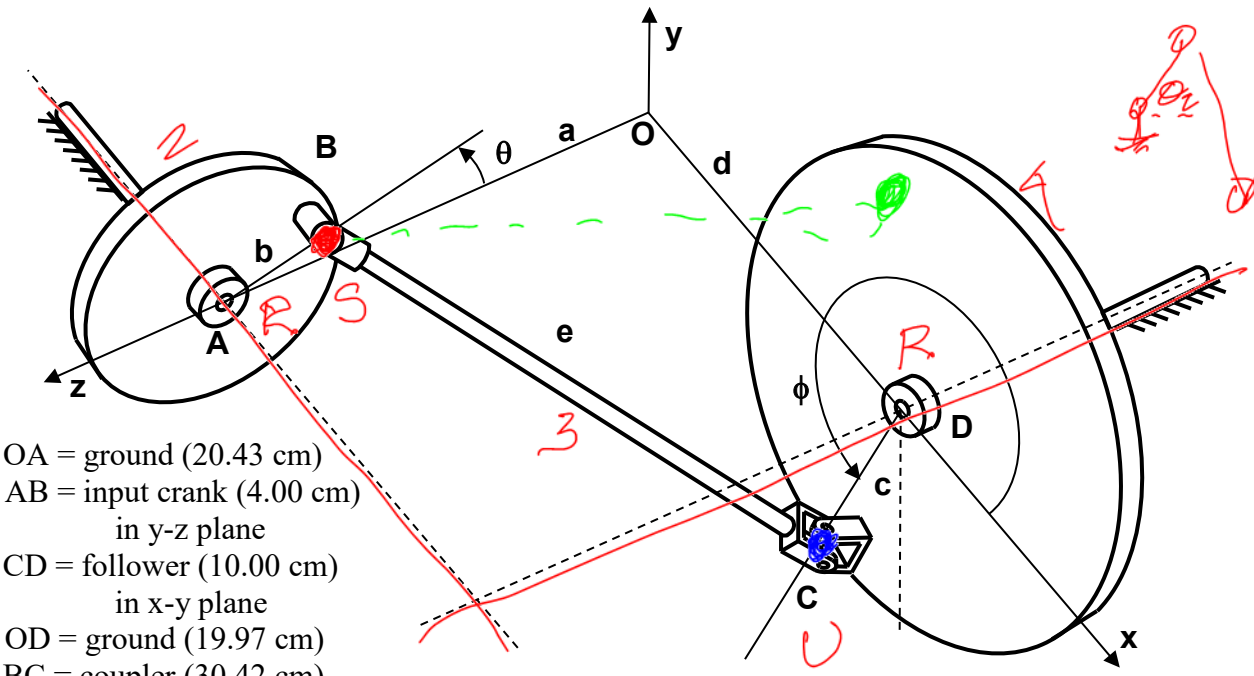
RSUR Geometric Method

A = revolute R B = spherical S C = universal U D = revolute R



a = OA = ground (20.43 cm)
b = AB = input crank (4.00 cm)
in y-z plane
c = CD = follower (10.00 cm)
in x-y plane
d = OD = ground (19.97 cm)
e = BC = coupler (30.42 cm)
θ = crank angle in y-z plane
ϕ = follower angle in x-y plane

Position solution - given θ, find ϕ

$x_B = 0 \qquad y_B = bS\theta \qquad z_B = a - bC\theta \qquad x_C = d + cC\phi \qquad y_C = cS\phi \qquad z_C = 0$

$$(x_B - x_C)^2 + (y_B - y_C)^2 + (z_B - z_C)^2 = e^2$$

$$(-d - cC\phi)^2 + (bS\theta - cS\phi)^2 + (a - bC\theta)^2 = e^2$$

$$d^2 + 2cdC\phi + c^2C^2\phi + b^2S^2\theta - 2bcS\theta S\phi + c^2S\phi^2 + a^2 - 2abC\theta + b^2C^2\theta = e^2$$

$$+ 2cdC\phi - 2bcS\theta S\phi = e^2 - a^2 - b^2 - c^2 - d^2 + 2abC\theta$$

$f = 2cd \qquad g = 2bcS\theta \qquad h = e^2 - a^2 - b^2 - c^2 - d^2 + 2abC\theta$

$$f\,C\phi - g\,S\phi = h$$

$u = \tan\frac{\phi}{2} \qquad S\phi = \frac{2u}{1+u^2} \qquad C\phi = \frac{1-u^2}{1+u^2} \qquad T\phi = \frac{2u}{1-u^2}$

tangent of half-angle

$$f\left(\frac{1-u^2}{1+u^2}\right)-g\left(\frac{2u}{1+u^2}\right)=h \qquad f-fu^2-2gu=h+hu^2$$

$$\boxed{(h+f)u^2+(2g)u+(h-f)=0}$$

quadratic

$$u=\frac{-g\pm\sqrt{f^2+g^2-h^2}}{h+f}=\tan\frac{\phi}{2}$$

Velocity solution - given $\dot{\theta}$, find $\dot{\phi}$

$$d/dt\left(+2cdC\phi-2bcS\theta S\phi=e^2-a^2-b^2-c^2-d^2+2abC\theta\right)$$

$$-2cd\dot{\phi}S\phi-2bc\dot{\theta}C\theta S\phi-2bc\dot{\phi}S\theta C\phi=-2ab\dot{\theta}S\theta$$

$$(cdS\phi+bcS\theta C\phi)\dot{\phi}=(abS\theta-bcC\theta S\phi)\dot{\theta}$$

$$\boxed{A=cdS\phi+bcS\theta C\phi} \qquad \boxed{B=abS\theta-bcC\theta S\phi}$$

$$A\dot{\phi}=B\dot{\theta}$$

$$\dot{\phi}=B\dot{\theta}/A$$

do not want $A=0$

Acceleration solution - given $\ddot{\theta}$, find $\ddot{\phi}$

$$d/dt\left((cdS\phi+bcS\theta C\phi)\dot{\phi}=(abS\theta-bcC\theta S\phi)\dot{\theta}\right)$$

$$(cdS\phi+bcS\theta C\phi)\ddot{\phi}+(cd\dot{\phi}C\phi+bc\dot{\theta}C\theta C\phi-bc\dot{\phi}S\theta S\phi)\dot{\phi}=$$
$$(abS\theta-bcC\theta S\phi)\ddot{\theta}+(ab\dot{\theta}C\theta+bc\dot{\theta}S\theta S\phi-bc\dot{\phi}C\theta C\phi)\dot{\theta}$$

$$(cdS\phi+bcS\theta C\phi)\ddot{\phi}-(bcS\theta S\phi-cdC\phi)\dot{\phi}^2=$$
$$(abS\theta-bcC\theta S\phi)\ddot{\theta}+(abC\theta+bcS\theta S\phi)\dot{\theta}^2-2bc\dot{\theta}\dot{\phi}C\theta C\phi$$

$$\boxed{C=abC\theta+bcS\theta S\phi} \qquad \boxed{D=bcS\theta S\phi-cdC\phi} \qquad \boxed{E=bcC\theta C\phi}$$

$$A\ddot{\phi}=B\ddot{\theta}+C\dot{\theta}^2+D\dot{\phi}^2-2E\dot{\theta}\dot{\phi}$$

$$\boxed{\ddot{\phi}=\left(B\ddot{\theta}+C\dot{\theta}^2+D\dot{\phi}^2-2E\dot{\theta}\dot{\phi}\right)/A}$$

