**Two-Dimensional Experimental Kinematics**

Digitize locations of landmarks  on body i for points k=1 to n at given time t

All points must be attached to body i

Use landmark weighting factor  if point k is available at time t. Use  if point k not available at given time t.

Determine  at given time t.

**Mean values**













**Velocity**





Point ICR is the instantaneous center of rotation for body i with respect to the ground. Note that the ICR is not attached to the body. Point P on the body coincident with ICR has zero velocity.

 for any point P attached to body i



**Acceleration**





Point IAP is the instantaneous acceleration pole for body i. Note that the IAP is not attached to the body. Point P on the body coincident with IAP has zero acceleration.

 for any point P attached to body i

 OLD

**Jerk**





Point IJP is the instantaneous jerk pole for the body. Note that the IJP is not attached to the body. Point P on the body coincident with IJP has zero jerk.

 for any point P attached to the body



**Snap**







Point ISP is the instantaneous snap pole for the body. Note that the ISP is not attached to the body. Point P on the body coincident with ISP has zero snap.

 for any point P attached to the body



**Centrode**

Location of the ICR measured relative to ground changes as body i moves and sweeps a locus called the fixed centrode for body i. The time derivative of the locus describes how the ICR moves. Tracking the location of the ICR relative to a coordinate frame fixed to body i provides a locus called the moving centrode. Motion of body i may be characterized as pure rolling of the moving centrode on the fixed centrode because body i instantaneously has zero velocity at each location of the ICR.



The second time derivative of the location of the ICR also changes as body i moves. First and second time derivatives of position along a locus may be combined to determine curvature  of the fixed centrode. If body i is part of a mechanism with mobility of one, curvature of the centrode at each location will be invariant to speed of the mechanism.





**Relative velocity**

For planar motion, the relative angular velocity of body j with respect to body i is the difference between the two angular velocities. The relative instantaneous center of rotation (RICR) for body j about body i describes a unique point that has zero relative velocity between the two bodies. Note that location of the RICR is measured with respect to the ground.





**Rigid Body**

Determine the velocity of point C on rigid body link 3. The rigid body and the velocity vectors are drawn to scale. Link 3 is NOT pinned to the ground. Show your work.

C

Xc = 27 mm

Yc = 121 mm

VB = 5.7 cm/sec

20º

B

XB = 90 mm

YB = 80 mm

3

XA = 43 mm

YA = 56 mm

A

25º

VA = 9.2 cm/sec

V3B\_wrt\_A



3

B



A



A

C

V3C\_wrt\_A

3





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VA3 = 9.2 cm/sec

point A = P1 fP1 = 1  from above

point B = P2 fP2 = 1 

use lm2kin2d per attached code

3 = +1.2422 rad/sec mm

% rbk.m - rigid body kinematics

% HJSIII, 14.01.13

clear

% constants

Rmat = [ 0 -1 ; 1 0 ];

% inputs

f = [ 1 1 ];

r = [ 43 90 ;

56 80 ];

rd = [ 83.38 53.56 ;

-38.88 19.5 ];

rdd = zeros(2,2);

rddd = zeros(2,2);

% call function

[ w, rICR, wd, rIAP, wdd, rIJP, rdICR, kappa ] = lm2kin2d( f, r, rd, rdd, rddd );

w

rICR

% find velocity of C

r3C = [ 27 121 ]';

r3Cd = w \* Rmat \* ( r3C - rICR )

% bottom of rbk

% t\_lm2kin2d.m - test 2D kinematics from landmark motion

% HJSIII, 14.01.13

clear

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

% example inputs - web cutter four bar

% Haug page 197 - not ME 581 web cutter

% B3 C3

f = [ 1 1 ];

r = [ 3.7588 3.9407 ;

1.3681 29.3675 ];

rd = [ -5.4874 22.5296 ;

15.0764 14.8943 ];

rdd = [ -60.4716 -42.8075 ;

-22.0098 -50.1604 ];

rddd = [ 88.2815 -673.2083 ;

-242.5514 -291.1768 ];

