

$$\sum \xi^A - \sum \xi^A = 0 ?$$

Two-Dimensional Constraints

General

$$\{\dot{\Phi}\} = \{0\}$$

$$\{\ddot{\Phi}\} = \{0\}$$

$$\{\ddot{\Phi}\} = [\Phi_q]\{\ddot{q}\} + \{\Phi_{tt}\} = \{0\}$$

$$[\Phi_q]\{\dot{q}\} = \{v\}$$

$$\{v\} = -\{\Phi_{tt}\}$$

constraints
residuals

$$\frac{d\{\dot{\Phi}\}_{ANT}}{dt} = 0$$

$$\begin{aligned} \{\dot{\Phi}\} &= \{0\} \\ \{\ddot{\Phi}\} &= \{0\} \\ \{\ddot{\Phi}\} &= [\Phi_q]\{\ddot{q}\} + \{\Phi_{tt}\} = \{0\} \\ \{\dot{\Phi}\} &= \{0\} \\ \{\ddot{\Phi}\} &= [\Phi_q]\{\ddot{q}\} + ([\Phi_q]\{\dot{q}\})_q\{\dot{q}\} + 2[\Phi_{qt}]\{\dot{q}\} + \{\Phi_{ttt}\} = \{0\} \end{aligned}$$

$$\{\ddot{q}\}$$

$q \times 1$

$$\begin{aligned} \{\dot{\Phi}\} &= \{\sum \Phi\}_{REV A} \\ \{\dot{\Phi}\} &= \{\sum \Phi\}_{REV B} \\ \{\dot{\Phi}\} &= \{\sum \Phi\}_{REV C} \\ \{\dot{\Phi}\} &= \{\sum \Phi\}_{PRIS C} \\ &\text{driver} \\ &\text{(one)} \end{aligned}$$

$$\begin{aligned} \text{vel } [\text{JAC}] \{\text{gen vel}\} &= \{\text{vel RHS}\} \\ \{\ddot{\Phi}\} &= \{\ddot{q}\} \\ \{\ddot{\Phi}\} &= [\Phi_q]\{\ddot{q}\} + ([\Phi_q]\{\dot{q}\})_q\{\dot{q}\} + 2[\Phi_{qt}]\{\dot{q}\} + \{\Phi_{ttt}\} = \{0\} \\ \rightarrow \dot{\phi}_2 - \dot{\phi}_{2\text{START}} - \omega_2 t &= 0 \\ \text{acc } [\text{JAC}] \{\text{gen acc}\} &= \{\text{acc RHS}\} \\ \{\ddot{\Phi}\} &= \{\ddot{q}\} \\ \{\ddot{\Phi}\} &= [\Phi_q]\{\ddot{q}\} + ([\Phi_q]\{\dot{q}\})_q\{\dot{q}\} + 2[\Phi_{qt}]\{\dot{q}\} + \{\Phi_{ttt}\} = \{0\} \\ \dot{\phi}_2 &= \dot{\phi}_{2\text{START}} + \omega_2 t \\ \{\ddot{\Phi}\} &= \{\ddot{q}\} \\ \{\ddot{\Phi}\} &= \{\ddot{q}\}_{JNTS} \\ \{\ddot{\Phi}\} &= \{\ddot{q}\}_{DRIVERS} \end{aligned}$$

$$\{\ddot{\Phi}\} = \{0\}$$

$$[\Phi_q]\{\ddot{q}\} = \{\sigma\}$$

$$\begin{aligned} \{\sigma\} &= -4([\Phi_q]\{\ddot{q}\})_q\{\dot{q}\} - 3([\Phi_q]\{\ddot{q}\})_q\{\ddot{q}\} - 6([\Phi_q]\{\dot{q}\})_q\{\dot{q}\} - \left(([\Phi_q]\{\dot{q}\})_q\{\dot{q}\} \right)_q\{\dot{q}\} \\ &- 4[\Phi_{qt}]\{\ddot{q}\} - 12([\Phi_{qt}]\{\dot{q}\})_q\{\dot{q}\} - 4([\Phi_{qt}]\{\dot{q}\})_q\{\ddot{q}\} - 6[\Phi_{qtt}]\{\ddot{q}\} - 6([\Phi_{qtt}]\{\dot{q}\})_q\{\dot{q}\} \\ &- 4[\Phi_{qttt}]\{\dot{q}\} - \{\Phi_{tttt}\} \end{aligned}$$

