

Forces cause accelerations

accelerations do not cause forces

### d'Alembert's Principle

An accelerating rigid body can be transformed into an equivalent quasi-static system by adding a fictitious imaginary "inertial force" and "inertial moment". The inertial force and inertial moment may be treated as an external force and external moment. This is particularly attractive for inverse dynamic problems.

Newton  $\sum \{F\} = m\{\ddot{r}_G\}$        $\sum M = J_G \dot{\omega}$        $J_G \dot{\omega}$

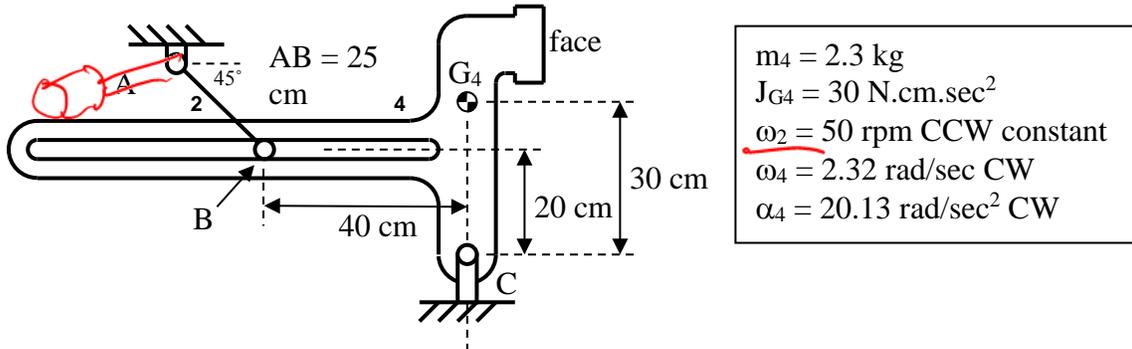
d'Alembert  $\{F_I\} = -m\{\ddot{r}_G\}$        $\sum \{F\} + \{F_I\} = 0$        $M_I = -J_G \dot{\omega}$        $\sum M + M_I = 0$        $I$

Unfortunately this concept promotes the incorrect causality that acceleration causes force.

Acceleration does not cause force. Force causes acceleration.

Inertial forces and inertial moments DO NOT EXIST. They are only fictitious algebraic constructs.

Pin B at the end of crank link 2 forms a pin-in-slot joint with the horizontal slot in hammer link 4 as shown below. The mechanism is drawn approximately to scale. The weight of crank link 2 is very small compared to the weight of hammer link 4. You may neglect the effects of friction at A, B and C. The hammer face is not yet in contact with its platen. Show your work.



Determine the magnitude and direction of motor torque  $T_{1\_on\_2}$  on crank 2 required to cause this motion at this position.

$T_{1\_on\_2} = \underline{451 \text{ N.cm CCW}}$

using virtual work and d'Alembert's Principle

$$T_{1\_on\_2} \circ \omega_2 + \bar{F}_{I\_on\_4} \circ \bar{V}_{G4} + M_{I\_on\_4} \circ \omega_4 = 0$$

assume  $T_{1\_on\_2}$  CCW

$\omega_2 = 50 \text{ rev /min} = 5.236 \text{ rad/sec CCW}$

$A_{G4}^T = (CG_4) \alpha_4 = 603.9 \text{ cm/sec}^2 \text{ right}$

$F_{I\_on\_4}^X = - m_4 A_{G4}^T = (2.3 \text{ kg}) (603.9 \text{ cm/sec}^2) (1 \text{ m} / 100 \text{ cm}) = 13.89 \text{ N left}$

$V_{G4}^X = (CG_4) \omega_4 = 69.6 \text{ cm/sec right}$

$A_{G4}^N = (CG_4) \omega_4^2 = 161.5 \text{ cm/sec}^2 \text{ down}$

$F_{I\_on\_4}^Y = - m_4 A_{G4}^N = 3.71 \text{ N up}$

$V_{G4}^Y = 0$

$m_4 g = 22.56 \text{ N down}$

$M_{I\_on\_4} = - J_{G4} \alpha_4 = (30 \text{ N.cm.sec}^2) (20.13 \text{ rad/sec}^2) = 603.9 \text{ N.cm CCW}$

$\bar{F}_{I \text{ on } 4} = - m_4 \bar{A}_{G4}$   
 $M_{I \text{ on } 4} = - J_{G4} \alpha_4$   
 $F_{2 \text{ on } 4}^Y = \uparrow$   
 $V_{B2/4} = \rightarrow$

$$\omega_4 = 2.32 \text{ rad/sec CW}$$

$$T_{1\_on\_2} \circ \omega_2 + \bar{F}_{1\_on\_4} \circ \bar{V}_{G4} + M_{1\_on\_4} \circ \omega_4 = 0$$

$$+ T_{1\_on\_2} (5.236 \text{ rad/sec}) - (13.89 \text{ N})(69.6 \text{ cm/sec}) - (603.9 \text{ N.cm}) (2.32 \text{ rad/sec}) = 0$$

