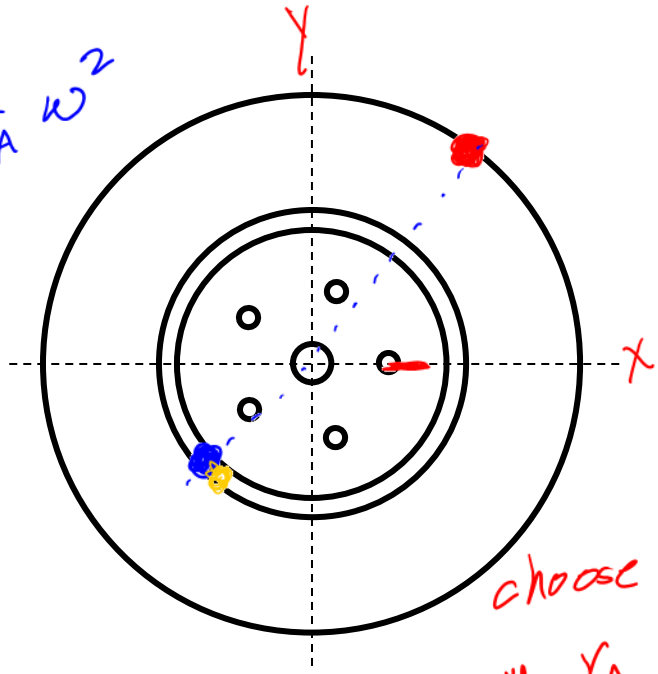
balancing mass A

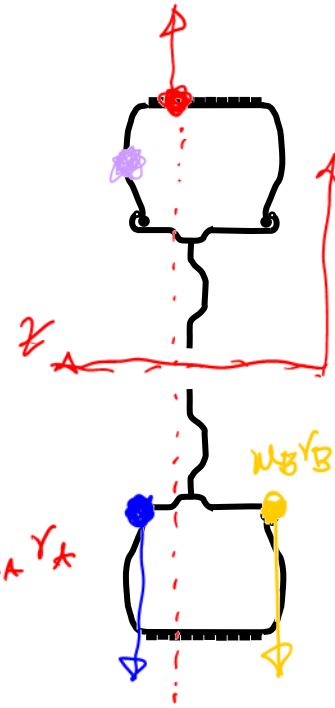
Multiplanar Balancing

$(mr)\omega^2$

F_I

$m_A r_A \omega^2$





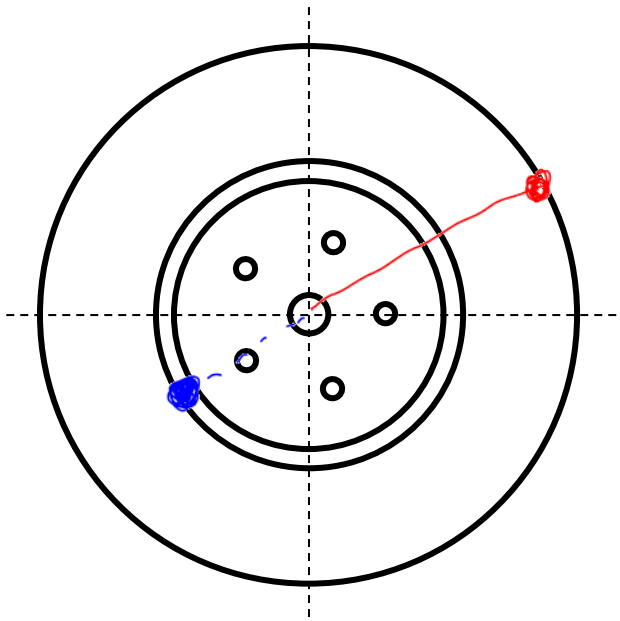
choose $m_A r_A$

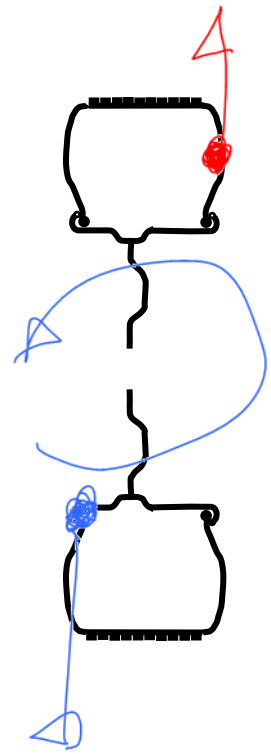
$mr = m_A r_A$

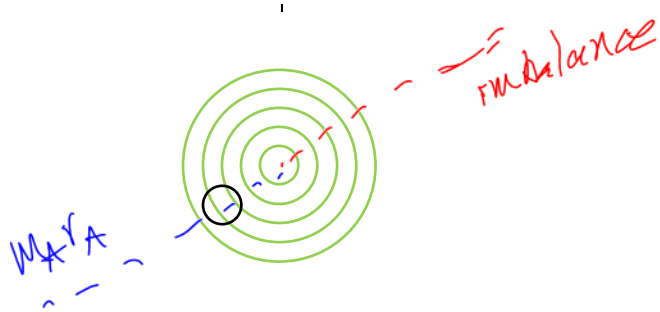
primary balancing

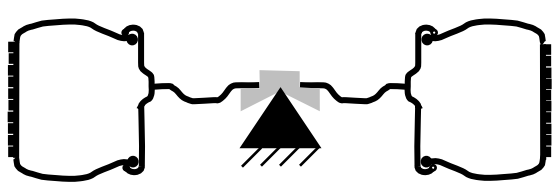
$mr = m_A r_A + m_B r_B$

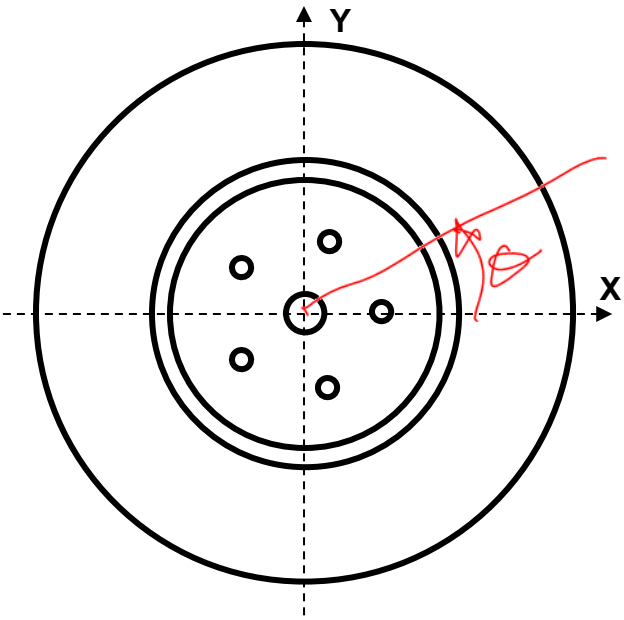