Jerk solution - given $\dddot{\theta}$, find $\dddot{\phi}$

$$d/dt\left(A\ddot{\phi} = B\ddot{\theta} + C\dot{\theta}^2 + D\dot{\phi}^2 - 2E\dot{\theta}\dot{\phi}\right)$$

$$A\dddot{\phi} + \dot{A}\ddot{\phi} = B\dddot{\theta} + \dot{B}\ddot{\theta} + 2C\dot{\theta}\ddot{\theta} + \dot{C}\dot{\theta}^2 + 2D\dot{\phi}\ddot{\phi} + \dot{D}\dot{\phi}^2 - 2E\ddot{\theta}\dot{\phi} - 2E\dot{\theta}\ddot{\phi} - 2\dot{E}\dot{\theta}\dot{\phi}$$

$$A = cdS\phi + bcS\theta C\phi \qquad \dot{A} = cd\dot{\phi}C\phi + bc\dot{\theta}C\theta C\phi - bc\dot{\phi}S\theta S\phi = E\dot{\theta} - D\dot{\phi}$$
$$B = abS\theta - bcC\theta S\phi \qquad \dot{B} = ab\dot{\theta}C\theta + bc\dot{\theta}S\theta S\phi - bc\dot{\phi}C\theta C\phi = C\dot{\theta} - E\dot{\phi}$$
$$C = abC\theta + bcS\theta S\phi \qquad \dot{C} = -ab\dot{\theta}S\theta + bc\dot{\theta}C\theta S\phi + bc\dot{\phi}S\theta C\phi = -B\dot{\theta} + F\dot{\phi}$$
$$D = bcS\theta S\phi - cdC\phi \qquad \dot{D} = bc\dot{\theta}C\theta S\phi + bc\dot{\phi}S\theta C\phi + cd\dot{\phi}S\phi = G\dot{\theta} + A\dot{\phi}$$
$$E = bcC\theta C\phi \qquad \dot{E} = -bc\dot{\theta}S\theta C\phi - bc\dot{\phi}C\theta S\phi = -F\dot{\theta} - G\dot{\phi}$$
$$F = bcS\theta C\phi \qquad \dot{F} = bc\dot{\theta}C\theta C\phi - bc\dot{\phi}S\theta S\phi = E\dot{\theta} - H\dot{\phi}$$
$$G = bcC\theta S\phi \qquad \dot{G} = -bc\dot{\theta}S\theta S\phi + bc\dot{\phi}C\theta C\phi = -H\dot{\theta} - E\dot{\phi}$$
$$H = bcS\theta S\phi$$

$$A\dddot{\phi} + \left(E\dot{\theta} - D\dot{\phi}\right)\ddot{\phi} = B\dddot{\theta} + \left(C\dot{\theta} - E\dot{\phi}\right)\ddot{\theta} + 2C\dot{\theta}\ddot{\theta} + \left(-B\dot{\theta} + bc\dot{\phi}S\theta C\phi\right)\dot{\theta}^2 + 2D\dot{\phi}\ddot{\phi}$$
$$+ \left(A\dot{\phi} + bc\dot{\theta}C\theta S\phi\right)\dot{\phi}^2 - 2E\ddot{\theta}\dot{\phi} - 2E\dot{\theta}\ddot{\phi} - 2\left(-bc\dot{\theta}S\theta C\phi - bc\dot{\phi}C\theta S\phi\right)\dot{\theta}\dot{\phi}$$

$$A\dddot{\phi} + E\dot{\theta}\ddot{\phi} - D\dot{\phi}\ddot{\phi} = B\dddot{\theta} + C\dot{\theta}\ddot{\theta} - E\dot{\phi}\ddot{\theta} + 2C\dot{\theta}\ddot{\theta} - B\dot{\theta}^3 + bc\dot{\theta}^2\dot{\phi}S\theta C\phi + 2D\dot{\phi}\ddot{\phi}$$
$$+ A\dot{\phi}^3 + bc\dot{\theta}\dot{\phi}^2C\theta S\phi - 2E\ddot{\theta}\dot{\phi} - 2E\dot{\theta}\ddot{\phi} + 2bc\dot{\theta}\dot{\phi}\left(\dot{\theta}S\theta C\phi + \dot{\phi}C\theta S\phi\right)$$

$$A\dddot{\phi} = B\left(\dddot{\theta} - \dot{\theta}^3\right) + 3C\dot{\theta}\ddot{\theta} + 3D\dot{\phi}\ddot{\phi} + A\dot{\phi}^3 - 3E\left(\ddot{\theta}\dot{\phi} + \dot{\theta}\ddot{\phi}\right) + 3bc\dot{\theta}\dot{\phi}\left(\dot{\theta}S\theta C\phi + \dot{\phi}C\theta S\phi\right)$$

$$A\dddot{\phi} = B\left(\dddot{\theta} - \dot{\theta}^3\right) + 3C\dot{\theta}\ddot{\theta} + 3D\dot{\phi}\ddot{\phi} + A\dot{\phi}^3 - 3E\left(\ddot{\theta}\dot{\phi} + \dot{\theta}\ddot{\phi}\right) + 3\dot{\theta}\dot{\phi}\left(F\dot{\theta} + G\dot{\phi}\right)$$

$$\dddot{\phi} = \left(B\left(\dddot{\theta} - \dot{\theta}^3\right) + 3C\dot{\theta}\ddot{\theta} + 3D\dot{\phi}\ddot{\phi} + A\dot{\phi}^3 - 3E\left(\ddot{\theta}\dot{\phi} + \dot{\theta}\ddot{\phi}\right) + 3\dot{\theta}\dot{\phi}\left(F\dot{\theta} + G\dot{\phi}\right)\right)/A$$

Snap solution - given $\ddddot{\theta}$, find $\ddddot{\phi}$

$$d/dt\left(A\dddot{\phi} = B\left(\dddot{\theta} - \dot{\theta}^3\right) + 3C\dot{\theta}\ddot{\theta} + 3D\dot{\phi}\ddot{\phi} + A\dot{\phi}^3 - 3E\left(\ddot{\theta}\dot{\phi} + \dot{\theta}\ddot{\phi}\right) + 3\dot{\theta}\dot{\phi}\left(F\dot{\theta} + G\dot{\phi}\right)\right)$$

$$\begin{aligned}
A\ddddot{\phi} + \dot{A}\dddot{\phi} = {} & B\left(\ddddot{\theta} - 3\dot{\theta}^2\ddot{\theta}\right) + \dot{B}\left(\dddot{\theta} - \dot{\theta}^3\right) + 3C\ddot{\theta}^2 + 3C\dot{\theta}\dddot{\theta} + 3\dot{C}\dot{\theta}\ddot{\theta} + 3D\ddot{\phi}^2 + 3D\dot{\phi}\dddot{\phi} + 3\dot{D}\dot{\phi}\ddot{\phi} \\
& + 3A\dot{\phi}^2\ddot{\phi} + \dot{A}\dot{\phi}^3 - 3E\left(\dddot{\theta}\dot{\phi} + \ddot{\theta}\ddot{\phi} + \ddot{\theta}\ddot{\phi} + \dot{\theta}\dddot{\phi}\right) - 3\dot{E}\left(\ddot{\theta}\dot{\phi} + \dot{\theta}\ddot{\phi}\right) \\
& + 3\ddot{\theta}\dot{\phi}\left(F\dot{\theta} + G\dot{\phi}\right) + 3\dot{\theta}\ddot{\phi}\left(F\dot{\theta} + G\dot{\phi}\right) + 3\dot{\theta}\dot{\phi}\left(F\ddot{\theta} + \dot{F}\dot{\theta} + G\ddot{\phi} + \dot{G}\dot{\phi}\right)
\end{aligned}$$

