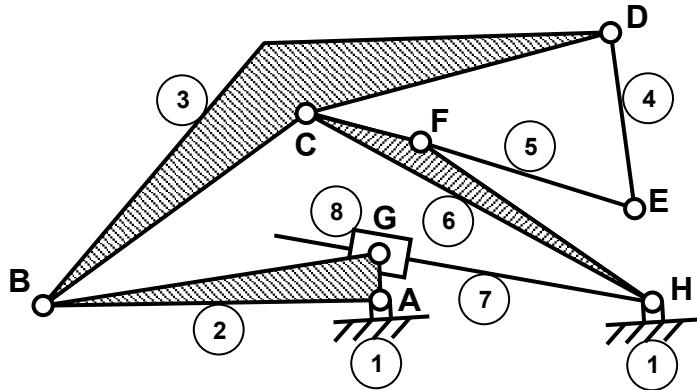
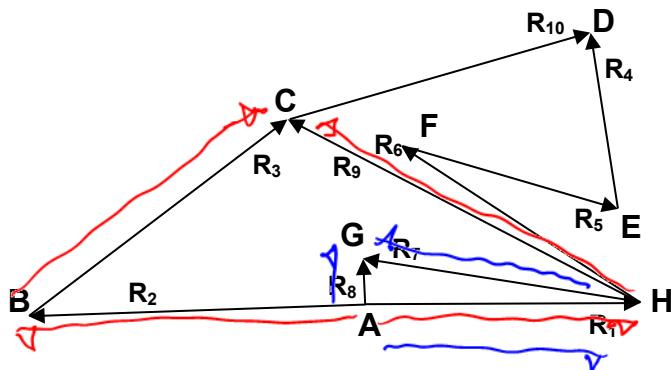