Scleronomous constraints

independent of time such as mechanical joints

$$\{\gamma\} \equiv -(\Phi_q \dot{q})_q \dot{q}$$

$$\{\eta\} \equiv \{\ddot{\gamma}\} - (\Phi_q \ddot{q})_q \dot{q}$$

$$\{\sigma\} \equiv \{\ddot{\eta}\} - (\Phi_q \ddot{q})_q \dot{q}$$

Mechanical joints
Joint 1
~~Joint 2~~
 $\phi_2 - \phi_{2, \text{start}} = 0$
 $\omega_2 t = 0$

Revolute

$$\cancel{2x1} \quad \{\Phi\}_{\text{REV}} = \{r_j\}^p - \{r_i\}^p = \{0_{2 \times 1}\}$$

Note: Haug uses $\{r_i\}^p - \{r_j\}^p$

Diagram of a revolute joint showing two coordinate frames, r_i and r_j, with axes of rotation.

$$\begin{aligned} [\Phi_{qi}]_{\text{REV}} &= - \begin{bmatrix} [I_2] & [B_i] \{s_i\}^p \end{bmatrix} \quad \cancel{2x3} \\ [\Phi_{qj}]_{\text{REV}} &= \begin{bmatrix} [I_2] & [B_j] \{s_j\}^p \end{bmatrix} \quad \cancel{2x3} \\ \{v\}_{\text{REV}} &= \{0_{2 \times 1}\} \quad \cancel{2x1} \end{aligned}$$

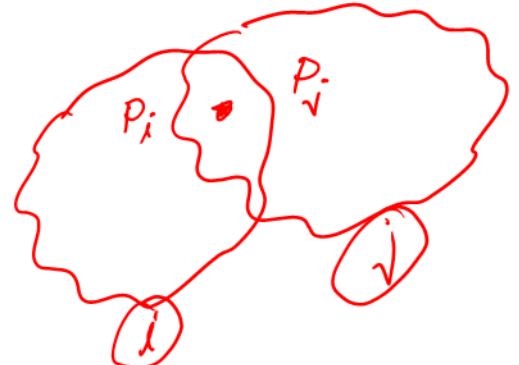
$$\{\gamma\}_{\text{REV}} = \dot{\phi}_j^2 [A_j] \{s_j\}^p - \dot{\phi}_i^2 [A_i] \{s_i\}^p \quad \cancel{2x1}$$

$$\{\eta\}_{\text{REV}} = \dot{\phi}_j^3 [B_j] \{s_j\}^p + 3 \dot{\phi}_j \ddot{\phi}_j [A_j] \{s_j\}^p - \dot{\phi}_i^3 [B_i] \{s_i\}^p - 3 \dot{\phi}_i \ddot{\phi}_i [A_i] \{s_i\}^p$$

$$\begin{aligned} \{\sigma\}_{\text{REV}} &= 6 \dot{\phi}_j^2 \ddot{\phi}_j [B_j] \{s_j\}^p + (4 \dot{\phi}_j \ddot{\phi}_j + 3 \ddot{\phi}_j^2 - \dot{\phi}_j^4) [A_j] \{s_j\}^p \\ &\quad - 6 \dot{\phi}_i^2 \ddot{\phi}_i [B_i] \{s_i\}^p - (4 \dot{\phi}_i \ddot{\phi}_i + 3 \ddot{\phi}_i^2 - \dot{\phi}_i^4) [A_i] \{s_i\}^p \end{aligned}$$

$$\{\kappa_i\}_{\text{REV}} = \begin{Bmatrix} [0_{2 \times 1}] \\ \{s_i\}^{pT} [A_i]^T \{\lambda\}_{\text{REV}} \dot{\phi}_i \end{Bmatrix}$$

$$\{\kappa_j\}_{\text{REV}} = \begin{Bmatrix} [0_{2 \times 1}] \\ -\{s_j\}^{pT} [A_j]^T \{\lambda\}_{\text{REV}} \dot{\phi}_j \end{Bmatrix}$$



$$\begin{aligned} x_j & \rightarrow \{x_j\}^p \\ y_j & \rightarrow \{y_j\}^p \\ \phi_i & \end{aligned}$$

$$\begin{aligned} x_j & \rightarrow \{x_j\}^p \\ y_j & \rightarrow \{y_j\}^p \\ \phi_i & \end{aligned}$$