$$\begin{aligned}
A &= cdS\phi + bcS\theta C\phi & \dot{A} &= cd\dot{\phi}C\phi + bc\dot{\theta}C\theta C\phi - bc\dot{\phi}S\theta S\phi = E\dot{\theta} - D\dot{\phi} \\
B &= abS\theta - bcC\theta S\phi & \dot{B} &= ab\dot{\theta}C\theta + bc\dot{\theta}S\theta S\phi - bc\dot{\phi}C\theta C\phi = C\dot{\theta} - E\dot{\phi} \\
C &= abC\theta + bcS\theta S\phi & \dot{C} &= -ab\dot{\theta}S\theta + bc\dot{\theta}C\theta S\phi + bc\dot{\phi}S\theta C\phi = -B\dot{\theta} + F\dot{\phi} \\
D &= bcS\theta S\phi - cdC\phi & \dot{D} &= bc\dot{\theta}C\theta S\phi + bc\dot{\phi}S\theta C\phi + cd\dot{\phi}S\phi = G\dot{\theta} + A\dot{\phi} \\
E &= bcC\theta C\phi & \dot{E} &= -bc\dot{\theta}S\theta C\phi - bc\dot{\phi}C\theta S\phi = -F\dot{\theta} - G\dot{\phi} \\
F &= bcS\theta C\phi & \dot{F} &= bc\dot{\theta}C\theta C\phi - bc\dot{\phi}S\theta S\phi = E\dot{\theta} - H\dot{\phi} \\
G &= bcC\theta S\phi & \dot{G} &= -bc\dot{\theta}S\theta S\phi + bc\dot{\phi}C\theta C\phi = -H\dot{\theta} - E\dot{\phi} \\
H &= bcS\theta S\phi
\end{aligned}$$

$$\begin{aligned}
A\ddddot{\phi} + \left(E\dot{\theta} - D\dot{\phi}\right)\dddot{\phi} = {} & B\left(\ddddot{\theta} - 3\dot{\theta}^2\ddot{\theta}\right) + \left(C\dot{\theta} - E\dot{\phi}\right)\left(\dddot{\theta} - \dot{\theta}^3\right) + 3C\ddot{\theta}^2 + 3C\dot{\theta}\dddot{\theta} + 3\left(-B\dot{\theta} + F\dot{\phi}\right)\dot{\theta}\ddot{\theta} \\
& + 3D\ddot{\phi}^2 + 3D\dot{\phi}\dddot{\phi} + 3\left(G\dot{\theta} + A\dot{\phi}\right)\dot{\phi}\ddot{\phi} + 3A\dot{\phi}^2\ddot{\phi} + \left(E\dot{\theta} - D\dot{\phi}\right)\dot{\phi}^3 \\
& - 3E\left(\dddot{\theta}\dot{\phi} + \ddot{\theta}\ddot{\phi} + \ddot{\theta}\ddot{\phi} + \dot{\theta}\dddot{\phi}\right) - 3\left(-F\dot{\theta} - G\dot{\phi}\right)\left(\ddot{\theta}\dot{\phi} + \dot{\theta}\ddot{\phi}\right) \\
& + 3\ddot{\theta}\dot{\phi}\left(F\dot{\theta} + G\dot{\phi}\right) + 3\dot{\theta}\ddot{\phi}\left(F\dot{\theta} + G\dot{\phi}\right) \\
& + 3\dot{\theta}\dot{\phi}\left(F\ddot{\theta} + \left(E\dot{\theta} - H\dot{\phi}\right)\dot{\theta} + G\ddot{\phi} + \left(-H\dot{\theta} - E\dot{\phi}\right)\dot{\phi}\right)
\end{aligned}$$

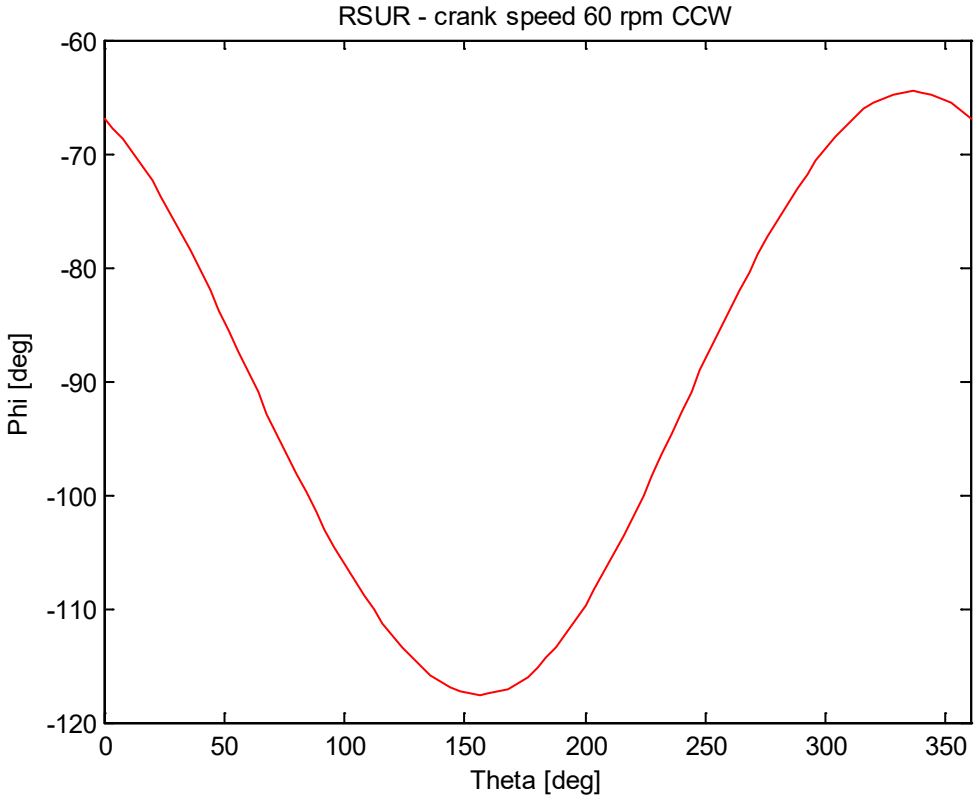
$$\begin{aligned}
A\ddddot{\phi} + E\dot{\theta}\dddot{\phi} - D\dot{\phi}\dddot{\phi} = {} & B\left(\ddddot{\theta} - 3\dot{\theta}^2\ddot{\theta}\right) + C\left(\dddot{\theta}\dot{\theta} - \dot{\theta}^4\right) - E\dddot{\theta}\dot{\phi} + E\dot{\theta}^3\dot{\phi} + 3C\ddot{\theta}^2 + 3C\dot{\theta}\dddot{\theta} - 3B\dot{\theta}^2\ddot{\theta} + 3F\dot{\phi}\dot{\theta}\ddot{\theta} \\
& + 3D\ddot{\phi}^2 + 3D\dot{\phi}\dddot{\phi} + 3G\dot{\theta}\dot{\phi}\ddot{\phi} + 3A\dot{\phi}^2\ddot{\phi} + 3A\dot{\phi}^2\ddot{\phi} + E\dot{\theta}\dot{\phi}^3 - D\dot{\phi}^4 \\
& - 3E\dddot{\theta}\dot{\phi} - 6E\ddot{\theta}\ddot{\phi} - 3E\dot{\theta}\dddot{\phi} + 3F\dot{\theta}\ddot{\theta}\dot{\phi} + 3F\dot{\theta}^2\ddot{\phi} + 3G\ddot{\theta}\dot{\phi}^2 + 3G\dot{\theta}\dot{\phi}\ddot{\phi} \\
& + 3F\dot{\theta}\ddot{\theta}\dot{\phi} + 3G\ddot{\theta}\dot{\phi}^2 + 3F\dot{\theta}^2\ddot{\phi} + 3G\dot{\theta}\dot{\phi}\ddot{\phi} \\
& + 3F\dot{\theta}\ddot{\theta}\dot{\phi} + 3E\dot{\theta}^3\dot{\phi} - 3H\dot{\theta}^2\dot{\phi}^2 + 3G\dot{\theta}\dot{\phi}\ddot{\phi} - 3H\dot{\theta}^2\dot{\phi}^2 - 3E\dot{\theta}\dot{\phi}^3
\end{aligned}$$