## POLYCENTRIC HINGE - SKELETAL DIAGRAM



## POLYCENTRIC HINGE - VECTORS

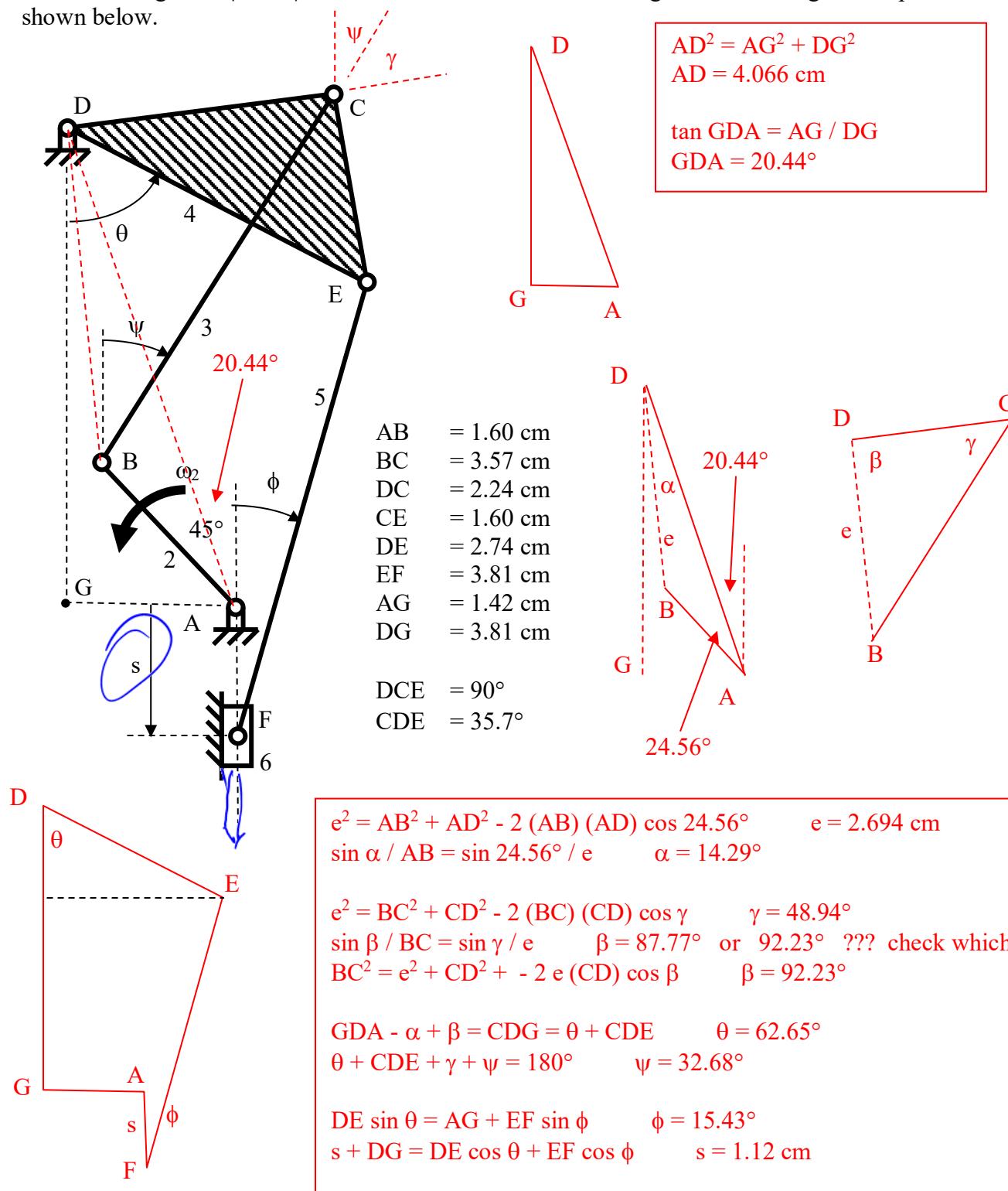


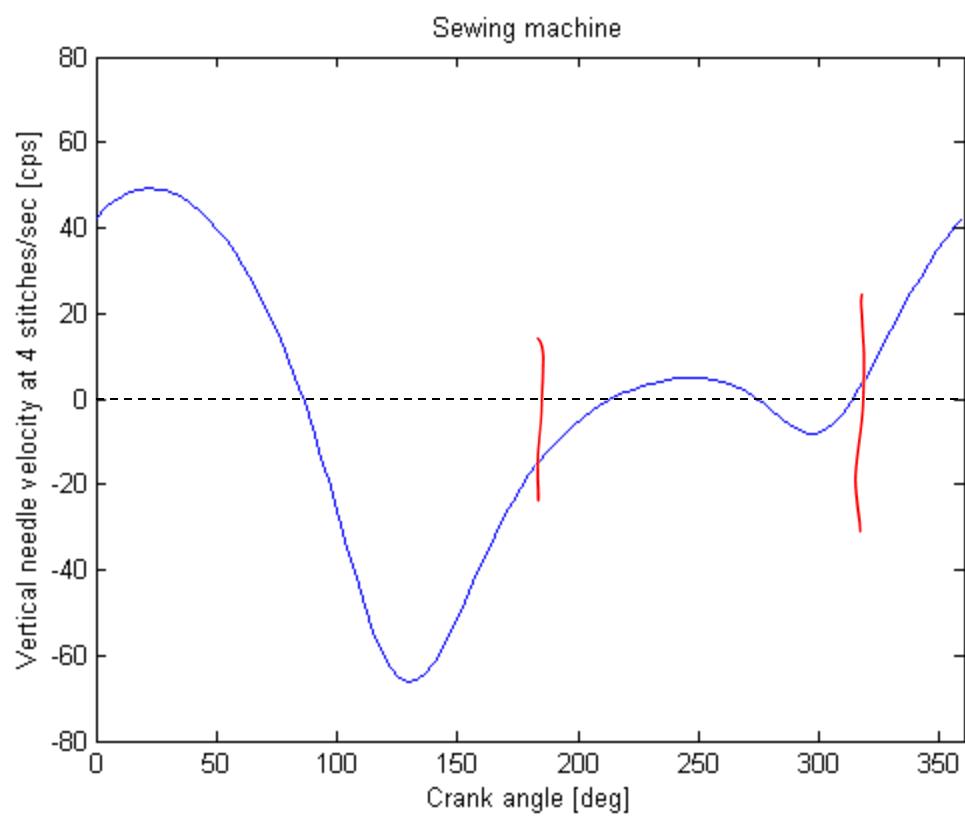
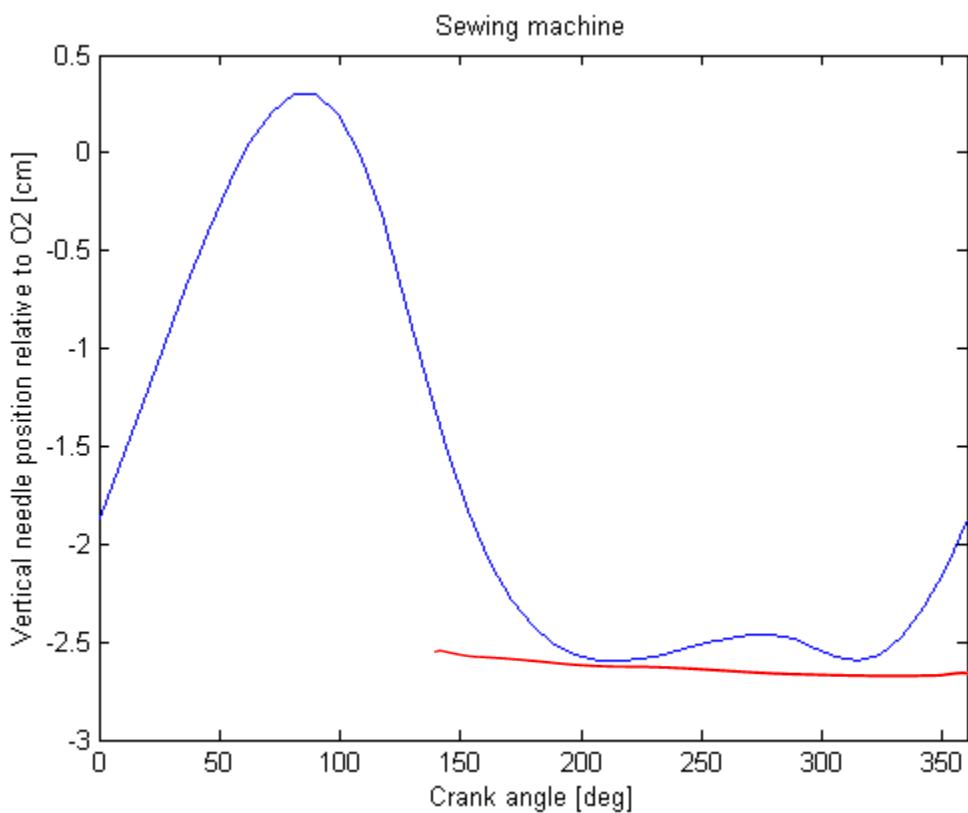
Vector	Position of	Length (mm)		Angle (deg)	
1	H wrt A	21.67	constant	0.00	constant
2	B wrt A	26.69	constant	181.61	var
3	C wrt B	26.00	constant	36.95	var
4	D wrt E	14.00	constant	99.38	driver
5	E wrt F	17.83	constant	-15.87	var
6	F wrt H	22.47	constant	146.31	$\theta_9 - 5.33^\circ$
7	G wrt H	22.03	var	171.56	var
8	G wrt A	3.23	constant	92.21	$\theta_2 - 89.40^\circ$
9	C wrt H	31.33	constant	151.64	var
10	D wrt C	24.62	constant	15.36	$\theta_3 - 21.59^\circ$