$$T_{1\_on\_2} = +452.2 \text{ N.cm}$$

matches Notes\_08\_01



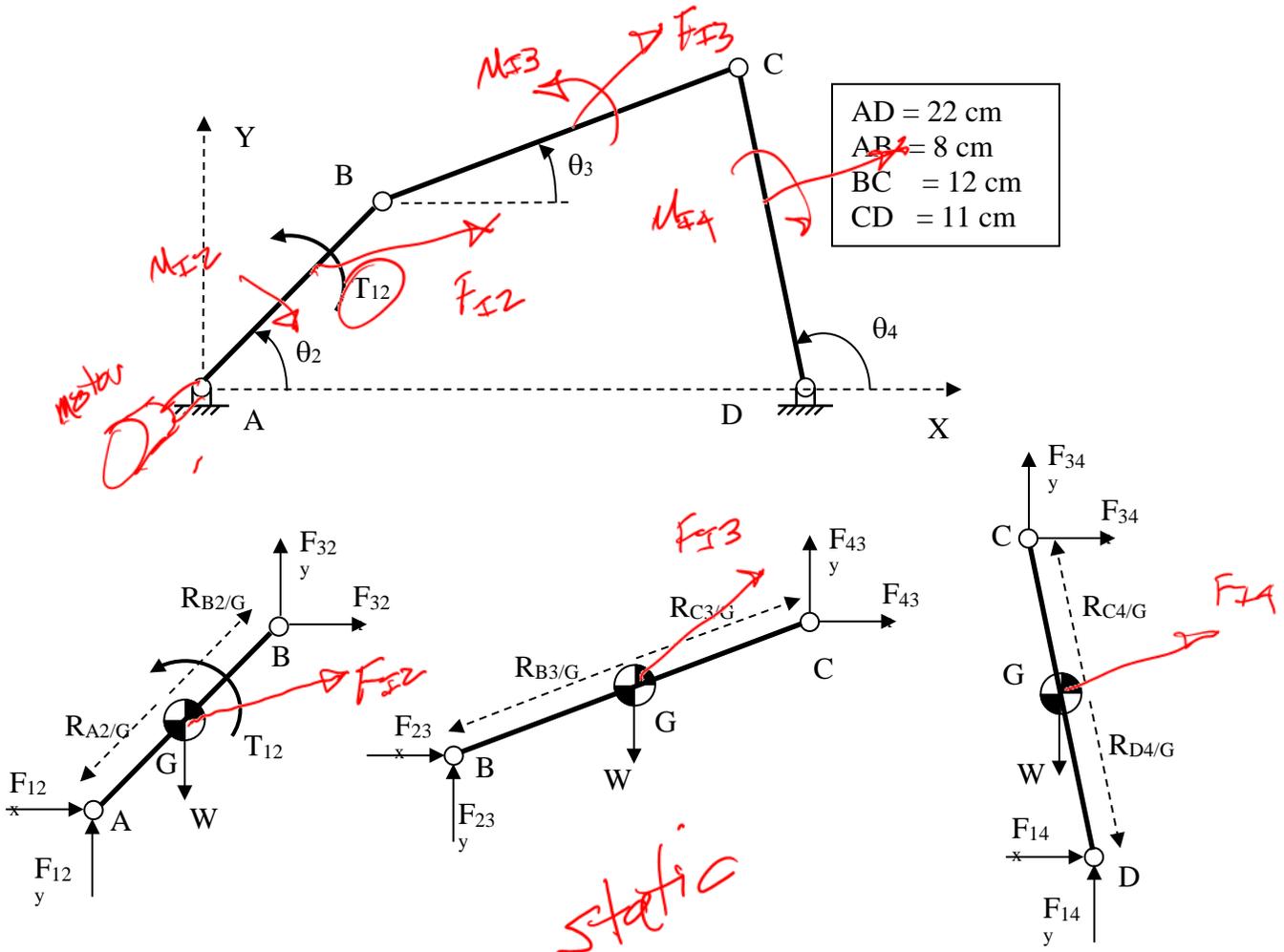
### Dynamic Force Analysis for Four Bar

The four bar linkage shown below operates in a vertical plane. Each link is a uniform bar with 2 cm by 2 cm square cross-section stainless steel. Assume that the masses of the bearings and the effects of friction are negligible. Do not neglect the effects of gravity.

|                                |                                      |  |
|--------------------------------|--------------------------------------|--|
| $\theta_2 = 45 \text{ deg}$    | $\omega_2 = 20 \text{ rad/s CW}$     | $\dot{\omega}_2 = 100 \text{ rad/s/s CCW}$   |
| $\theta_3 = 20 \text{ deg}$    | $\omega_3 = 12.82 \text{ rad/s CCW}$ | $\dot{\omega}_3 = 39.6 \text{ rad/s/s CW}$   |
| $\theta_4 = 117.4 \text{ deg}$ | $\omega_4 = 6.20 \text{ rad/s CW}$   | $\dot{\omega}_4 = 482.5 \text{ rad/s/s CCW}$ |