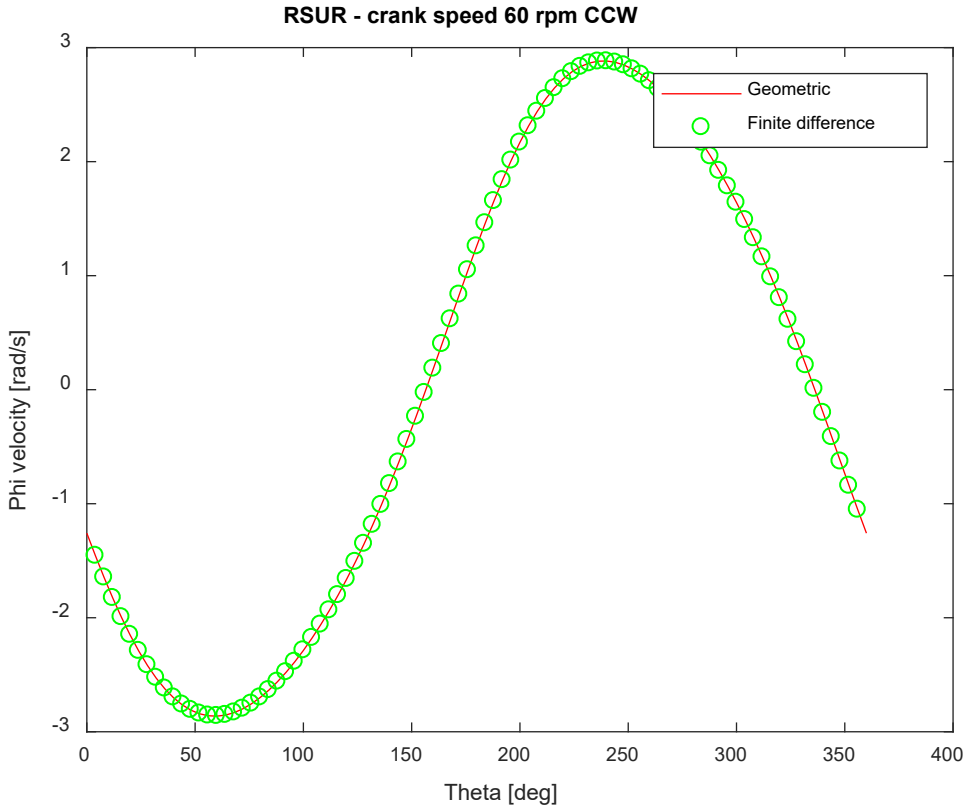
$$\begin{aligned}
A\ddddot{\phi} = {} & B\left(\ddddot{\theta} - 6\dot{\theta}^2\ddot{\theta}\right) + C\left(4\dddot{\theta}\dot{\theta} + 3\ddot{\theta}^2 - \dot{\theta}^4\right) + D\left(4\dddot{\phi}\dot{\phi} + 3\ddot{\phi}^2 - \dot{\phi}^4\right) + 6A\dot{\phi}^2\ddot{\phi} \\
& - E\dot{\theta}\dddot{\theta} - E\dddot{\theta}\dot{\phi} + E\dot{\theta}^3\dot{\phi} + E\dot{\theta}\dot{\phi}^3 - 3E\dddot{\theta}\dot{\phi} - 6E\ddot{\theta}\ddot{\phi} - 3E\dot{\theta}\dddot{\phi} + 3E\dot{\theta}^3\dot{\phi} - 3E\dot{\theta}\dot{\phi}^3 \\
& + 6F\left(2\dot{\theta}\ddot{\theta}\dot{\phi} + \dot{\theta}^2\ddot{\phi}\right) + 6G\left(\ddot{\theta}\dot{\phi}^2 + 2\dot{\theta}\dot{\phi}\ddot{\phi}\right) - 6H\dot{\theta}^2\dot{\phi}^2
\end{aligned}$$

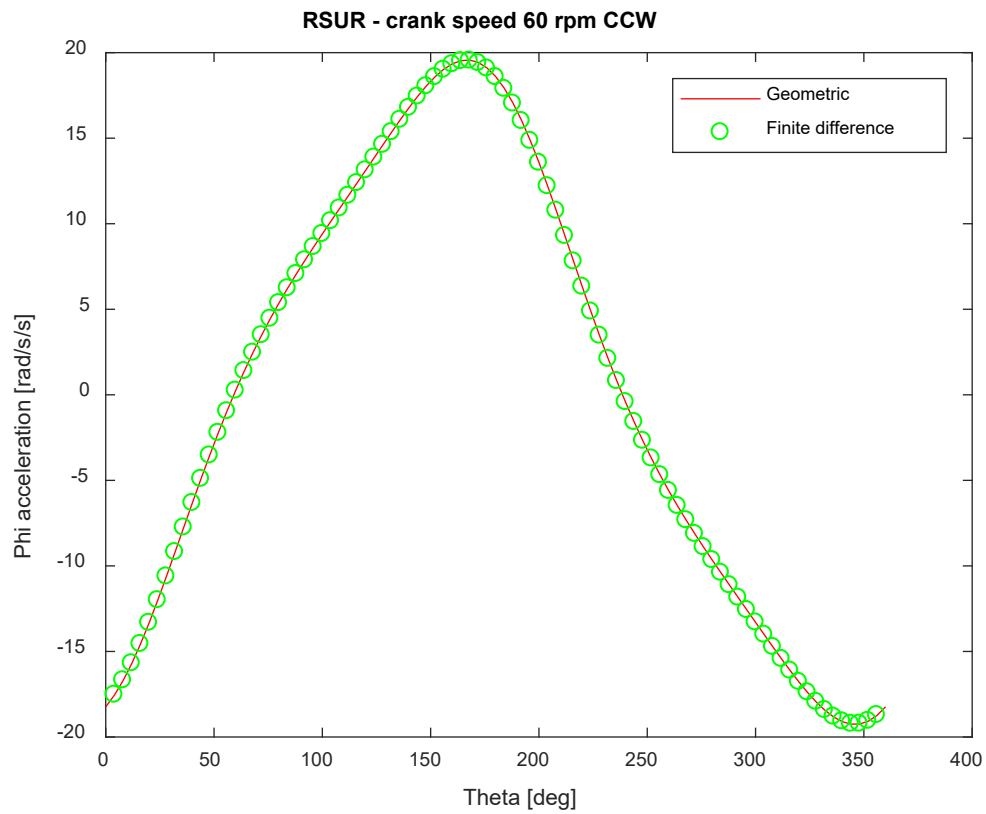
$$\begin{aligned}
A\ddddot{\phi} = {} & B\left(\ddddot{\theta} - 6\dot{\theta}^2\ddot{\theta}\right) + C\left(4\dddot{\theta}\dot{\theta} + 3\ddot{\theta}^2 - \dot{\theta}^4\right) + D\left(4\dddot{\phi}\dot{\phi} + 3\ddot{\phi}^2 - \dot{\phi}^4\right) + 6A\dot{\phi}^2\ddot{\phi} \\
& - 4E\dddot{\theta}\dot{\phi} - 6E\ddot{\theta}\ddot{\phi} + 4E\dot{\theta}^3\dot{\phi} + E\dot{\theta}\dot{\phi}^3 - 4E\dot{\theta}\dddot{\phi} - 3E\dot{\theta}\dot{\phi}^3 \\
& + 6F\left(2\dot{\theta}\ddot{\theta}\dot{\phi} + \dot{\theta}^2\ddot{\phi}\right) + 6G\left(\ddot{\theta}\dot{\phi}^2 + 2\dot{\theta}\dot{\phi}\ddot{\phi}\right) - 6H\dot{\theta}^2\dot{\phi}^2
\end{aligned}$$