$$\begin{aligned} \text{---} \rightarrow \bar{R}_2 + \bar{R}_3 - \bar{R}_9 - \bar{R}_1 &= 0 \\ \bar{R}_2 + \bar{R}_3 + \bar{R}_{10} - \bar{R}_4 - \bar{R}_5 - \bar{R}_6 - \bar{R}_1 &= 0 \\ \bar{R}_8 - \bar{R}_7 - \bar{R}_1 &= 0 \end{aligned}$$

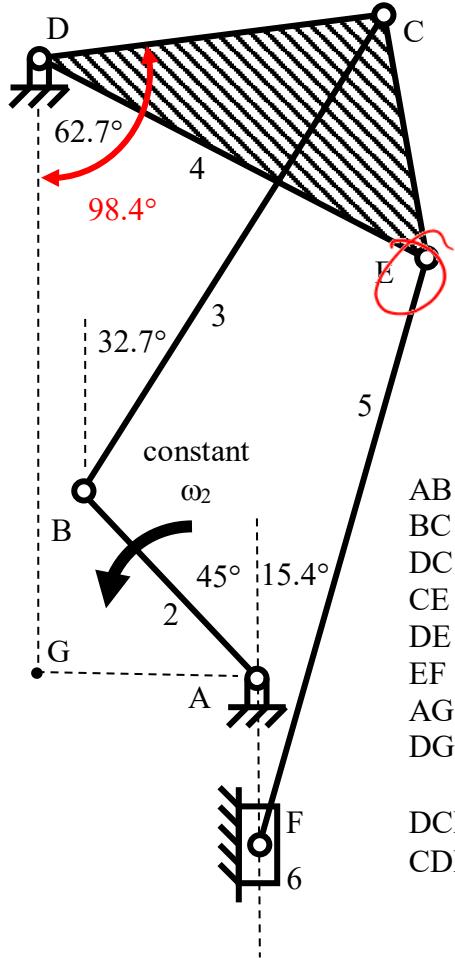
## Sewing Machine

Determine angles  $\theta$ ,  $\phi$  and  $\psi$  as well as distance  $s$  for this sewing machine linkage at the position shown below.

