)}{2} + \frac{2}{9}(1+v)\sqrt{2(1+v)} \right] \quad \text{Norton Eq. 7.12b}$$

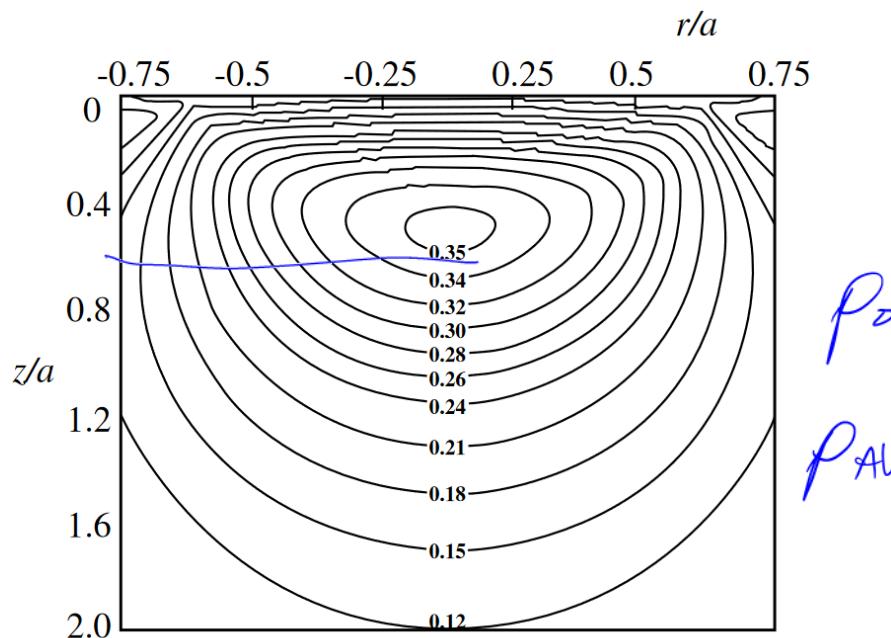
$$\text{aluminum plate } v_2 = 0.34 \quad \tau_{\text{max}} = 0.3237 \quad p_{\text{max}} = 1659 \text{ MPa} \quad \text{Eq. 7.12b}$$

do not use
diameter

von Mises $\sigma' = \sqrt{3} \tau_{\text{MAX}} = \underline{\underline{2874 \text{ MPa}}}$

strongest aluminum in Norton Table A-2 7075 heat treated $S_Y = 503 \text{ MPa}$

Brinell tester will dent strongest aluminum $\sigma' > S_Y$



$$P_0 = \frac{F}{\pi a^2}$$

$$P_{\text{AUG}}$$

FIGURE 3.7 Contours of maximum shear stress normalized by Hertz stress p_0 , beneath nominal circular point contact of radius a in material with $v = 0.3$.

A ball and socket joint uses a 10 mm DIA steel ball (4340 with HB 430) inside a 10.1 mm DIA phosphor bronze socket (CA510 with HRB 95). Determine factor of safety for the ball and for the socket in static yield at a load of 2000 N.

~~do not use diagram~~

Table A-1 for E and v

Table A-10 for 4340

Table A-4 for phosphor bronze

$$\text{steel ball (convex)} \quad R_1 = +5 \text{ mm} \quad v_1 = 0.28 \quad E_1 = 206.8 \text{ GPa} \quad S_Y = 1365 \text{ MPa}$$

$$\text{phosphor bronze socket (concave)} \quad R_2 = -5.05 \text{ mm} \quad v_2 = 0.33 \quad E_2 = 110.3 \text{ GPa} \quad S_Y = 552 \text{ MPa}$$

$$m_1 = (1-v_1^2) / E_1 = 0.004456 \text{ GPa}^{-1} \quad \text{Eq. 7.9a}$$

$$m_2 = (1-v_2^2) / E_2 = 0.008079 \text{ GPa}^{-1}$$

$$B = (1/R_1 + 1/R_2) / 2 = 0.0009901 \text{ mm}^{-1} \quad \text{Eq. 7.9b}$$

$$a^3 = 0.375 \frac{m_1 + m_2}{B} F \quad \text{Eq. 7.9d}$$

$$a^3 = 0.375 \left(\frac{(0.004456 + 0.008079 \text{ m}^2)}{10^9 \text{ N}} \right) \left(\frac{1000 \text{ mm}}{\text{m}} \right)^2 \left(\frac{\text{mm}}{0.0009901} \right) (2000 \text{ N}) \quad \text{Eq. 7.9d}$$

$$a = 2.118 \text{ mm}$$

$$p_{\max} = \frac{3}{2} \frac{F}{\pi a^2} = \left(\frac{3}{2} \right) \frac{2000 \text{ N}}{\pi (2.118 \text{ mm})^2} = 212.9 \text{ MPa} \quad \text{Eq. 7.8b}$$

ball

$$v_1 = 0.28 \quad \tau_{\max} = \frac{p_{\max}}{2} \left(\frac{1-2v_1}{2} + \frac{2}{9}(1+v_1)\sqrt{2(1+v_1)} \right) = 0.3376 \quad p_{\max} = 71.88 \text{ MPa} \quad \text{Eq. 7.12b}$$