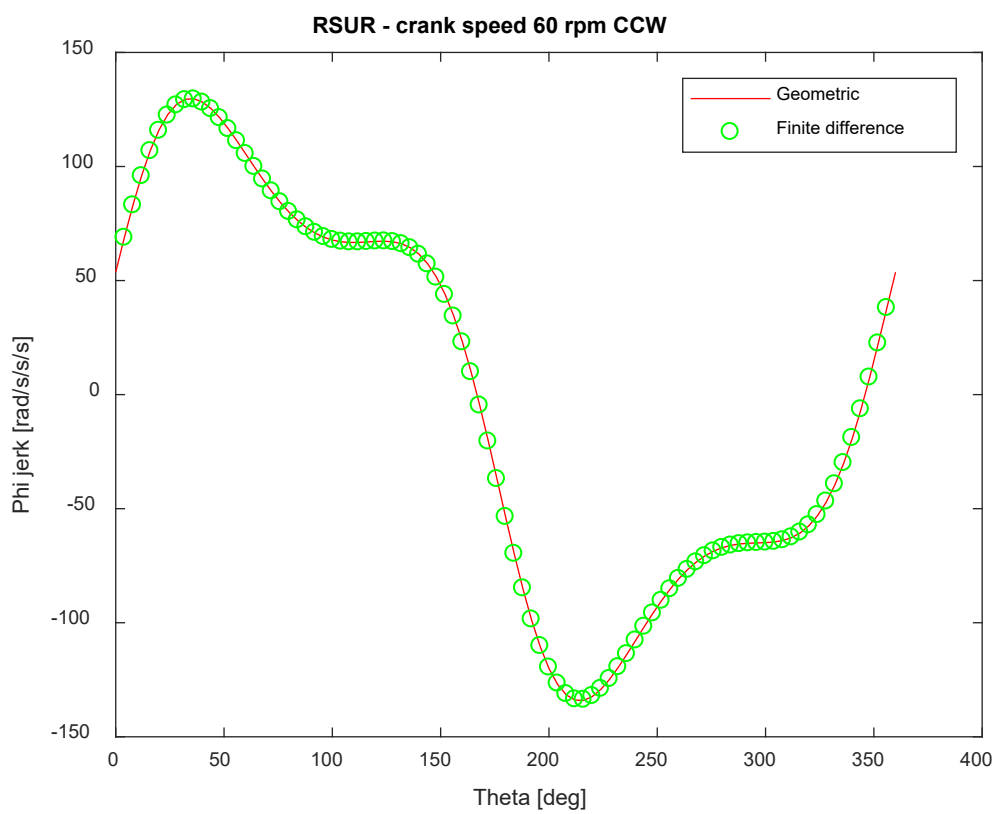
$$A\dddot{\phi} = B\left(\dddot{\theta} - 6\dot{\theta}^2\ddot{\theta}\right) + C\left(4\dddot{\theta}\dot{\theta} + 3\ddot{\theta}^2 - \dot{\theta}^4\right) + D\left(4\dddot{\phi}\dot{\phi} + 3\ddot{\phi}^2 - \dot{\phi}^4\right) + 6A\dot{\phi}^2\ddot{\phi}$$
$$- 2E\left(2\dddot{\theta}\dot{\phi} + 3\ddot{\theta}\ddot{\phi} - 2\dot{\theta}^3\dot{\phi} + \dot{\theta}\dot{\phi}^3 + 2\dot{\theta}\dddot{\phi}\right) + 6F\left(2\dot{\theta}\ddot{\theta}\dot{\phi} + \dot{\theta}^2\ddot{\phi}\right) + 6G\left(\ddot{\theta}\dot{\phi}^2 + 2\dot{\theta}\dot{\phi}\ddot{\phi}\right) - 6H\dot{\theta}^2\dot{\phi}^2$$

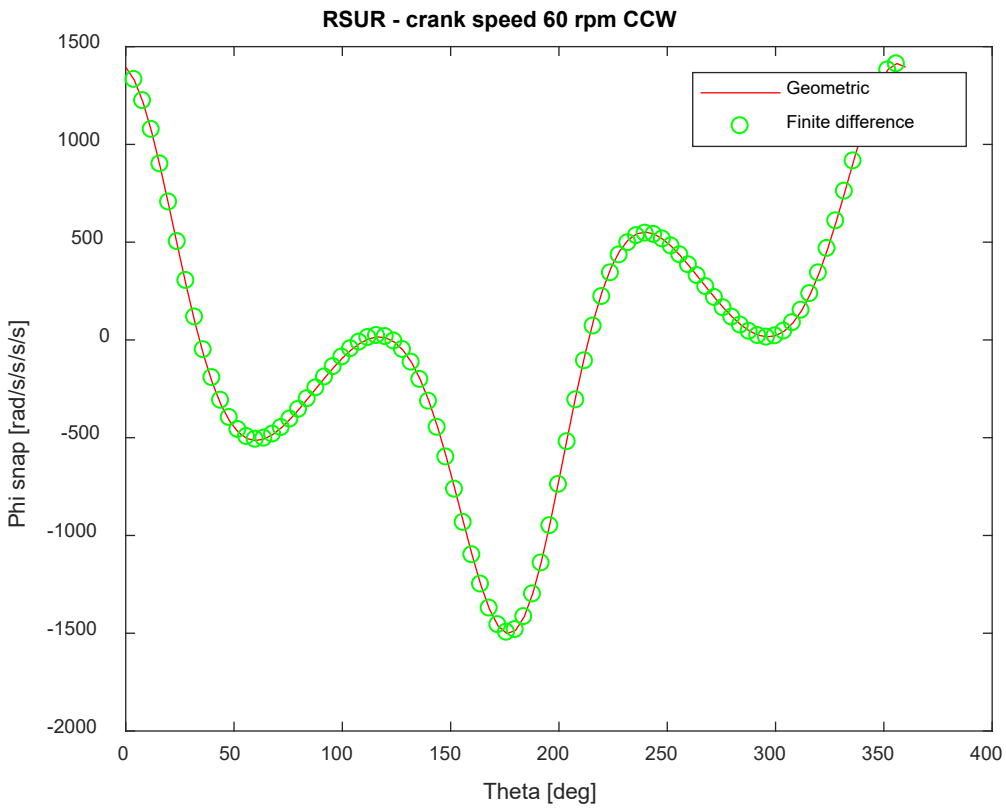
$$\dddot{\phi} = \begin{pmatrix} B\left(\dddot{\theta} - 6\dot{\theta}^2\ddot{\theta}\right) + C\left(4\dddot{\theta}\dot{\theta} + 3\ddot{\theta}^2 - \dot{\theta}^4\right) + D\left(4\dddot{\phi}\dot{\phi} + 3\ddot{\phi}^2 - \dot{\phi}^4\right) + 6A\dot{\phi}^2\ddot{\phi} \\ + E\left(-4\dddot{\theta}\dot{\phi} - 6\ddot{\theta}\ddot{\phi} + 4\dot{\theta}^3\dot{\phi} - 2\dot{\theta}\dot{\phi}^3 - 4\dot{\theta}\dddot{\phi}\right) + 6F\left(2\dot{\theta}\ddot{\theta}\dot{\phi} + \dot{\theta}^2\ddot{\phi}\right) + 6G\left(\ddot{\theta}\dot{\phi}^2 + 2\dot{\theta}\dot{\phi}\ddot{\phi}\right) - 6H\dot{\theta}^2\dot{\phi}^2 \end{pmatrix} / A$$