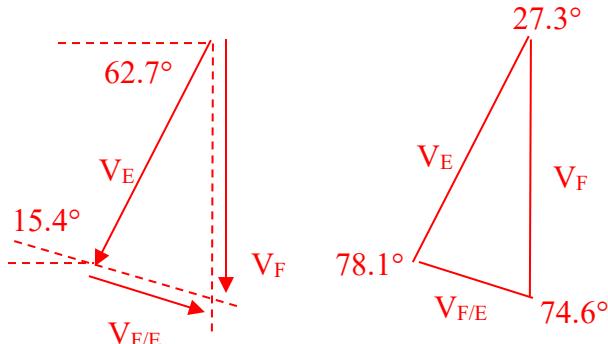
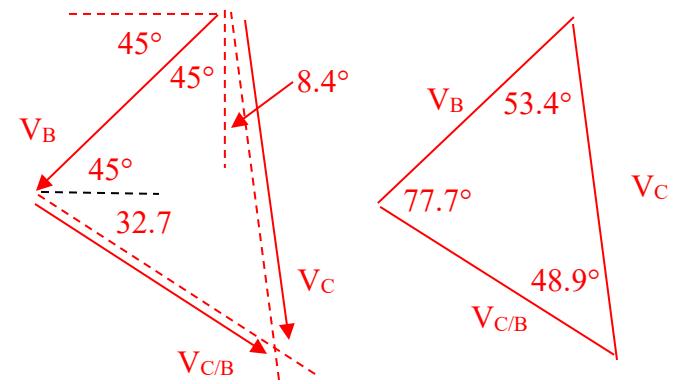




Determine the angular velocity of links 2, 3, 4 and 5 as well as the velocity of needle 6 for the sewing machine linkage as shown below when sewing at 4 stitches per second constant speed.



$$\begin{aligned} AB &= 1.60 \text{ cm} \\ BC &= 3.57 \text{ cm} \\ DC &= 2.24 \text{ cm} \\ CE &= 1.60 \text{ cm} \\ DE &= 2.74 \text{ cm} \\ EF &= 3.81 \text{ cm} \\ AG &= 1.42 \text{ cm} \\ DG &= 3.81 \text{ cm} \\ \\ DCE &= 90^\circ \\ CDE &= 35.7^\circ \end{aligned}$$



$V_E = DE \omega_4 = 63.77 \text{ cps}$

$\bar{V}_F$	$= \bar{V}_E + \bar{V}_{F/E}$	
?	$DE \omega_4$	?
vertical	$\perp DE$	$\perp EF$

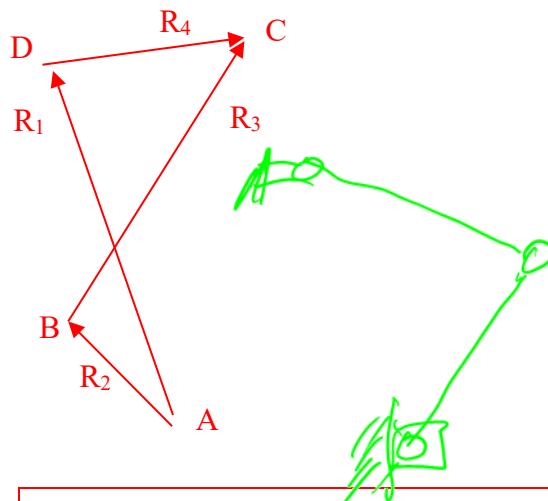
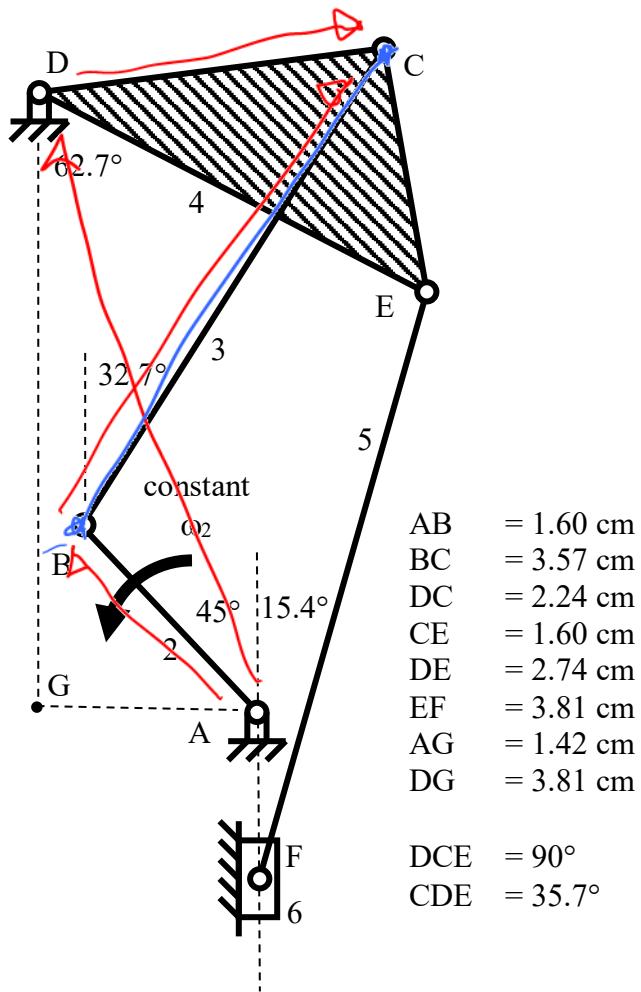
$V_E / \sin 74.6^\circ = V_F / \sin 78.1^\circ = V_{F/E} / \sin 27.3^\circ$