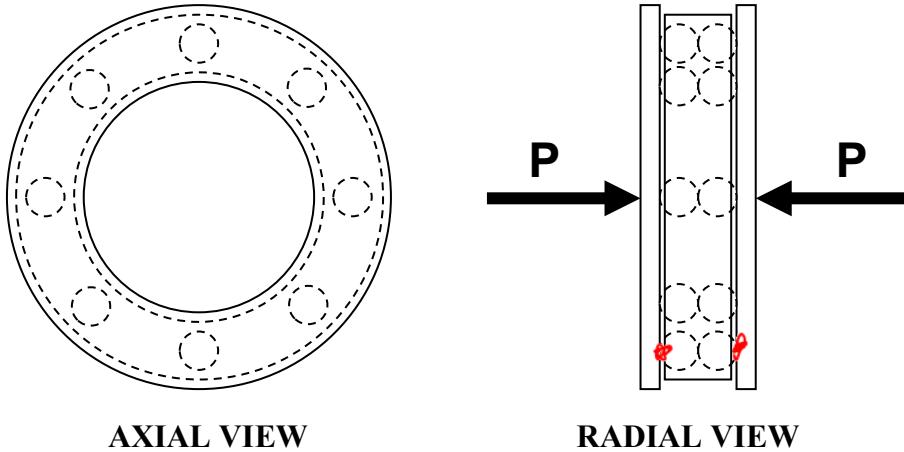
$$\sigma' = \sqrt{3} \tau = 124.5 \text{ MPa} \quad N_{FS} = S_y / \sigma' = 10.96$$

socket

$$v_2 = 0.33 \quad \tau_{\max} = \frac{p_{\max}}{2} \left(\frac{1-2v_2}{2} + \frac{2}{9}(1+v_2)\sqrt{2(1+v_2)} \right) = 0.3260 \quad p_{\max} = 69.4 \text{ MPa} \quad \text{Eq. 7.12b}$$

$$\sigma' = \sqrt{3} \tau = 120.2 \text{ MPa} \quad N_{FS} = S_y / \sigma' = 4.59$$

Sixteen steel balls (0.156 inch DIA, S-5 steel with HRC 59) are used in torque adjusters for cordless electric drills. The balls are stacked in pairs inside eight axial holes within a 6/6 nylon ring spacer and compressed between two flat steel disks (4340 steel with HB 430) by a compression spring as shown below. The axial holes in the nylon ring do not constrain axial motion of the balls. Determine factor of safety in the balls and in the plates for compressive load $P = 30 \text{ lbf}$. The eight pairs of balls share load P equally.



contact between two steel spheres S-5 HRC 59 $S_Y = 280 \text{ ksi}$ Table A-10

$$d = 0.156 \text{ in} \quad F = P / 8 = 3.75 \text{ lbf}$$

$$\underline{\underline{R_1 = R_2 = d / 2 = 0.078 \text{ in}}} \quad \text{steel} \quad v_1 = v_2 = 0.28 \quad E_1 = E_2 = 30 \times 10^6 \text{ psi}$$

$$m_1 = m_2 = (1-v^2) / E = 3.072 \times 10^{-8} \text{ in}^2/\text{lbf} \quad \text{Eq. 7.9a}$$

$$B = (1/R_1 + 1/R_2) / 2 = 12.82 \text{ in}^{-1} \quad \text{Eq. 7.9b}$$

$$a^3 = 0.375 \frac{m_1 + m_2}{B} F = 0.375 \left(\frac{6.144 \times 10^{-8} \text{ in}^2}{\text{lbf}} \right) \left(\frac{\text{in}}{12.82} \right) (3.75 \text{ lbf}) \quad \text{Eq. 7.9d}$$

$$a = 0.001889 \text{ in}$$

$$p_{\max} = \frac{3}{2} \frac{F}{\pi a^2} = \left(\frac{3}{2} \right) \frac{3.75 \text{ lbf}}{\pi (0.001889 \text{ in})^2} = 501.8 \text{ ksi} \quad \text{Eq. 7.8b}$$

$$\tau_{\max} = \frac{p_{\max}}{2} \left(\frac{1-2v}{2} + \frac{2}{9} (1+v) \sqrt{2(1+v)} \right) = 0.3376 p_{\max} = 169.4 \text{ ksi} \quad \text{Eq. 7.12b}$$

