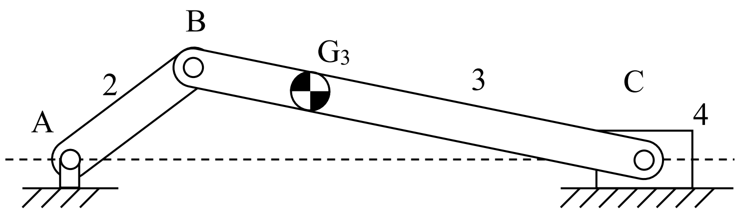
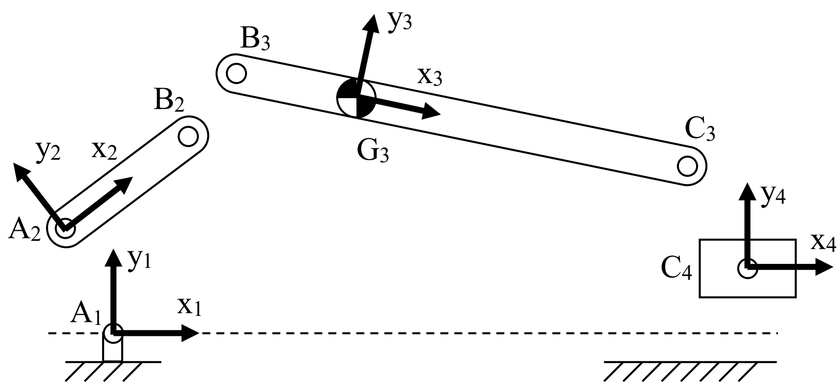
ME 481 - H06 **Name ________________________**

Use generalized coordinates {q} and joint constraints {Φ} for the slider crank shown below.

$$\{q\} = \begin{Bmatrix} x_2 \\ y_2 \\ \phi_2 \\ x_3 \\ y_3 \\ \phi_3 \\ x_4 \\ y_4 \\ \phi_4 \end{Bmatrix} = \begin{Bmatrix} \{r_2\} \\ \phi_2 \\ \{r_3\} \\ \phi_3 \\ \{r_4\} \\ \phi_4 \end{Bmatrix} \qquad \{\Phi\} = \begin{Bmatrix} \{r_2\}^A - \{r_1\}^A \\ \{r_3\}^B - \{r_2\}^B \\ \{r_4\}^C - \{r_3\}^C \\ \phi_4 \\ y_4 \\ \phi_2 - \omega_2 t \end{Bmatrix}$$



AB = R = 0.985 inch
BC = L = 4.33 inch
BG_3 = 1.1 inch
G_2 is at A_2 (balanced crank)
G_3 is on centerline of link 3
G_4 is at C_4 (simple piston model)
x_2 axis along centerline of link 2
x_3 axis along centerline of link 3
ω_2 = 1000 rpm CCW constant



BLUEPRINT INFORMATION

$$\{s_2\}'^A = \begin{Bmatrix} 0 \\ 0 \end{Bmatrix} \qquad \{s_2\}'^B = \begin{Bmatrix} R \\ 0 \end{Bmatrix} \qquad \{s_3\}'^B = \begin{Bmatrix} -BG_3 \\ 0 \end{Bmatrix} \qquad \{s_3\}'^C = \begin{Bmatrix} L - BG_3 \\ 0 \end{Bmatrix}$$

$$\{s_4\}'^C = \begin{Bmatrix} 0 \\ 0 \end{Bmatrix} \qquad \{r_1\}^A = \begin{Bmatrix} 0 \\ 0 \end{Bmatrix}$$

example for B3 $\quad \{r_3\}^B = \{r_3\} + [A_3]\{s_3\}'^B \qquad [A_3] = \begin{bmatrix} \cos\phi_3 & -\sin\phi_3 \\ \sin\phi_3 & \cos\phi_3 \end{bmatrix}$

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1) Evaluate residuals $\{\Phi\}$ for rough estimates of generalized coordinates $\{q\}$ at t = 0.005 sec shown below. Comment on the relative precision of $\{\Phi\}$ versus $\{q\}$.

$$\{q\} = \begin{Bmatrix} 0 \text{ inch} \\ 0 \text{ inch} \\ 0.4363 \text{ rad } (25°) \\ 2.0 \text{ inch} \\ 0.5 \text{ inch} \\ -0.1745 \text{ rad } (-10°) \\ 5.0 \text{ inch} \\ 0 \text{ inch} \\ 0 \text{ rad } (0°) \end{Bmatrix} \qquad \{\Phi\} = \begin{Bmatrix} \_\_\_\_\_\_\_\_ \\ \_\_\_\_\_\_\_\_ \\ \_\_\_\_\_\_\_\_ \\ \_\_\_\_\_\_\_\_ \\ \_\_\_\_\_\_\_\_ \\ \_\_\_\_\_\_\_\_ \\ \_\_\_\_\_\_\_\_ \\ \_\_\_\_\_\_\_\_ \\ \_\_\_\_\_\_\_\_ \end{Bmatrix}$$