$V_F = 64.72 \text{ cps}$

$V_{F/E} = 30.34 \text{ cps}$

$\omega_5 = V_{F/E} / EF = 7.96 \text{ rad/sec CCW}$

Determine the angular velocity of links 2, 3, 4 and 5 as well as the velocity of needle 6 for the sewing machine linkage as shown below when sewing at 4 stitches per second constant speed.



	r [cm]	$\theta$ [deg]
1	4.07	110.4
2	1.60	135.0
3	3.57	57.3
4	2.24	8.4

$$\omega_3 = -r_2 \omega_2 \sin(\theta_2 - \theta_4) / r_3 \sin(\theta_3 - \theta_4) = -12.00 \text{ rad/s}$$

$$\omega_4 = -r_2 \omega_2 \sin(\theta_2 - \theta_3) / r_4 \sin(\theta_3 - \theta_4) = -23.27 \text{ rad/s}$$

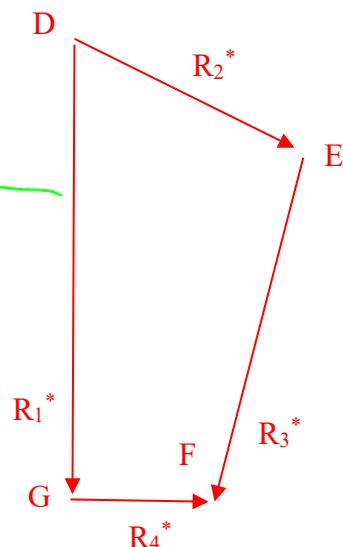
$\omega_2^* = \omega_4 = -23.27 \text{ rad/sec}$		
	$r^* [\text{cm}]$	$\theta^* [\text{deg}]$
1	4.93	270
2	2.74	332.7
3	3.81	254.6
4	1.42	0

$$\omega_3^* = - r_2^* \omega_2^* \cos(\theta_2^* - \theta_1^*) / r_3^* \cos(\theta_3^* - \theta_1^*) = + 7.96 \text{ rad/s}$$

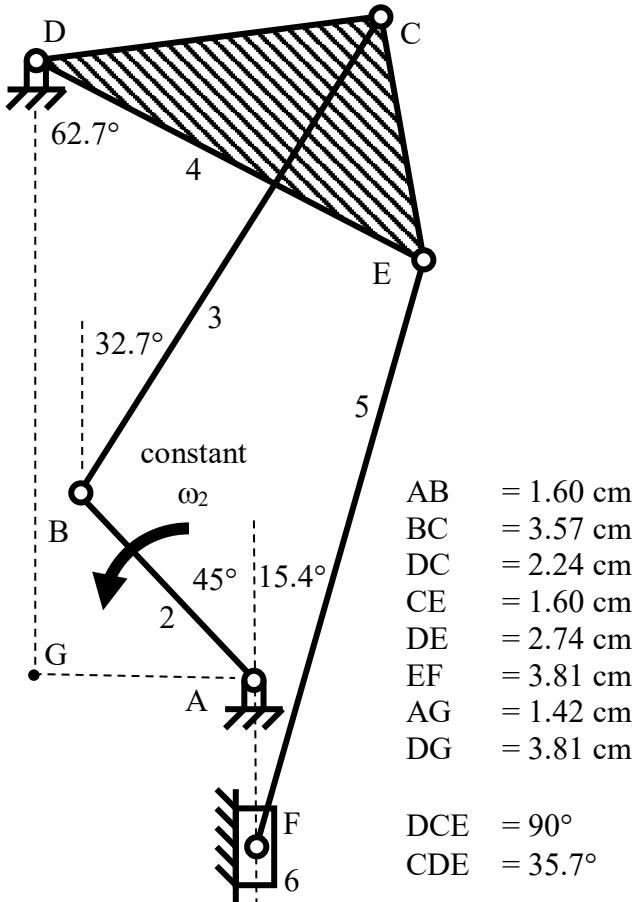
$$\dot{r}_1^* = - r_2^* \omega_2^* \sin(\theta_2^* - \theta_3^*) / \cos(\theta_3^* - \theta_1^*) = + 64.71 \text{ cps}$$

$$\omega_5 = \omega_5^* = 7.96 \text{ rad/sec CCW}$$

$$\overline{V}_E = 64.71 \text{ cps down}$$



Determine the angular velocity of links 2, 3, 4 and 5 as well as the velocity of needle 6 for the sewing machine linkage as shown below when sewing at 4 stitches per second constant speed.