$$\sigma' = \sqrt{3} \tau = 293.4 \text{ ksi}$$

sphere on sphere $N_{FS} = S_Y / \sigma' = 0.95$

plate

contact between steel sphere and steel plate

$$\underline{R_1 = 0.078 \text{ in}} \quad R_2 = \infty \quad v_1 = v_2 = 0.28 \quad E_1 = E_2 = 30 \times 10^6 \text{ psi}$$

$$m_1 = m_2 = (1-v^2) / E = 3.072 \times 10^{-8} \text{ in}^2/\text{lbf} \quad \text{Eq. 7.9a}$$

$$B = (1/R_1 + 1/R_2) / 2 = 6.4103 \text{ in}^{-1} \quad \text{Eq. 7.9b}$$

$$a^3 = 0.375 \frac{m_1 + m_2}{B} F \quad \text{Eq. 7.9d}$$

$$a = 0.002380 \text{ in}$$

$$p_{\max} = \frac{3}{2} \frac{F}{\pi a^2} = 316.1 \text{ ksi} \quad \text{Eq. 7.8b}$$

$$\tau_{\max} = \frac{p_{\max}}{2} \left(\frac{1-2v}{2} + \frac{2}{9}(1+v)\sqrt{2(1+v)} \right) = 0.3376 \text{ p}_{\max} = 106.7 \text{ ksi} \quad \text{Eq. 7.12b}$$

$$\sigma' = \sqrt{3} \tau = 184.8 \text{ ksi}$$

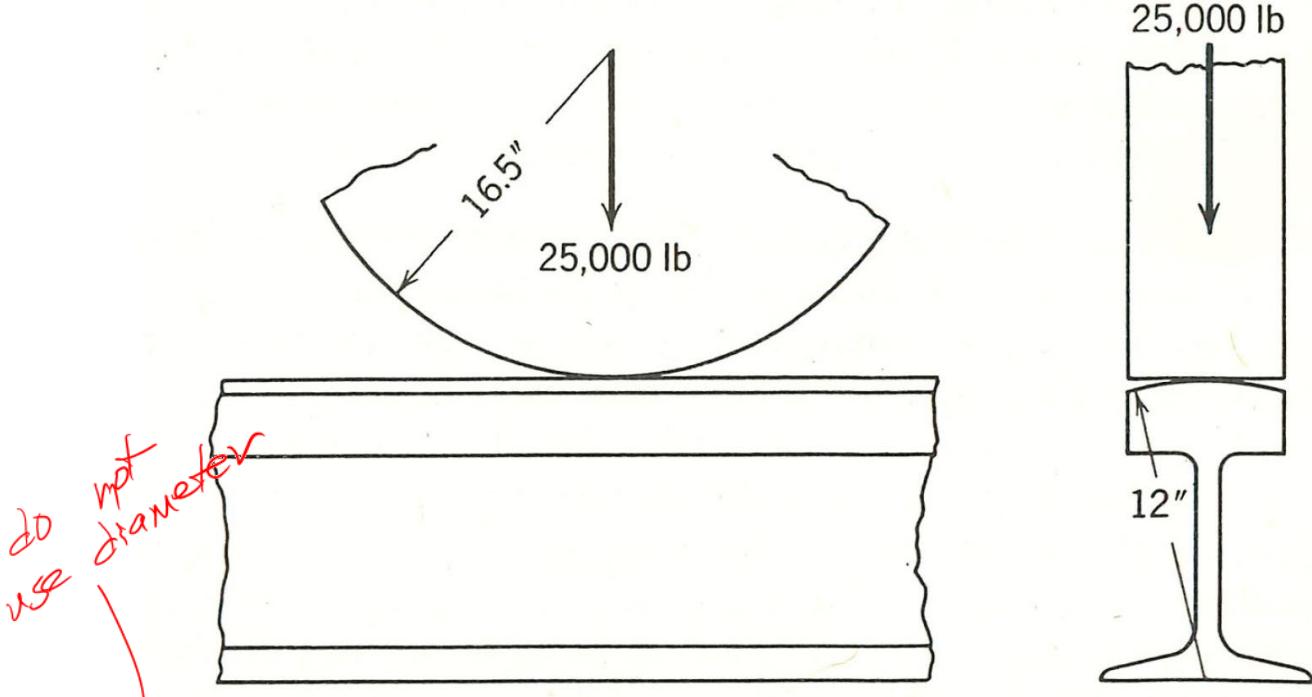
sphere on flat plate sphere S-5 HRC 59 $S_Y = 280 \text{ ksi}$ Table A-10

$$N_{FS} = S_Y / \sigma' = 1.52$$

sphere on flat plate plate 4340 with HB 486 $S_Y = 230 \text{ ksi}$ Table A-10

$$N_{FS} = S_Y / \sigma' = 1.24$$

A steel railway car wheel with 33 inch DIA rolls on a steel rail whose top surface has a cross section radius of 12 inches as shown below. The wheel load on the rail is 25,000 lbf. Assume width of the rail is 6 inches and **ignore the curvature of the rail cross section**. Determine width of the contact patch and maximum von Mises stress in the rail and wheel.