|                          |                                   |                              |
|--------------------------|-----------------------------------|------------------------------|
| $m_2 = 0.248 \text{ kg}$ | $J_{G2}' = 1.405 \text{ kg.cm}^2$ | $\rho = 7.75 \text{ g/cm}^3$ |
| $m_3 = 0.372 \text{ kg}$ | $J_{G3}' = 4.588 \text{ kg.cm}^2$ |                              |
| $m_4 = 0.341 \text{ kg}$ | $J_{G4}' = 3.552 \text{ kg.cm}^2$ |                              |

|  |   |                 |
|--|---|-----------------|
| $\{\ddot{r}_2\}^G = 56.57 - j 56.57 \text{ cps}$ | $\{\ddot{r}_2\}^G = -1414.2 - j 848.5 \text{ cps}^2$  | <i>opposite</i> |
| $\{\ddot{r}_3\}^G = 86.83 - j 40.86 \text{ cps}$ | $\{\ddot{r}_3\}^G = -3672.5 - j 2260.9 \text{ cps}^2$ |                 |
| $\{\ddot{r}_4\}^G = 30.27 - j 15.71 \text{ cps}$ | $\{\ddot{r}_4\}^G = -2262.4 - j 1412.4 \text{ cps}^2$ |                 |



Weights are true external forces caused by gravity. d'Alembert inertial forces and moments may be treated as if they were external forces/moments acting on the links.

$$\begin{aligned} \{W_2\} &= -j \, 2.433 \, \text{N} & \{F_{12}\} &= -m_2 \{\ddot{r}_2\}^G = +3.507 + j \, 2.104 \, \text{N} & M_{I2} &= -J_{G2}' \alpha_2 = -1.405 \, \text{N.cm} \\ \{W_3\} &= -j \, 3.649 \, \text{N} & \{F_{13}\} &= -m_3 \{\ddot{r}_3\}^G = +13.662 + j \, 8.411 \, \text{N} & M_{I3} &= -J_{G3}' \alpha_3 = +1.817 \, \text{N.cm} \\ \{W_4\} &= -j \, 3.345 \, \text{N} & \{F_{14}\} &= -m_4 \{\ddot{r}_4\}^G = +7.715 + j \, 4.816 \, \text{N} & M_{I4} &= -J_{G4}' \alpha_4 = 17.138 \, \text{N.cm} \end{aligned}$$

$$\text{Actual power at A and D is zero} \quad \{F_{12}\} \circ \{\dot{r}_2\}^A = 0 \quad \{F_{14}\} \circ \{\dot{r}_4\}^D = 0$$

$$\text{Actual power at B is zero} \quad \{F_{32}\} \circ \{\dot{r}_2\}^B + \{F_{23}\} \circ \{\dot{r}_3\}^B = 0$$

$$\{F_{32}\} = -\{F_{23}\}$$

$$\text{Actual power at C is zero} \quad \{F_{43}\} \circ \{\dot{r}_3\}^C + \{F_{34}\} \circ \{\dot{r}_4\}^C = 0$$

$$\text{Must include actual power at G} \quad \{W_i\} \circ \{\dot{r}_i\}^G$$

$$\text{Must include d'Alembert power at G} \quad \{F_{ii}\} \circ \{\dot{r}_i\}^G + M_{ii} \omega_i$$

$$(+)$$

$$\text{Must include actual power into crank} \quad T_{12} \circ \omega_2$$

$$\begin{aligned} &+ \{F_{12}\} \circ \{\dot{r}_2\}^G + \{F_{13}\} \circ \{\dot{r}_3\}^G + \{F_{14}\} \circ \{\dot{r}_4\}^G + M_{I2} \omega_2 + M_{I3} \omega_3 + M_{I4} \omega_4 \\ &+ \{W_2\} \circ \{\dot{r}_2\}^G + \{W_3\} \circ \{\dot{r}_3\}^G + \{W_4\} \circ \{\dot{r}_4\}^G + T_{12} \omega_2 = 0 \end{aligned}$$

$$\{F_{12}\} \circ \{\dot{r}_2\}^G = +79.368 \, \text{N.cm/s}$$

$$\{F_{13}\} \circ \{\dot{r}_3\}^G = +842.598 \, \text{N.cm/s}$$

$$\{F_{14}\} \circ \{\dot{r}_4\}^G = +309.192 \, \text{N.cm/s}$$

$$M_{I2} \omega_2 = +28.100 \, \text{N.cm/s}$$

$$M_{I3} \omega_3 = +23.292 \, \text{N.cm/s}$$

$$M_{I4} \omega_4 = +106.258 \, \text{N.cm/s}$$

$$\{W_2\} \circ \{\dot{r}_2\}^G = +137.635 \, \text{N.cm/s}$$

$$\{W_3\} \circ \{\dot{r}_3\}^G = +149.098 \, \text{N.cm/s}$$

$$\{W_4\} \circ \{\dot{r}_4\}^G = -52.550 \, \text{N.cm/s}$$

$$T_{12} \omega_2 = -20 \, T_{12} \, \text{rad/s}$$

$$1622.89 \, \text{N.cm/s} - 20 \, T_{12} \, \text{rad/s} = 0$$

$$T_{12} = +81.14 \, \text{N.cm}$$

matches Notes\_08\_02