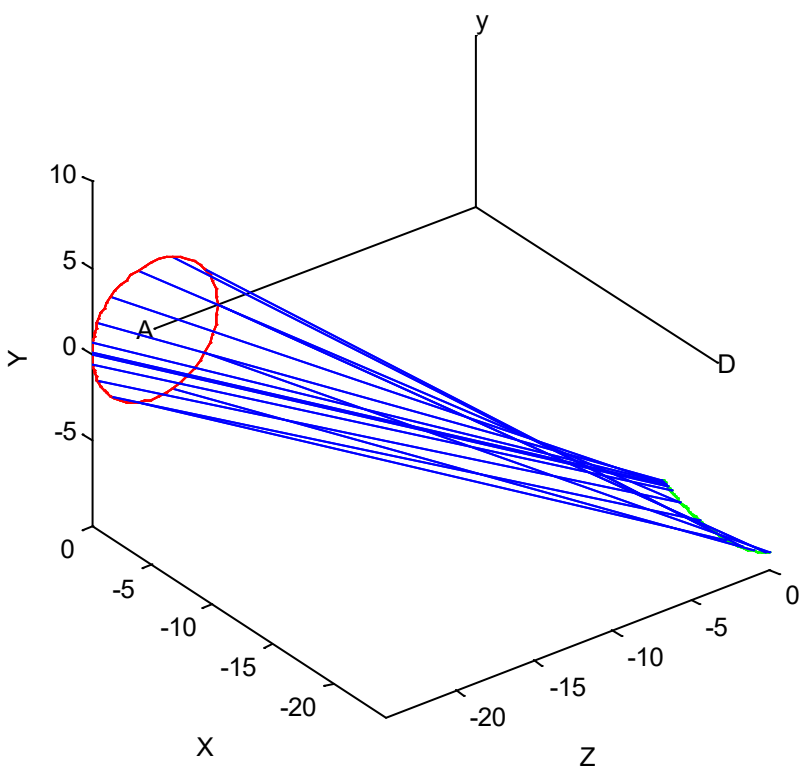

RSUR - crank speed 60 rpm CCW
Phi [deg]
Theta [deg]
-60
-70
-80
-90
-100
-110
-120
0
50
100
150
200
250
300
350


RSUR - crank speed 60 rpm CCW
Geometric
Finite difference
Phi velocity [rad/s]
Theta [deg]
3
2
1
0
-1
-2
-3
0
50
100
150
200
250
300
350
400


RSUR - crank speed 60 rpm CCW
Geometric
Finite difference
Phi acceleration [rad/s/s]
Theta [deg]
20
15
10
5
0
-5
-10
-15
-20
0
50
100
150
200
250
300
350
400


RSUR - crank speed 60 rpm CCW
Geometric
Finite difference
Phi jerk [rad/s/s/s]
Theta [deg]
150
100
50
0
-50
-100
-150
0
50
100
150
200
250
300
350
400


RSUR - crank speed 60 rpm CCW
Geometric
Finite difference
Phi snap [rad/s/s/s/s]
Theta [deg]
1500
1000
500
0
-500
-1000
-1500
-2000
0
50
100
150
200
250
300
350
400