Double revolute

$$\Phi_{REV_REV} = \{d_{ij}\}^T \{d_{ij}\} - L^2 = 0 \quad L = \text{constant length}$$

$$\text{for } \{d_{ij}\} = \{r_j\}^p - \{r_i\}^p$$

$$\text{and } \{\dot{d}_{ij}\} = \{\dot{r}_j\}^p - \{\dot{r}_i\}^p$$

$$\text{and } \{\ddot{d}_{ij}\} = \{\ddot{r}_j\}^p - \{\ddot{r}_i\}^p$$

$$\text{and } \{\dddot{d}_{ij}\} = \{\dddot{r}_j\}^p - \{\dddot{r}_i\}^p$$

$$[\Phi_{qi}]_{REV_REV} = 2\{d_{ij}\}^T [\Phi_{qi}]_{REV}$$

$$[\Phi_{qj}]_{REV_REV} = 2\{d_{ij}\}^T [\Phi_{qj}]_{REV}$$

$$v_{REV_REV} = 0$$

$$\gamma_{REV_REV} = 2\{d_{ij}\}^T \{\gamma\}_{REV} - 2\{\dot{d}_{ij}\}^T \{\dot{d}_{ij}\}$$

$$\eta_{REV_REV} = 2\{d_{ij}\}^T \{\eta\}_{REV} - 6\{\dot{d}_{ij}\}^T \{\ddot{d}_{ij}\}$$

$$\sigma_{REV_REV} = 2\{d_{ij}\}^T \{\sigma\}_{REV} - 8\{\dot{d}_{ij}\}^T \{\ddot{d}_{ij}\} - 6\{\ddot{d}_{ij}\}^T \{\ddot{d}_{ij}\}$$

$$\{\kappa_i\}_{REV_REV} = -2\lambda_{REV_REV} \left\{ \begin{array}{c} \{\dot{d}_{ij}\} \\ \{s_i\}^{\text{PT}} \left([B_i]^T \{\dot{d}_{ij}\} - [A_i]^T \{d_{ij}\} \right) \dot{\phi}_i \end{array} \right\}$$

$$\{\kappa_j\}_{REV_REV} = 2\lambda_{REV_REV} \left\{ \begin{array}{c} \{\dot{d}_{ij}\} \\ \{s_j\}^{\text{PT}} \left([B_j]^T \{\dot{d}_{ij}\} - [A_j]^T \{d_{ij}\} \right) \dot{\phi}_j \end{array} \right\}$$

Parallel vectors (planar parallel-1)

$\{a_i\}$ parallel to $\{a_j\}$

$$\Phi_{PARALLEL} = \{a_i\}^T [R]^T \{a_j\} = 0$$

$$\text{for } \{a_i\} = \{r_i\}^q - \{r_i\}^p \quad \text{and} \quad \{a_j\} = \{r_j\}^q - \{r_j\}^p$$

$$[\Phi_{qi}]_{PARALLEL} = \begin{bmatrix} \{0_{1x2}\} & -\{a_i\}^T \{a_j\} \end{bmatrix} \quad \{a_i\}^T \{a_j\} = \pm \text{norm}\{a_i\} \times \text{norm}\{a_j\} = \text{constant}$$

$$[\Phi_{qj}]_{\text{PARALLEL}} = [0_{1 \times 2}] + \{a_i\}^T \{a_j\}$$

$$v_{\text{PARALLEL}} = 0$$

$$\gamma_{\text{PARALLEL}} = 0$$

$$\eta_{\text{PARALLEL}} = 0$$

$$\sigma_{\text{PARALLEL}} = 0$$

$$\{\kappa_i\}_{\text{PARALLEL}} = \{0_{3 \times 1}\}$$

$$\{\kappa_j\}_{\text{PARALLEL}} = \{0_{3 \times 1}\}$$

Pin-in-slot (planar parallel-2)

$$\{a_i\} \text{ parallel to } \{d_{ij}\}$$

$$\Phi_{\text{PIN_SLOT}} = \{a_i\}^T [R]^T \{d_{ij}\} = 0$$

$$\text{for } \{d_{ij}\} = \{r_j\}^P - \{r_i\}^P \quad \text{and} \quad \{a_i\} = \{r_i\}^Q - \{r_i\}^P$$

and $\{\dot{d}_{ij}\} \quad \{\ddot{d}_{ij}\} \quad \{\ddot{d}_{ij}\}$ from above