secondary imbalance moment

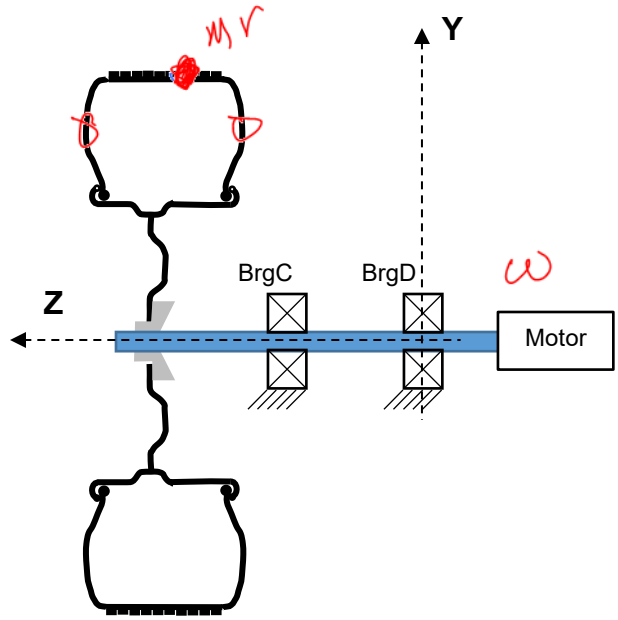


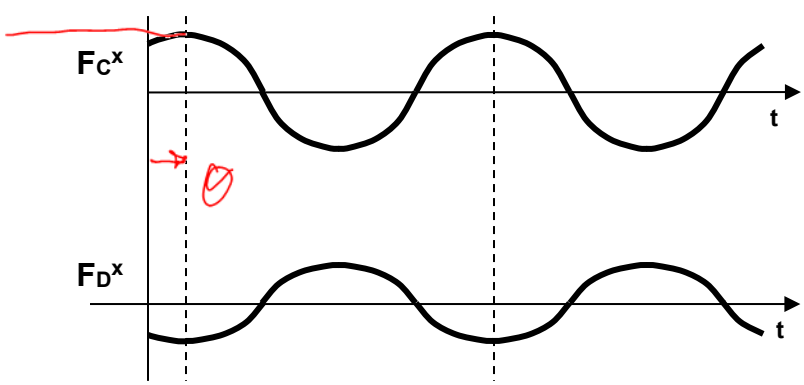



mUnbalance
$m_A r_A$









$\theta = \omega t$

imbalance

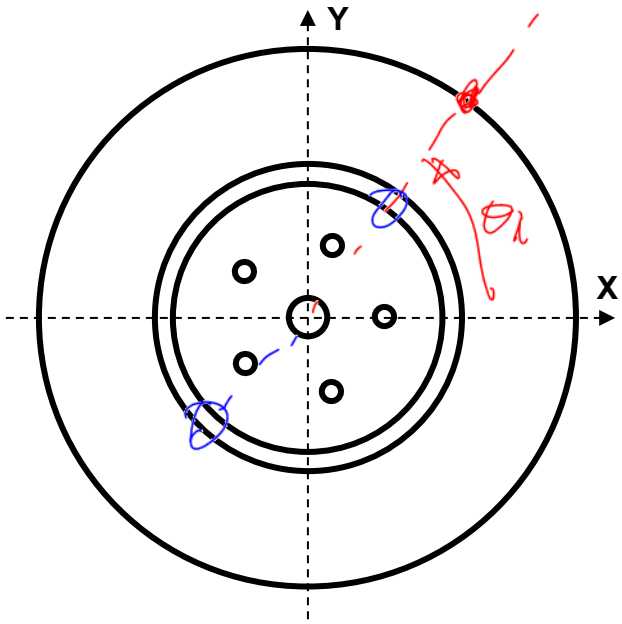
$$F_C^{\,X} + F_D^{\,X} = m_i\, r_i\, \omega^2 \qquad m_i\, r_i = \frac{F_C^{\,X} + F_D^{\,X}}{\omega^2}$$

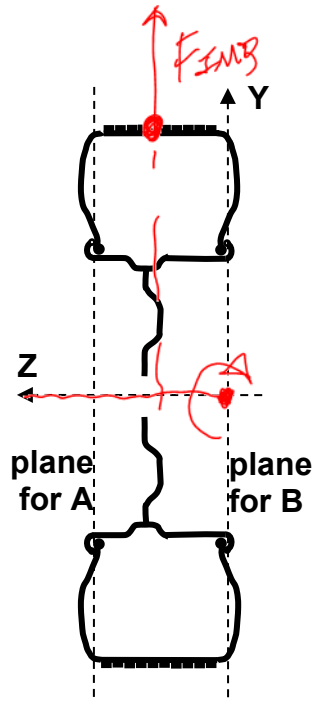
$$F_C^{\,X}\, z_C = z_i\, m_i\, r_i\, \omega^2 \qquad z_i = z_C \frac{F_C^{\,X}}{F_C^{\,X} + F_D^{\,X}}$$

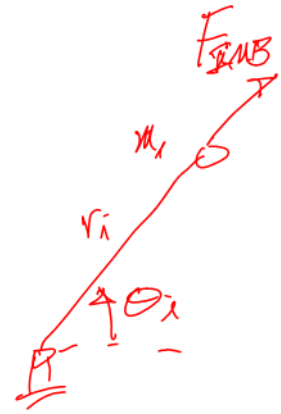
$m_i r_i$

z_i

θ_i







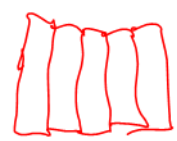
know
$m_i r_i$
θ_i
z_i

in-plane primary balancing (static balancing)

$F_{IMB_i}{}^{X} = m_i\ r_i\ \omega^2\ \cos\theta_i$ $\qquad$ $F_{IMB_i}{}^{Y} = m_i\ r_i\ \omega^2\ \sin\theta_i$

$F_{BAL_A}{}^{X} = m_A\ r_A\ \omega^2\ \cos\theta_A$ $\qquad$ $F_{BAL_A}{}^{Y} = m_A\ r_A\ \omega^2\ \sin\theta_A$