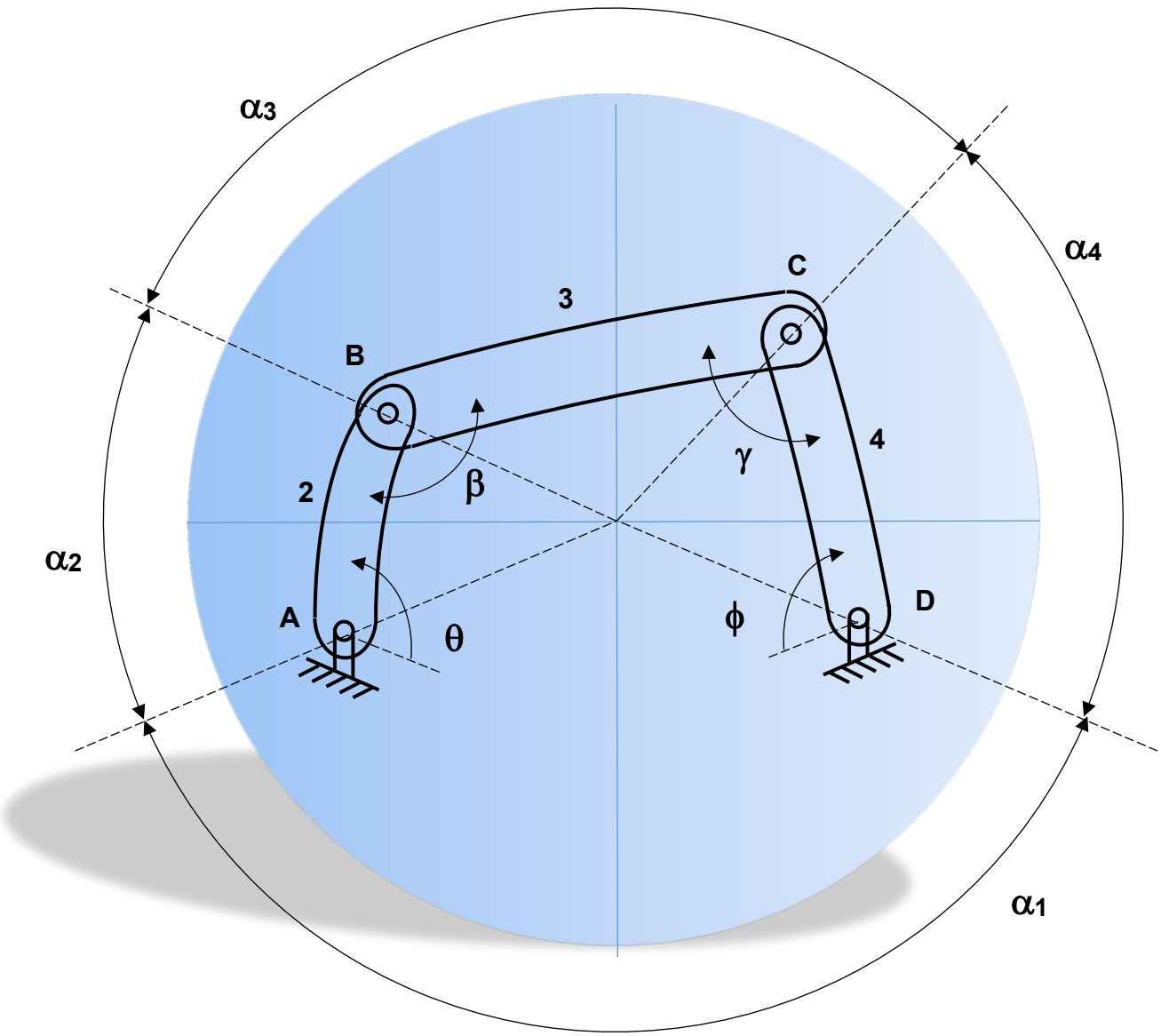

y
A
D
Y
X
Z
10
5
0
-5
0
-5
-10
-15
-20
0
-5
-10
-15
-20

```
% rsur_geometric.m - geometric solution for RSUR mechanism
% HJSIII, 19.04.13

clear

% constants
d2r = pi / 180;

% mechanism dimensions - units = [cm]
a = 20.43;
b =  4.00;
c = 10.00;
d = 19.97;
e = 30.42;

% constant 60 rpm CCW = 1 rev/sec = 2 pi rad/sec
w2 = 60 * 2*pi /60;  % rad/sec
th_start = 0;

% time for one revolution
tend = 1 / (w2/2/pi);

% 180 steps
dt = tend / 90;

% allocate space to save results
keep = [];

% time loop
for t = 0 : dt : tend;

% constant speed rotation
  th = th_start + w2*t;
  thd = w2;
  thdd = 0;
  thddd = 0;
  thdddd = 0;

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
% geometric solution

% position
  f = 2*c*d;
  g = 2*b*c*sin(th);
  h = e*e - a*a - b*b - c*c - d*d + 2*a*b*cos(th);
  u1 = (-g - sqrt( f*f + g*g - h*h )) / (h+f);
  u2 = (-g + sqrt( f*f + g*g - h*h )) / (h+f);
  phi  = 2*atan(u1);
  phi2 = 2*atan(u2);  % alternate assembly configuration

% general terms
  A =  c*d*sin(phi) +b*c*sin(th)*cos(phi);
  B =  a*b*sin(th)  -b*c*cos(th)*sin(phi);
  C =  a*b*cos(th)  +b*c*sin(th)*sin(phi);
  D = -c*d*cos(phi) +b*c*sin(th)*sin(phi);
  E =  b*c*cos(th)*cos(phi);
  F =  b*c*sin(th)*cos(phi);
  G =  b*c*cos(th)*sin(phi);
  H =  b*c*sin(th)*sin(phi);

% velocity
  phid = B * thd /A;

% acceleration
  phidd = ( B*thdd +C*thd*thd +D*phid*phid -2*E*thd*phid ) /A;

% jerk
  phiddd = ( B*(thddd-thd*thd*thd) +3*C*thd*thdd +3*D*phid*phidd ...
           +A*phid*phid*phid -3*E*(thdd*phid+thd*phidd) ...
           +3*thd*phid*(F*thd+G*phid) ) /A;

% snap
  phidddd = ( B*(thdddd-6*thd*thd*thdd) +C*(4*thddd*thd+3*thdd*thdd-thd*thd*thd*thd) ...
```

```
            +D*(4*phiddd*phid +3*phidd*phidd -phid*phid*phid*phid) +6*A*phid*phid*phidd ...
            -2*E*(2*thddd*phid +3*thdd*phidd -2*thd*thd*thd*phid ...
                 +thd*phid*phid*phid +2*thd*phiddd) ...
            +6*F*(2*thd*thdd*phid +thd*thd*phidd) ...
            +6*G*(thdd*phid*phid  +2*thd*phid*phidd) -6*H*thd*thd*phid*phid ) /A;

% save results
  keep = [ keep ; th thd thdd thddd thdddd phi phid phidd phiddd phidddd ];
end

% values for plotting
th      = keep(:,1);
th_deg  = th /d2r;
thd     = keep(:,2);
thdd    = keep(:,3);
thddd   = keep(:,4);
thdddd  = keep(:,5);

phi     = keep(:,6);
phi_deg = phi /d2r;
phid    = keep(:,7);
phidd   = keep(:,8);
phiddd  = keep(:,9);
phidddd = keep(:,10);

% simple finite difference approximations
phid_fd    = ( [diff(phi);NaN]    + [NaN;diff(phi)] )    /2 /dt;
phidd_fd   = ( [diff(phid);NaN]   + [NaN;diff(phid)] )   /2 /dt;
phiddd_fd  = ( [diff(phidd);NaN]  + [NaN;diff(phidd)] )  /2 /dt;
phidddd_fd = ( [diff(phiddd);NaN] + [NaN;diff(phiddd)] ) /2 /dt;

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
% plot position solution
figure( 1 )
  clf
  plot( th_deg,phi_deg,'r' )
  title( 'RSUR - crank speed 60 rpm CCW' )
  xlabel( 'Theta [deg]' )
  ylabel( 'Phi [deg]' )
  axis( [ 0 360  -120 -60 ] )

% plot velocity solution
figure( 2 )
  clf
  plot( th_deg,phid,'r', th_deg,phid_fd,'go' )
  title( 'RSUR - crank speed 60 rpm CCW' )
  xlabel( 'Theta [deg]' )
  ylabel( 'Phi velocity [rad/s]' )
%  axis( [ 0 360  -3 3 ] )
  legend( 'Geometric', 'Finite difference' )

% plot acceleration solution
figure( 3 )
  clf
  plot( th_deg,phidd,'r', th_deg,phidd_fd,'go' )
  title( 'RSUR - crank speed 60 rpm CCW' )
  xlabel( 'Theta [deg]' )
  ylabel( 'Phi acceleration [rad/s/s]' )
%  axis( [ 0 360  -20 20 ] )
  legend( 'Geometric', 'Finite difference' )

% plot jerk solution
figure( 4 )
  clf
  plot( th_deg,phiddd,'r', th_deg,phiddd_fd,'go' )
  title( 'RSUR - crank speed 60 rpm CCW' )
  xlabel( 'Theta [deg]' )
  ylabel( 'Phi jerk [rad/s/s/s]' )
%  axis( [ 0 360  -150 150 ] )
  legend( 'Geometric', 'Finite difference' )

% plot snap solution
figure( 5 )
```

```
  clf
  plot( th_deg,phidddd,'r', th_deg,phidddd_fd,'go' )
  title( 'RSUR - crank speed 60 rpm CCW' )
  xlabel( 'Theta [deg]' )
  ylabel( 'Phi snap [rad/s/s/s/s]' )
%  axis( [ 0 360  -150 150 ] )
  legend( 'Geometric', 'Finite difference' )

% 3D sketch
n = length( th );
rA = [ 0             0           a           ] ;
rB = [ zeros(n,1)    b*sin(th)   a-b*cos(th) ] ;
rC = [ d+c*cos(phi)  c*sin(phi)  zeros(n,1)  ] ;
rD = [ d             0           0           ] ;
figure( 6 )
  clf
  plot3( -rB(:,3), -rB(:,1), rB(:,2), 'r' )
  hold on
  plot3( -rC(:,3), -rC(:,1), rC(:,2), 'g' )
  xlabel( 'Z' )
  ylabel( 'X' )
  zlabel( 'Y' )

  ax = [ 0   0   0   ;
         rD          ;  % X axis
         NaN NaN NaN ;
         0   0   0   ;
         0   10  0   ;  % Y axis
         NaN NaN NaN ;
         0   0   0   ;
         rA          ]; % Z axis
  plot3( -ax(:,3), -ax(:,1), ax(:,2), 'k' )
  text(  -ax(2,3), -ax(2,1), ax(2,2), 'D' )
  text(  -ax(5,3), -ax(5,1), ax(5,2), 'y', 'VerticalAlignment', 'bottom' )
  text(  -ax(8,3), -ax(8,1), ax(8,2), 'A', 'HorizontalAlignment', 'right' )

  axis equal

  nskip = 10;
  for i = 1 : nskip : n,
    plot3( [ -rB(i,3) -rC(i,3) ], [ -rB(i,1) -rC(i,1) ], [ rB(i,2) rC(i,2) ], 'b' )
  end

% swivel angle for S joint
sCwrtB = rC - rB;
sBwrtA = rB - ones(n,1)*rA;
swivel_ang = 90 - acos( diag( sBwrtA*sCwrtB' )/b/e )/d2r;


% bottom of rsur_geometric
```