$$\bar{R}_2 + \bar{R}_3 - \bar{R}_4 - \bar{R}_7 - \bar{R}_1 = 0$$

$$r_2 e^{j\theta_2} + r_3 e^{j\theta_3} - r_4 e^{j\theta_4} - r_7 e^{j\theta_7} - r_1 e^{j\theta_1} = 0$$

$$j r_2 \dot{\theta}_2 e^{j\theta_2} + j r_3 \dot{\theta}_3 e^{j\theta_3} - j r_4 \dot{\theta}_4 e^{j\theta_4} = 0$$

~~REAL:~~  $-r_2 \dot{\theta}_2 \sin \theta_2 - r_3 \dot{\theta}_3 \sin \theta_3 + r_4 \dot{\theta}_4 \sin \theta_4 = 0$

~~IMAG:~~  $j r_2 \dot{\theta}_2 \cos \theta_2 + j r_3 \dot{\theta}_3 \cos \theta_3 - j r_4 \dot{\theta}_4 \cos \theta_4 = 0$

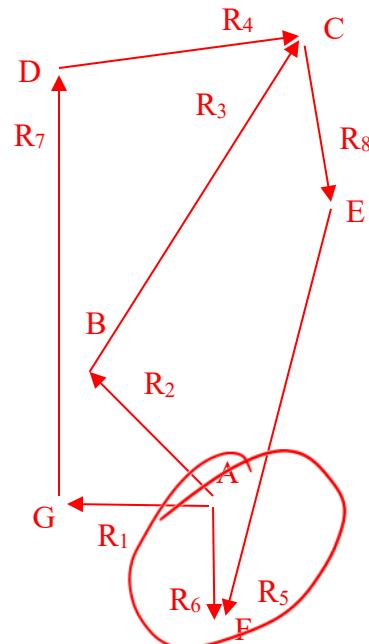
$$\bar{R}_2 + \bar{R}_3 + \bar{R}_8 + \bar{R}_5 - \bar{R}_6 = 0$$

$$r_2 e^{j\theta_2} + r_3 e^{j\theta_3} + r_8 e^{j\theta_8} + r_5 e^{j\theta_5} - r_6 e^{j\theta_6} = 0$$

$$j r_2 \dot{\theta}_2 e^{j\theta_2} + j r_3 \dot{\theta}_3 e^{j\theta_3} + j r_8 \dot{\theta}_8 e^{j\theta_8} + j r_5 \dot{\theta}_5 e^{j\theta_5} - j r_6 \dot{\theta}_6 e^{j\theta_6} = 0 \quad \text{FOR} \quad \dot{\theta}_8 = \dot{\theta}_4$$

~~REAL:~~  $-r_2 \dot{\theta}_2 \sin \theta_2 - r_3 \dot{\theta}_3 \sin \theta_3 - r_8 \dot{\theta}_8 \sin \theta_8 - r_5 \dot{\theta}_5 \sin \theta_5 + r_6 \dot{\theta}_6 \cos \theta_6 = 0$

~~IMAG:~~  $j r_2 \dot{\theta}_2 \cos \theta_2 + j r_3 \dot{\theta}_3 \cos \theta_3 + j r_8 \dot{\theta}_8 \cos \theta_8 + j r_5 \dot{\theta}_5 \cos \theta_5 - j r_6 \dot{\theta}_6 \cos \theta_6 = 0$



	r [cm]	θ [deg]
1	1.42	constant
2	1.60	constant
3	3.57	constant
4	2.24	constant
5	3.81	constant
6	1.12	variable
7	3.81	constant
8	1.60	constant