$$[\Phi_{qi}]_{\text{PIN_SLOT}} = \{a_i\}^T [R]^T [\Phi_{qi}]_{\text{REV}} - [0_{1 \times 2}] \quad \{a_i\}^T \{d_{ij}\}$$

$$[\Phi_{qj}]_{\text{PIN_SLOT}} = \{a_i\}^T [R]^T [\Phi_{qj}]_{\text{REV}}$$

$$v_{\text{PIN_SLOT}} = 0$$

$$\gamma_{\text{PIN_SLOT}} = \{a_i\}^T (2 \dot{\phi}_i \{\dot{d}_{ij}\} + [R]^T (\dot{\phi}_i^2 \{d_{ij}\} + \{\gamma\}_{\text{REV}}))$$

$$\eta_{\text{PIN_SLOT}} = \{a_i\}^T (3 \dot{\phi}_i \{\dot{d}_{ij}\} + 3 \ddot{\phi}_i \{\dot{d}_{ij}\} - \dot{\phi}_i^3 \{d_{ij}\} + [R]^T (3 \dot{\phi}_i^2 \{\dot{d}_{ij}\} + 3 \dot{\phi}_i \ddot{\phi}_i \{d_{ij}\} + \{\eta\}_{\text{REV}}))$$

$$\sigma_{\text{PIN_SLOT}} = \{a_i\}^T \left(\begin{array}{l} 4 \dot{\phi}_i \{\ddot{d}_{ij}\} + 6 \ddot{\phi}_i \{\dot{d}_{ij}\} + 4 (\ddot{\phi}_i - \dot{\phi}_i^3) \{\dot{d}_{ij}\} - 6 \dot{\phi}_i^2 \ddot{\phi}_i \{d_{ij}\} \\ + [R]^T (6 \dot{\phi}_i^2 \{\dot{d}_{ij}\} + 12 \dot{\phi}_i \ddot{\phi}_i \{\dot{d}_{ij}\} + (4 \dot{\phi}_i \ddot{\phi}_i + 3 \dot{\phi}_i^2 - \dot{\phi}_i^4) \{d_{ij}\} + \{\sigma\}_{\text{REV}}) \end{array} \right)$$

$$\{\kappa_i\}_{\text{PIN_SLOT}} = \lambda_{\text{PIN_SLOT}} \left\{ \begin{array}{l} \dot{\phi}_i \{a_i\} \\ - \{\dot{d}_{ij}\}^T \{a_i\} \end{array} \right\}$$

$$\{\kappa_j\}_{PIN} = \lambda_{PIN} \left\{ \begin{matrix} -\{a_i\}\dot{\phi}_j \\ \{s_j\}^T [B_j]^T \{a_i\} (\dot{\phi}_j - \dot{\phi}_i) \end{matrix} \right\}$$

Relative angle driver

$$\Phi_{ANGLE} = \phi_j - \phi_i - C - f(t) = 0 \quad C = \text{cons tan } t$$

$$\begin{aligned} [\Phi_{qi}]_{ANGLE} &= [0 \ 0 \ -1] \\ [\Phi_{ji}]_{ANGLE} &= [0 \ 0 \ 1] \end{aligned}$$

$$v_{ANGLE} = f_t$$

$$\gamma_{ANGLE} = f_{tt}$$

$$\eta_{ANGLE} = f_{ttt}$$

$$\sigma_{ANGLE} = f_{tttt}$$

$$\{\kappa_i\}_{ANGLE} = \{0_{3x1}\}$$

$$\{\kappa_j\}_{ANGLE} = \{0_{3x1}\}$$

Gear pair driver (chain/sprockets, belt/pulleys)

$$\Phi_{GEAR} = \phi_j - K\phi_i - C = 0 \quad K = \text{cons tan } t, C = \text{cons tan } t$$

external gears $K = -\rho_i / \rho_j$, internal gears $K = +\rho_i / \rho_j$

$$\begin{aligned} [\Phi_{qi}]_{GEAR} &= [0 \ 0 \ -K] \\ [\Phi_{qj}]_{GEAR} &= [0 \ 0 \ 1] \end{aligned}$$

$$v_{GEAR} = 0$$

$$\gamma_{GEAR} = 0$$

$$\eta_{GEAR} = 0$$

$$\sigma_{GEAR} = 0$$

Gear pair on rotating link k

$$\Phi_{\text{GEAR_ON_K}} = (\phi_j - \phi_k) - K(\phi_i - \phi_k) - C = 0 \quad K = \text{cons tan } t, C = \text{cons tan } t \quad \text{from above}$$