% expected outputs

w\_test = -1.0006;

rICR\_test = [ 18.8257 ; 6.8520 ];

wd\_test = -0.6374;

rIAP\_test = [ -49.1784 ; 13.0846 ];

wdd\_test = 26.1823;

rIJP\_test = [ 12.8647 ; 3.9747 ];

rdICR\_test = [ -37.0807 ; 72.0169 ];

kappa\_test = -0.0151 ;

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

% test function

[ w, rICR, wd, rIAP, wdd, rIJP, rdICR, kappa ] = lm2kin2d( f, r, rd, rdd, rddd );

w

rICR

wd

rIAP

wdd

rIJP

rdICR

kappa

% bottom of t\_lm2kin2d

function [ w, rICR, wd, rIAP, wdd, rIJP, rdICR, kappa ] = lm2kin2d( f, r, rd, rdd, rddd )

% 2D instantaneous kinematics of a rigid body from landmark motion

% HJSIII, 14.01.13

%

% USAGE

% function [ w, rICR, wd, rIAP, wdd, rIJP, rdICR, kappa ] = lm2kin2d( f, r, rd, rdd, rddd )

% INPUTS

% f - 1xn vector of weights - f(j)=1 means data valid, f(j)=0 means data not available

% r - 2xn matrix of x,y landmark location

% rd - 2xn matrix of x,y landmark velocity

% rdd - 2xn matrix of x,y landmark acceleration

% rddd - 2xn matrix of x,y landmark jerk

%

% OUTPUTS

% rICR - 2x1 location of instantaneous center of rotation

% w - angular velocity

% rIAP - 2x1 location of instantaneous acceleration pole

% wd - angular acceleration

% rIJP - 2x1 location of instantaneous jerk pole

% wdd - angular jerk

% rdICR - 2x1 time derivative of location of instantaneous center

% kappa - curvature of centrode

% constants

Rmat = [ 0 -1 ; 1 0 ];

% mean values

[ nr, n ] = size( r );

fmat = diag( f );

sf = sum( f' );

rm = sum( fmat\*r' )' /sf;

rdm = sum( fmat\*rd' )' /sf;

rddm = sum( fmat\*rdd' )' /sf;

rdddm = sum( fmat\*rddd' )' /sf;

% centered location

rc = r - rm\*ones(1,n);

S = trace( rc \* fmat \* rc' );

% velocity

vmat = rd \* fmat \* rc';

w = ( vmat(2,1) - vmat(1,2) ) /S;

wsk = w \* Rmat;

rICR = rm - inv(wsk) \* rdm;

% acceleration

amat = rdd \* fmat \* rc';

wd = ( amat(2,1) - amat(1,2) ) /S;

wdsk = wd \* Rmat;

beta = wdsk + wsk\*wsk;

rIAP = rm - inv(beta) \* rddm;

% jerk

jmat = rddd \* fmat \* rc';

wdd = w\*w\*w + ( jmat(2,1) - jmat(1,2) ) /S;

wddsk = wdd \* Rmat;

eta = wddsk + 3\*wsk\*wdsk + wsk\*wsk\*wsk;

rIJP = rm - inv(eta) \* rdddm;

% snap

%rddddm = sum( fmat\*rdddd' )' /sf;

%smat = rdddd \* fmat \* rc';

%wddd = 6\*w\*w\*wd + ( smat(2,1) - smat(1,2) ) /S;

%wdddsk = wddd \* Rmat;

%sigma = wdddsk + 6\*wsk\*wsk\*wdsk + 4\*wsk\*wddsk + 3\*wdsk\*wdsk + wsk\*wsk\*wsk\*wsk;

%rISP = rm - inv(sigma) \* rddddm;

% centrode

rdICR = ( wsk\*rddm - beta\*rdm ) /w/w;

nrdICR = norm( rdICR );

sk1 = [ 0 (w\*wdd-2\*wd\*wd) ; -(w\*wdd-2\*wd\*wd) 0 ];

sk2 = [ w\*w\*w 2\*w\*wd ; -2\*w\*wd w\*w\*w ];

sk3 = [ 0 -w\*w ; w\*w 0 ];

rddICR = ( sk1\*rdm + sk2\*rddm + sk3\*rdddm ) /w/w/w;

kappa = ( rdICR(1)\*rddICR(2) - rdICR(2)\*rddICR(1) ) /nrdICR/nrdICR/nrdICR;

% bottom of lm2kin2d