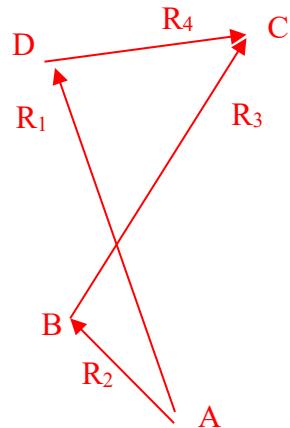
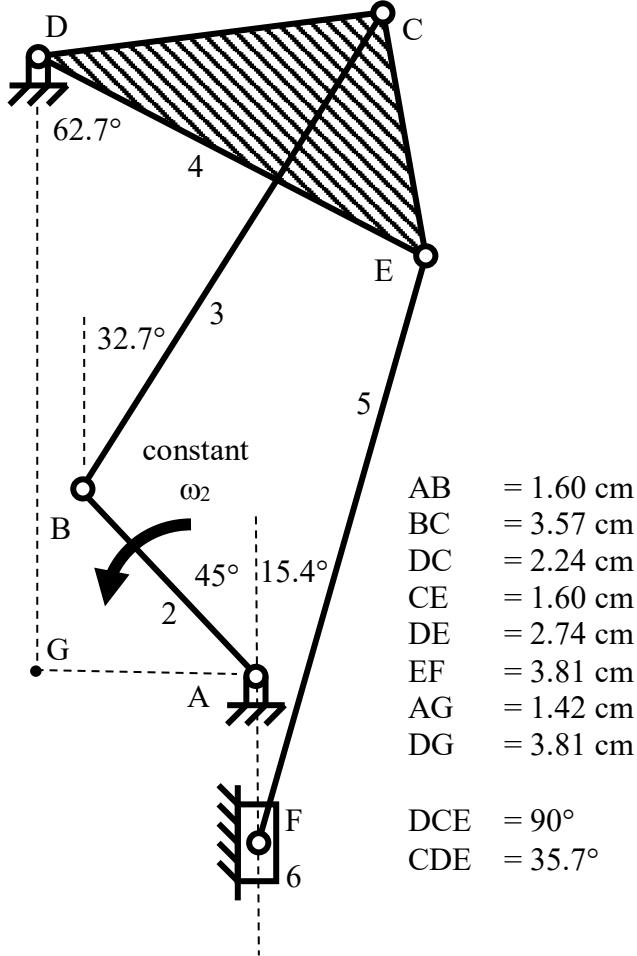
$$\begin{bmatrix} -r_3 \sin \theta_3 & r_4 \sin \theta_4 & 0 & 0 \\ r_3 \cos \theta_3 & -r_4 \cos \theta_4 & 0 & 0 \\ -r_3 \sin \theta_3 & -r_8 \sin \theta_8 & -r_5 \sin \theta_5 & 0 \\ r_3 \cos \theta_3 & r_8 \cos \theta_8 & r_5 \cos \theta_5 & 1 \end{bmatrix} \begin{Bmatrix} \dot{\theta}_3 \\ \dot{\theta}_4 \\ \dot{\theta}_5 \\ \dot{r}_6 \end{Bmatrix} = \begin{Bmatrix} r_2 \dot{\theta}_2 \sin \theta_2 \\ -r_2 \dot{\theta}_2 \cos \theta_2 \\ r_2 \dot{\theta}_2 \sin \theta_2 \\ -r_2 \dot{\theta}_2 \cos \theta_2 \end{Bmatrix}$$

$$\begin{bmatrix} -3.004 & 0.327 & 0 & 0 \\ 1.929 & -2.216 & 0 & 0 \\ -3.004 & 1.583 & 3.673 & 0 \\ 1.929 & 0.234 & -1.012 & 1 \end{bmatrix} \begin{Bmatrix} \dot{\theta}_3 \\ \dot{\theta}_4 \\ \dot{\theta}_5 \\ \dot{r}_6 \end{Bmatrix} = \begin{Bmatrix} 28.43 \\ 28.43 \\ 28.43 \\ 28.43 \end{Bmatrix}$$

using MATLAB

$$\begin{Bmatrix} \dot{\theta}_3 \\ \dot{\theta}_4 \\ \dot{\theta}_5 \\ \dot{r}_6 \end{Bmatrix} = \begin{Bmatrix} -12.00 \text{ rad/sec} \\ -23.27 \text{ rad/sec} \\ +7.96 \text{ rad/sec} \\ +65.07 \text{ cm/sec} \end{Bmatrix}$$

Determine the angular acceleration of links 2, 3, 4 and 5 as well as the acceleration of needle 6 for the sewing machine linkage as shown below when sewing at 4 stitches per second constant speed.



$$\omega_2 = +8\pi \text{ rad/sec} \quad \alpha_2 = 0 \quad \text{angular velocities from velocity solution}$$

	r [cm]	θ [deg]	dot{θ} [rad/sec]	r dot{θ}² [cpss]	ddot{θ} [rad/s/s]	r ddot{θ} [cpss]
1	4.07	110.4				
2	1.60	135.0	+25.13	+1010.6	0	0
3	3.57	57.3	-12.00	+514.1	?	
4	2.24	8.4	-23.27	+1212.9	?	

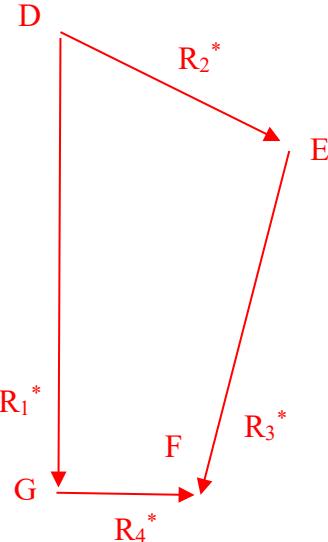
$$\begin{bmatrix} -r_3 \sin \theta_3 & r_4 \sin \theta_4 \\ r_3 \cos \theta_3 & -r_4 \cos \theta_4 \end{bmatrix} \begin{Bmatrix} \ddot{\theta}_3 \\ \ddot{\theta}_4 \end{Bmatrix} = \begin{Bmatrix} r_2 \ddot{\theta}_2 \sin \theta_2 + r_2 \dot{\theta}_2^2 \cos \theta_2 + r_3 \dot{\theta}_3^2 \cos \theta_3 - r_4 \dot{\theta}_4^2 \cos \theta_4 \\ -r_2 \ddot{\theta}_2 \cos \theta_2 + r_2 \dot{\theta}_2^2 \sin \theta_2 + r_3 \dot{\theta}_3^2 \sin \theta_3 - r_4 \dot{\theta}_4^2 \sin \theta_4 \end{Bmatrix}$$