$F_{BAL_B}{}^{X} = m_B\ r_B\ \omega^2\ \cos\theta_B$ $\qquad$ $F_{IMB_B}{}^{Y} = m_B\ r_{Bi}\ \omega^2\ \sin\theta_B$



forces in X $\qquad \sum(m_i\ r_i\ \cos\theta_i) + m_A\ r_A\ \cos\theta_A + m_B\ r_B\ \cos\theta_B = 0$ $\qquad$ Eq. 1

forces in Y $\qquad \sum(m_i\ r_i\ \sin\theta_i) + m_A\ r_A\ \sin\theta_A + m_B\ r_B\ \sin\theta_B = 0$ $\qquad$ Eq. 2

multiple imbalances

find $m_A\ \theta_A$ $\quad m_B\ \theta_B$
know $r_A\ z_A$ $\quad r_B\ z_B$

4 unknowns

secondary dynamic balancing to prevent shaking moments

$M_{IMB_i}{}^{Y} = +F_{IMB_i}{}^{X}(z_i) = m_i\ r_i\ z_i\ \omega^2\ \cos\theta_i$

$M_{IMB_i}{}^{X} = -F_{IMB_i}{}^{Y}(z_i) = -m_i\ r_i\ z_i \omega^2\ \sin\theta_i$

moments in Y $\qquad \sum(m_i\ r_i\ z_i\ \cos\theta_i) + m_A\ r_A\ z_A\ \cos\theta_A = 0$ $\qquad z_B = 0$ $\qquad$ Eq. 3

moments in X $\qquad \sum(m_i\ r_i\ z_i\ \sin\theta_i) + m_A\ r_A\ z_A\ \sin\theta_A = 0$ $\qquad z_B = 0$ $\qquad$ Eq. 4

judicious

$m_A > 0 \quad \theta_A = \theta_i + 180^\circ$

$m_A < 0 \quad \theta_A = \theta_i$

moment

$$\begin{cases} m_A\ r_A\ z_A\ \sin\theta_A = -\sum(m_i\ r_i\ z_i\ \sin\theta_i) \\ m_A\ r_A\ z_A\ \cos\theta_A = -\sum(m_i\ r_i\ z_i\ \cos\theta_i) \end{cases}$$

$$\tan\theta_A = \frac{\sum(m_i\ r_i\ z_i\ \sin\theta_i)}{\sum(m_i\ r_i\ z_i\ \cos\theta_i)}$$

for only one imbalance $\tan\theta_A = \tan\theta_i$

θ_A

$$(m_A\ r_A\ z_A\ \sin\theta_A)^2 + (m_A\ r_A\ z_A\ \cos\theta_A)^2 = (\sum(m_i\ r_i\ z_i\ \sin\theta_i))^2 + (\sum(m_i\ r_i\ z_i\ \cos\theta_i))^2$$

$$(m_A\ r_A\ z_A)^2 = (\sum(m_i\ r_i\ z_i\ \sin\theta_i))^2 + (\sum(m_i\ r_i\ z_i\ \cos\theta_i))^2$$

$$(m_A\ r_A) = \frac{\sqrt{(\sum(m_i\ r_i\ z_i\ \sin\theta_i))^2 + (\sum(m_i\ r_i\ z_i\ \cos\theta_i))^2}}{z_A}$$

m_A

for only one imbalance $(m_A\ r_A) = (m_i\ r_i)\dfrac{z_i}{z_A}$

force

$$\begin{cases} m_B\ r_B\ \sin\theta_B = -\sum(m_i\ r_i\ \sin\theta_i) - m_A\ r_A\ \sin\theta_A \\ m_B\ r_B\ \cos\theta_B = -\sum(m_i\ r_i\ \cos\theta_i) - m_A\ r_A\ \cos\theta_A \end{cases}$$

θ_B

$$\tan\theta_B = \frac{\sum(m_i\ r_i\ \sin\theta_i) + m_A\ r_A\ \sin\theta_A}{\sum(m_i\ r_i\ \cos\theta_i) + m_A\ r_A\ \cos\theta_A}$$

m_B

$$(m_B\ r_B) = \sqrt{(\sum(m_i\ r_i\ \sin\theta_i) + m_A\ r_A\ \sin\theta_A)^2 + (\sum(m_i\ r_i\ \cos\theta_i) + m_A\ r_A\ \cos\theta_A)^2}$$