**Two-Dimensional Inverse Dynamics**

**Kinematically driven**

**Must use centroidal coordinate frames !**









**Single body**





**System of multiple bodies**



















**Virtual work**







 for kinematic consistency 

 subject to 

**Virtual work for one revolute**

















 **OK**

**Lagrange multiplier theorem**

general problem  subject to 

 using Lagrange multipliers  for any arbitrary 

virtual work  subject to 



 for arbitrary size but kinematically consistent





 each row in  is multiplied times corresponding column in 

each row in  corresponds to matching row in  and 

**Lagrange multipliers**











**Equations of motion (EOM)**







nq = number of generalized coordinates

nk = number of kinematic constraints

nd = number of driver constraints

nc = total number of constraints (nc = nk + nd)

**Inverse dynamics – kinematically driven**

solve kinematics   must have full rank nc = nq

compute constraint forces 



**Inverse dynamics – simultaneous EOM matrix**







**Statics**







**Lagrange multipliers for specific constraints**









**Revolute**

** Note:** Haug uses ****









**check body i**  **OK**

**check body j**  **OK**

**Double revolute**

****









**Parallel vectors (planar parallel-1)**











**Pin-in-slot (planar parallel-2)**











**Relative angle driver**

****









**Gear pair (chain/sprockets, belt/pulleys)**

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**Gear pair on rotating link k**

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