2) Use geometric equations to determine better estimates for $\{q\}$ at time t = 0.005 sec. Then evaluate new residuals. Comment on precision of $\{q\}$ and $\{\Phi\}$ between parts 1) and 2).

$$\{q\} = \begin{Bmatrix} 0 \text{ inch} \\ 0 \text{ inch} \\ \omega_2 t = \_\_\_\_\_\_ \\ \_\_\_\_\_\_\_\_ \\ \_\_\_\_\_\_\_\_ \\ \_\_\_\_\_\_\_\_ \\ \_\_\_\_\_\_\_\_ \\ 0 \text{ inch} \\ 0 \text{ rad } (0°) \end{Bmatrix} \qquad \{\Phi\} = \begin{Bmatrix} \_\_\_\_\_\_\_\_ \\ \_\_\_\_\_\_\_\_ \\ \_\_\_\_\_\_\_\_ \\ \_\_\_\_\_\_\_\_ \\ \_\_\_\_\_\_\_\_ \\ \_\_\_\_\_\_\_\_ \\ \_\_\_\_\_\_\_\_ \\ \_\_\_\_\_\_\_\_ \\ \_\_\_\_\_\_\_\_ \end{Bmatrix}$$

3) Evaluate the Jacobian $\lfloor \Phi_q \rfloor$ for your better estimate of $\{q\}$ at t = 0.005 sec.

$$[\Phi_q] = \begin{bmatrix} \_\_\_ & \_\_\_ & \_\_\_ & \_\_\_ & \_\_\_ & \_\_\_ & \_\_\_ & \_\_\_ & \_\_\_ \\ \_\_\_ & \_\_\_ & \_\_\_ & \_\_\_ & \_\_\_ & \_\_\_ & \_\_\_ & \_\_\_ & \_\_\_ \\ \_\_\_ & \_\_\_ & \_\_\_ & \_\_\_ & \_\_\_ & \_\_\_ & \_\_\_ & \_\_\_ & \_\_\_ \\ \_\_\_ & \_\_\_ & \_\_\_ & \_\_\_ & \_\_\_ & \_\_\_ & \_\_\_ & \_\_\_ & \_\_\_ \\ \_\_\_ & \_\_\_ & \_\_\_ & \_\_\_ & \_\_\_ & \_\_\_ & \_\_\_ & \_\_\_ & \_\_\_ \\ \_\_\_ & \_\_\_ & \_\_\_ & \_\_\_ & \_\_\_ & \_\_\_ & \_\_\_ & \_\_\_ & \_\_\_ \\ \_\_\_ & \_\_\_ & \_\_\_ & \_\_\_ & \_\_\_ & \_\_\_ & \_\_\_ & \_\_\_ & \_\_\_ \\ \_\_\_ & \_\_\_ & \_\_\_ & \_\_\_ & \_\_\_ & \_\_\_ & \_\_\_ & \_\_\_ & \_\_\_ \\ \_\_\_ & \_\_\_ & \_\_\_ & \_\_\_ & \_\_\_ & \_\_\_ & \_\_\_ & \_\_\_ & \_\_\_ \end{bmatrix}$$

4) Use your code to perform a Newton-Raphson position solution at t = 0.010 sec. Calculate piston position x_4 and determinant of the Jacobian. Validate with geometric equations.

x_4 (Newton-Raphson) ____________ $\det\left[\Phi_q\right]$ ____________

x_4 (geometric) ____________

5) Compute piston velocity $\dot{x}_4$ and acceleration $\ddot{x}_4$ at t = 0.010 sec using a matrix solution with right-hand-side (RHS) vectors $\{\nu\}$ and $\{\gamma\}$. Validate with geometric equations.

$\dot{x}_4$ (matrix) ____________ $\ddot{x}_4$ (matrix) ____________

$\dot{x}_4$ (geometric) ____________ $\ddot{x}_4$ (geometric) ____________

EXTRA CREDIT
Place a loop around your solution for part 5) using $0 \le t \le 0.06$ sec and provide MATLAB graphs for piston position x_4, velocity $\dot{x}_4$ and acceleration $\ddot{x}_4$ as functions of crank angle ϕ_2. Validate using results from geometric equations on the same MATLAB graphs.

EXTRA EXTRA CREDIT
Modify your slider crank code for part 5) to analyze the four bar in Notes_04_05. This should only require modifying the last three rows in your constraint vector, your Jacobian matrix and your acceleration RHS vector.

use	$\phi_2 = 65°$	$\dot{\phi}_2 = 10$ rad/sec CW	$\ddot{\phi}_2 = 2$ rad/sec^2 CCW
validation	$\phi_3 = 13.151°$	$\dot{\phi}_3 =$ _______________	$\ddot{\phi}_3 = +7.0627$ rad/sec^2
	$\phi_4 = -65.173°$	$\dot{\phi}_4 = -5.3533$ rad/sec	$\ddot{\phi}_4 =$ _______________