Spherical Four Bar Geometric



revolutes A B C D all intersect at same point

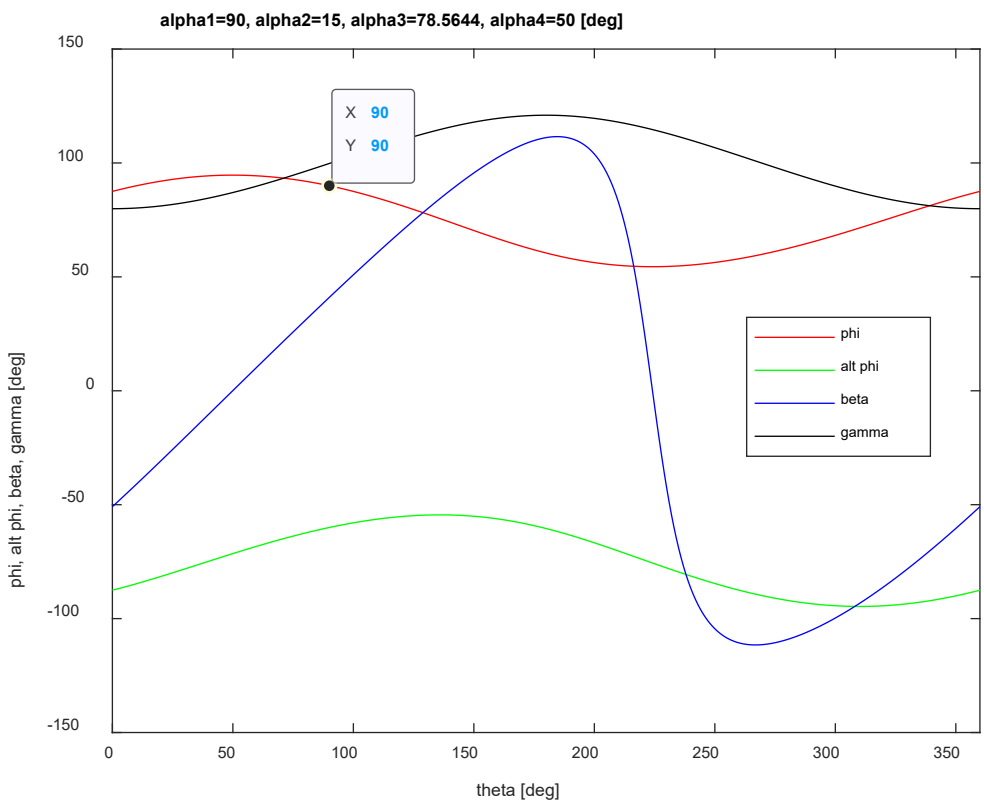
α_i = angle subtended by link i

θ = angle from plane for revolutes A D to center line for link 2
ϕ = angle from plane for revolutes A D to center line for link 4
β = angle from center line for link 2 to center line for link 3
γ = angle from center line for link 3 to center line for link 4

tangent of half-angle solution

$$U = \sin\alpha_2 \sin\alpha_4 \sin\theta$$
$$V = \cos\alpha_2 \sin\alpha_4 \sin\alpha_1 - \sin\alpha_2 \sin\alpha_4 \cos\alpha_1 \cos\theta$$
$$W = \sin\alpha_2 \cos\alpha_4 \sin\alpha_1 \cos\theta + \cos\alpha_2 \cos\alpha_4 \cos\alpha_1 - \cos\alpha_3$$
$$\phi = 2\,\mathrm{a\,tan}\left(\left(-U \pm \sqrt{U^2 + V^2 - W^2}\right) / (W - V)\right)$$

test values produce phi = 90 deg when theta = 90 deg




```
% t_sfb.m - test spherical four bar
% HJSIII, 20.04.28

clear

% constatnts
d2r = pi / 180;

% mechanism constants
r = 4;           % radius [inch]
a1 = 90 * d2r;
a2 = 15 * d2r;
a3 = 78.5644 * d2r;  % calculated for a1=90, theta=90, phi=90
a4 = 50 * d2r;

% test for a1 = 90 deg
theta = 90 * d2r;
phi = 90 * d2r;
```

```
rA = [ 0  0  r ]';
rB = [ 0  r*sin(a2)  r*cos(a2) ]';
rC = [ r*cos(a4)  r*sin(a4)  0 ]';
rD = [ r  0  0 ]';

dBC = norm( rC - rB );
a3_calc1 = acos( 1 - dBC^2/(2*r*r) );
a3_calc2 = 2 * asin( dBC /2 /r );

% rotate theta and calculate phi
theta = ( 0 : 360 )' * d2r;
U = sin(a2)*sin(a4)*sin(theta);
V = cos(a2)*sin(a4)*sin(a1) - sin(a2)*sin(a4)*cos(a1)*cos(theta);
W = sin(a2)*cos(a4)*sin(a1)*cos(theta) + cos(a2)*cos(a4)*cos(a1) - cos(a3);
phi     = 2 * atan( (-U - sqrt( U.*U + V.*V - W.*W )) ./ (W-V) );
phi_alt = 2 * atan( (-U + sqrt( U.*U + V.*V - W.*W )) ./ (W-V) );

% 2002 Cervantes-Sanchez - beta ????
beta  = acos( (sin(a4)*sin(a1)*cos(phi)   + cos(a4)*cos(a1) - cos(a2)*cos(a3)) / (sin(a2)*sin(a3)) );
gamma = acos( (sin(a2)*sin(a1)*cos(theta) + cos(a2)*cos(a1) - cos(a3)*cos(a4)) / (sin(a3)*sin(a4)) );

% 2018 Sun - theta2 = beta, theta4 = phi
X1 = sin(a2)*sin(theta);
Y1 = -sin(a1)*cos(a2) + cos(a1)*sin(a2)*cos(theta);
Z1 = cos(a1)*cos(a2) + sin(a1)*sin(a2)*cos(theta);
theta4 = 2*atan2( Z1*cos(a4)-Y1*sin(a4)-cos(a3) , ...
                 -X1*sin(a4)+sqrt( (sin(a4)*sin(a3))^2 - (Z1-cos(a4)*cos(a3)).^2 ) );

X4 = sin(a4)*sin(theta4);
Y4 = -sin(a1)*cos(a4) - cos(a1)*sin(a4)*cos(theta4);
Z4 = cos(a1)*cos(a4) - sin(a1)*sin(a4)*cos(theta4);
theta2 = pi - 2*atan2( cos(a2)*(X4.*sin(theta)-Y4.*cos(theta)) - Z4*sin(a2) + sin(a3) , ...
                      -X4.*cos(theta) - Y4.*sin(theta) );
beta = theta2;

% show results
figure( 1 )
  clf
  plot( theta/d2r,phi/d2r,'r',  theta/d2r,phi_alt/d2r,'g', ...
        theta/d2r,beta/d2r,'b', theta/d2r,gamma/d2r,'k'        )
  xlabel( 'theta [deg]' )
  ylabel( 'phi, alt phi, beta, gamma [deg]' )
  title( 'alpha1=90, alpha2=15, alpha3=78.5644, alpha4=50 [deg]' )
  axis( [ 0 360  -150 150 ] )
  legend( 'phi', 'alt phi', 'beta', 'gamma' )

% bottom - t_sfb
```