$$R_1 = 16.5 \text{ in} \quad R_2 = \infty \quad \text{steel} \quad v_1 = v_2 = 0.28 \quad E_1 = E_2 = 30 \times 10^6 \text{ psi}$$

$$m_1 = m_2 = (1-v^2)/E = 3.072 \times 10^{-8} \text{ in}^2/\text{lbf} \quad \text{Eq. 7.9a}$$

$$B = (1/R_1 + 1/R_2)/2 = 0.0303 \text{ in}^{-1} \quad \text{Eq. 7.9b}$$

$$a = \sqrt{\frac{2(m_1 + m_2)}{\pi B}} \left(\frac{F}{L} \right) = \sqrt{\frac{2(3.072 \times 10^{-8} \text{ in}^2 + 3.072 \times 10^{-8} \text{ in}^2)}{\pi \text{ lbf}}} \left(\frac{\text{in}}{0.0303} \right) \left(\frac{25,000 \text{ lbf}}{6 \text{ in}} \right)$$

$$a = 0.0733 \text{ in} \quad \text{Eq. 7.15b}$$

$$\text{width} = 2a = 0.1467 \text{ in}$$

$$\text{rectangular contact area} = 2aL = 0.8802 \text{ in}^2$$

$$p_{\max} = \frac{2F}{\pi a L} = \frac{2(25,000 \text{ lbf})}{\pi(0.0733 \text{ in})(6 \text{ in})} = 36.19 \text{ ksi} \quad \text{Eq. 7.14b}$$

$$\tau_{\max} = 0.304 p_{\max} = 11.0 \text{ ksi} \quad \text{Eq. 7.17b} \quad \text{steel}$$

Σ

do not compare SUT
to S_y or S_u

cylinder on flat plate $\sigma' = \sqrt{3} \tau = 19.06 \text{ ksi}$

include radius of rail cross section – General Contact Section 7.10

steel $v_1 = v_2 = 0.28$ $E_1 = E_2 = 30 \times 10^6 \text{ psi}$

$$m_1 = m_2 = (1 - v^2) / E = 3.072 \times 10^{-8} \text{ in}^2/\text{lbf} \quad \text{Eq. 7.9a}$$

crossed cylinders

$$R_1 = 16.5 \text{ in} \quad R_1' = \infty \text{ in} \quad R_2 = 12 \text{ in} \quad R_2' = \infty \text{ in} \quad \theta = 90^\circ$$

$$A = \frac{1}{2} \left(\frac{1}{R_1} + \frac{1}{R_1'} + \frac{1}{R_2} + \frac{1}{R_2'} \right) = 0.07197 \text{ in}^{-1} \quad \text{Eq. 7.19a}$$

$$B = \frac{1}{2} \sqrt{\left(\frac{1}{R_1} - \frac{1}{R_1'} \right)^2 + \left(\frac{1}{R_2} - \frac{1}{R_2'} \right)^2 + 2 \left(\frac{1}{R_1} - \frac{1}{R_1'} \right) \left(\frac{1}{R_2} - \frac{1}{R_2'} \right) \cos 2\theta} = 0.01136 \text{ in}^{-1}$$

Eq. 7.19b

$$\cos \phi = \frac{B}{A} \quad \phi = 80.92^\circ \quad \text{Eq. 7.19c}$$

$$a = k_a \sqrt[3]{\frac{3F(m_1 + m_2)}{4A}} = 0.2812 \text{ in} \quad \text{Eq. 7.19d} \quad k_a = 1.1157 \quad \text{interpolated Table 7-5}$$

$$b = k_b \sqrt[3]{\frac{3F(m_1 + m_2)}{4A}} = 0.2274 \text{ in} \quad \text{Eq. 7.19d} \quad k_b = 0.9024 \quad \text{interpolated Table 7-5}$$

elliptic contact area = $\pi a b = 0.2009 \text{ in}^2$

$$p_{MAX} = \frac{3}{2} \frac{F}{\pi a b} = \frac{3}{2} \frac{(25,000 \text{ lbf})}{\pi (0.2812 \text{ in})(0.2274 \text{ in})} = 186.7 \text{ ksi} \quad \text{Eq. 7.18b}$$

$\tau_{max} \sim 0.34 p_{max} = 63.48 \text{ ksi}$ page 482 Norton 5th edition

crossed cylinders $\sigma' = \sqrt{3} \tau = 110.0 \text{ ksi}$