$$\begin{bmatrix} -3.004 & 0.327 \\ 1.929 & -2.216 \end{bmatrix} \begin{Bmatrix} \ddot{\theta}_3 \\ \ddot{\theta}_4 \end{Bmatrix} = \begin{Bmatrix} -1636.8 \text{ cpss} \\ 970.0 \text{ cpss} \end{Bmatrix} \quad \begin{Bmatrix} \ddot{\theta}_3 \\ \ddot{\theta}_4 \end{Bmatrix} = \begin{bmatrix} -0.3677 & -0.0543 \\ -0.3201 & -0.4985 \end{bmatrix} \begin{Bmatrix} -1636.8 \\ 970.0 \end{Bmatrix} = \begin{Bmatrix} 549.3 \text{ rad/s/s} \\ 40.4 \text{ rad/s/s} \end{Bmatrix}$$

closed form

$$\ddot{\theta}_3 = (-r_2 \ddot{\theta}_2 \sin(\theta_2 - \theta_4) - r_2 \dot{\theta}_2^2 \cos(\theta_2 - \theta_4) - r_3 \dot{\theta}_3^2 \cos(\theta_3 - \theta_4) + r_4 \dot{\theta}_4^2) / r_3 \sin(\theta_3 - \theta_4) = 549.3 \text{ rad/s/s}$$

$$\ddot{\theta}_4 = (-r_2 \ddot{\theta}_2 \sin(\theta_2 - \theta_3) - r_2 \dot{\theta}_2^2 \cos(\theta_2 - \theta_3) - r_3 \dot{\theta}_3^2 + r_4 \dot{\theta}_4^2 \cos(\theta_4 - \theta_3)) / r_4 \sin(\theta_3 - \theta_4) = 40.4 \text{ rad/s/s}$$



$$\omega_2^* = \omega_4 = -23.27 \text{ rad/sec} \quad \alpha_2^* = \alpha_4 = +40.4 \text{ rad/s/s} \quad \omega_3^* = \omega_5 = -23.27 \text{ rad/sec}$$

	$r^* \text{ [cm]}$	$\theta^* \text{ [deg]}$	$\dot{\theta}^* \text{ [rad/sec]}$	$r\dot{\theta}^2 \text{ [cpss]}$	$\ddot{\theta}^* \text{ [rad/s/s]}$	$r\ddot{\theta}^* \text{ [cpss]}$
1	4.93	270				
2	2.74	332.7	-23.27	+1483.7	+40.4	+110.7
3	3.81	254.6	+7.96	+241.4	?	
4	1.42	0				

$$\begin{bmatrix} -r_3 \sin \theta_3 & -\cos \theta_1 \\ r_3 \cos \theta_3 & -\sin \theta_1 \end{bmatrix} \begin{Bmatrix} \ddot{\theta}_3 \\ \ddot{r}_1 \end{Bmatrix} = \begin{Bmatrix} r_2 \ddot{\theta}_2 \sin \theta_2 + r_2 \dot{\theta}_2^2 \cos \theta_2 + r_3 \dot{\theta}_3^2 \cos \theta_3 \\ -r_2 \ddot{\theta}_2 \cos \theta_2 + r_2 \dot{\theta}_2^2 \sin \theta_2 + r_3 \dot{\theta}_3^2 \sin \theta_3 \end{Bmatrix}$$

$$\begin{bmatrix} -3.673 & 0 \\ -1.012 & 1 \end{bmatrix} \begin{Bmatrix} \ddot{\theta}_3 \\ \ddot{r}_1 \end{Bmatrix} = \begin{Bmatrix} +1203.6 \text{ cpss} \\ -1011.6 \text{ cpss} \end{Bmatrix} \quad \begin{Bmatrix} \ddot{\theta}_3 \\ \ddot{r}_1 \end{Bmatrix} = \begin{bmatrix} 0.2723 & 0 \\ 0.2755 & 1 \end{bmatrix} \begin{Bmatrix} +1203.6 \text{ cpss} \\ -1011.6 \text{ cpss} \end{Bmatrix} = \begin{Bmatrix} +227.7 \text{ rad/s/s} \\ -680.0 \text{ cm/s/s} \end{Bmatrix}$$