$$[\Phi_{qi}]_{\text{GEAR_ON_K}} = [0 \ 0 \ -K]$$

$$[\Phi_{qj}]_{\text{GEAR_ON_K}} = [0 \ 0 \ 1]$$

$$[\Phi_{qk}]_{\text{GEAR_ON_K}} = [0 \ 0 \ (K-1)]$$

$$v_{\text{GEAR_ON_K}} = 0$$

$$\gamma_{\text{GEAR_ON_K}} = 0$$

$$\eta_{\text{GEAR_ON_K}} = 0$$

$$\sigma_{\text{GEAR_ON_K}} = 0$$

Relative coordinate driver (translation, rotation, gears, pure rolling)

$$\Phi_{\text{RCD}} = q_j - Kq_i - C - f(t) = 0 \quad K = \text{cons tan } t, C = \text{cons tan } t$$

$$[\Phi_{qi}]_{\text{RCD}} = [-K \ 0 \ 0] \quad q_i = x_i$$

$$[\Phi_{qi}]_{\text{RCD}} = [0 \ -K \ 0] \quad q_i = y_i$$

$$[\Phi_{qi}]_{\text{RCD}} = [0 \ 0 \ -K] \quad q_i = \phi_i$$

$$[\Phi_{qj}]_{\text{RCD}} = [1 \ 0 \ 0] \quad q_j = x_j$$

$$[\Phi_{qj}]_{\text{RCD}} = [0 \ 1 \ 0] \quad q_j = y_j$$

$$[\Phi_{qj}]_{\text{RCD}} = [0 \ 0 \ 1] \quad q_j = \phi_j$$

$$v_{\text{RCD}} = f_t$$

$$\gamma_{\text{RCD}} = f_{tt}$$

$$\eta_{\text{RCD}} = f_{ttt}$$

$$\sigma_{\text{RCD}} = f_{tttt}$$

Planar parallel-2 distance driver (see pin-in-slot)

$$\Phi_{PP2DD} = \{a_i\}^T \{d_{ij}\} / L - f(t) = 0 \quad L = |\{a_i\}| = \text{cons tan t length}$$

$$\begin{aligned} [\Phi_{qi}]_{PP2DD} &= (\{a_i\}^T [\Phi_{qi}]_{REV} + [\{0_{1x2}\} \quad \{a_i\}^T [R]^T \{d_{ij}\}]) / L \\ [\Phi_{qj}]_{PP2DD} &= \{a_i\}^T [\Phi_{qj}]_{REV} / L \end{aligned}$$

$$v_{PP2DD} = f_t$$

$$\gamma_{PP2DD} = \{a_i\}^T (-2 \dot{\phi}_i [R]^T \{d_{ij}\} + \dot{\phi}_i^2 \{d_{ij}\} + \{\gamma\}_{REV}) / L + f_{tt}$$

$$\eta_{PP2DD} = \{a_i\}^T (-[R]^T (3 \dot{\phi}_i \{d_{ij}\} + 3 \ddot{\phi}_i \{d_{ij}\} - \dot{\phi}_i^3 \{d_{ij}\}) + 3 \dot{\phi}_i^2 \{d_{ij}\} + 3 \dot{\phi}_i \ddot{\phi}_i \{d_{ij}\} + \{\eta\}_{REV}) / L + f_{ttt}$$

$$\sigma_{PP2DD} = \{a_i\}^T \left(\begin{array}{l} -[R]^T (4 \dot{\phi}_i \{d_{ij}\} + 6 \ddot{\phi}_i \{d_{ij}\} + 4 (\ddot{\phi}_i - \dot{\phi}_i^3) \{d_{ij}\} - 6 \dot{\phi}_i^2 \ddot{\phi}_i \{d_{ij}\}) \\ + 6 \dot{\phi}_i^2 \{d_{ij}\} + 12 \dot{\phi}_i \ddot{\phi}_i \{d_{ij}\} + (4\dot{\phi}_i \ddot{\phi}_i + 3\ddot{\phi}_i^2 - \dot{\phi}_i^4) \{d_{ij}\} + \{\sigma\}_{REV} \end{array} \right) / L + f_{tttt}$$

Pure rolling along planar parallel-2 distance

$$\Phi_{ROLL} = \{a_i\}^T \{d_{ij}\} / L - \rho(\phi_j - \phi_i) - C = 0$$

$L = |\{a_i\}| = \text{cons tan t length}$, $\rho = \text{rolling radius}$, $C = \text{cons tan t}$

$$\begin{aligned} [\Phi_{qi}]_{ROLL} &= (\{a_i\}^T [\Phi_{qi}]_{REV} + [\{0_{1x2}\} \quad \{a_i\}^T [R]^T \{d_{ij}\} + \rho L]) / L \\ [\Phi_{qj}]_{ROLL} &= (\{a_i\}^T [\Phi_{qj}]_{REV} - [\{0_{1x2}\} \quad \rho L]) / L \end{aligned}$$

$$v_{ROLL} = 0$$

$$\gamma_{ROLL} = \gamma_{PP2DD} \quad \text{for } f_{tt} = 0$$

$$\eta_{ROLL} = \eta_{PP2DD} \quad \text{for } f_{ttt} = 0$$

$$\sigma_{ROLL} = \sigma_{PP2DD} \quad \text{for } f_{tttt} = 0$$

Planar relative distance driver (see double revolute)

$$\Phi_{PRDD} = \{d_{ij}\}^T \{d_{ij}\} - (f(t))^2 = 0 \quad f(t) > 0$$

$$\left[\Phi_{qi} \right]_{PRDD} = \left[\Phi_{qi} \right]_{REV_REV}$$

$$v_{PRDD} = 2 f f_t$$

$$\gamma_{PRDD} = \gamma_{REV_REV} + 2 f_t^2 + 2 f f_{tt}$$

$$\eta_{PRDD} = \eta_{REV_REV} + 6 f_t f_{tt} + 2 f f_{ttt}$$

$$\sigma_{PRDD} = \sigma_{REV_REV} + 6 f_{tt}^2 + 8 f_t f_{ttt} + 2 f f_{ttt}$$

Acceleration Right-hand Side for Revolute

$$\{\gamma\} \equiv -(\Phi_q \dot{q})_q \dot{q} - 2[\Phi_q]_t \dot{q} - \{\Phi_{tt}\}$$

$$\{\Phi\}_{REV} = \{r_j\}^p - \{r_i\}^p = \{0_{2x1}\}$$

$$\{q_i\} = \begin{Bmatrix} \{r_i\} \\ \phi_i \end{Bmatrix} \quad \dot{q}_i = \begin{Bmatrix} \{r_i\} \\ \dot{\phi}_i \end{Bmatrix}$$

$$[\Phi_{qi}]_{REV} = -[[I_2] \quad [B_i] \{s_i\}^p]$$

$$[\Phi_{qi}] \dot{q}_i = -[[I_2] \quad [B_i] \{s_i\}^p] \begin{Bmatrix} \{r_i\} \\ \dot{\phi}_i \end{Bmatrix} = -\{r_i\} - \dot{\phi}_i [B_i] \{s_i\}^p$$

$$([\Phi_{qi}] \dot{q}_i)_q = [[0_{2x2}] \quad \dot{\phi}_i [A_i] \{s_i\}^p] \quad [B_i]_{\phi i} = -[A_i]$$

$$([\Phi_{qi}] \dot{q}_i)_{qi} \dot{q}_i = [[0_{2x2}] \quad \dot{\phi}_i [A_i] \{s_i\}^p] \begin{Bmatrix} \{r_i\} \\ \dot{\phi}_i \end{Bmatrix} = \dot{\phi}_i^2 [A_i] \{s_i\}^p$$

$$[\Phi_{qi}]_{REV} = [[I_2] \quad [B_i] \{s_i\}^p]$$

$$[\Phi_{qi}]_t = [0_{2x3}] \quad [\Phi_{qi}]_t \dot{q}_i = \{0_{2x1}\}$$

$$\{\Phi_t\} = \{0_{2x1}\} \quad \{\Phi_{tt}\} = \{0_{2x1}\}$$

$$\{\gamma\} \equiv -(\Phi_q \dot{q})_q \dot{q} - 2[\Phi_q]_t \dot{q} - \{\Phi_{tt}\}$$

$$\{\gamma\}_{REV} = -\dot{\phi}_i^2 [A_i] \{s_i\}^p \quad \text{for body } i$$

$$\{\gamma\}_{REV} = \dot{\phi}_j^2 [A_j] \{s_j\}^p \quad \text{for body } j$$

$$\{\gamma\}_{REV} = \dot{\phi}_j^2 [A_j] \{s_j\}^p - \dot{\phi}_i^2 [A_i] \{s_i\}^p$$