Gyroscope System

Introduction
For this lab, there are three experiments, each with separate sub-parts to perform. For experiment 1, the goal is to determine 3 unknown inertias (the inertia of Body C around the Y axis is denoted $J_C$, Body A around the Z axis has inertia $K_A$, and Body C around the X axis has inertia $I_C$). Experiment 2 consists of finding the hardware gains of the system $k_{g1}$ and $k_{g2}$. Finally, Experiment 3 illustrates the unique properties of gyroscopes, specifically nutation and precession.

![Figure 1: ECP Control Moment Gyro Experiment](image)

Hardware
The Control Moment Gyro Experiment consists of the CMG mechanism and its actuators and sensors. The design features two high torque density (rare earth magnet type) DC servo motors for control effort transmission, high resolution encoders for gimbal (rotating ring) angle feedback, and low friction slip rings for signal and motor power transmission across all gimbals. It also includes inertial switches for high gimbal speed detection and
safety shutdown and electromechanical brakes to facilitate changing dynamic degrees of freedom as well as securing the system during safety shutdown.

The plant, shown in Figure 1, consists of a high inertia brass rotor suspended in an assembly with four angular degrees of freedom. The rotor spin torque is provided by a rare earth magnet type DC motor (motor#1) whose angular position is measured by a 2000 count per revolution optical encoder (encoder #1). The motor drives the rotor through a 3.33:1 reduction ratio that amplifies both the torque and encoder resolution by this factor. In this laboratory, the other degrees of freedom will be locked using the electromagnetic brake switches on the control box. Thus, the system has a single motor/encoder for position and speed control, the most frequently observed applications in practice.

**Safety**

When implementing a controller, use a ruler or other non-sharp object to nudge the various elements of the system to verify that there is no unstable control condition and that the system is safe to manipulate. If there is an instability or large control signal, **immediately abort the control**. In these experiments, do not move axes 3 and 4 while their brakes are engaged. This leads to premature wear out of the brakes.

**Hardware/Software Equipment Check**

Before starting the lab, confirm that the hardware is working by performing the following steps:

**Step 1:** With power switched off to the Control Box, enter the ECP program by double clicking on its icon. One should see the Background Screen at this point. Gently rotate the inner gimbal ring (the one that encloses the brass rotor). Observe changing readings in the Encoder 2 position and possibly small changes in the Encoder 1 (rotor) position. The Control Loop Status should indicate "OPEN" and the Motor 1 Status, Motor 2 Status, and Servo Time Limit should all indicate "OK".

**Step 2:** Now press the black "ON" button to turn on the power to the Control Box. Notice that the green power indicator LED is lit, but the motors should remain in a disabled state. Turn off the Axis 3 and 4 Brakes via the toggle switches on the Control Box and **safety check** the controller as per the instructions in Appendix B on the course website. Now move Axes 3 and 4. Observe the corresponding Encoder position values change on the Background Screen. Encoder 1 position will usually change as one moves axis 3.
Experiment 1: System Identification

In the following tests, students will measure three moments of inertias using the principle of conservation of angular momentum. The other inertia values are provided, which are more easily determined by mass and geometry properties of the parts.

Read this section before beginning the procedure below. This will clearly explain which axis corresponds to which inertia in the procedures. Follow the procedures to determine the moment of inertia about the specified axis, and thus complete Table 1.2.

This part of the lab will use conservation of angular momentum to determine a moment of inertia with the following equation:

\[ J_1 \omega_{1o} + J_2 \omega_{2o} = J_1 \omega_{1f} + J_2 \omega_{2f} \]  \hspace{1cm} (A)

where \( J_1 \) and \( J_2 \) are the inertias of two bodies rotating about a common axis, and the subscripts “o” and “f” denote two time instants, say original and final. Equation (A) is valid only if there is no external moments acting on the system of the two moving bodies.

Figure 2: Coordinate System Orientations
Figure 2 above is the orientation setting with which inertias of various bodies are defined. In the figure, four bodies are labeled, A, B, C, and D. Also, three coordinate directions are labeled, 1, 2 and 3 for each body: the inertias of each body about its respective axes 1, 2, and 3 are further named I, J and K. An incomplete table of I, J and K values is given on the next page with unknowns to be filled in after the completion of the experiment.

In the first part of the lab, students are expected to determine inertia $J_C$, which is the inertia of body C about coordinate-direction 2 defined in Fig. 2. In this procedure, one may assume that body D rotates as one piece and Bodies C and B together rotate as one piece. Both rotate about coordinate-direction 2 or axis J. The inertia of Body D about direction 2 is given in Table 1-2 as $J_D$. The inertia of Body B about the same axis is also given in the table as $J_B$. Now, if one substitutes $J_D$ as $J_1$ in Eq. (A), then $J_2$ in the same equation would be equal to $J_B + J_C$ in this case. Thus, one can determine $J_C$ using the equation along with the measurement data of the two angular velocities in this equation.

Subsequently, students will repeat the work above for two similar experiments to determine $K_A$ and $I_C$ and enter the results in Table 1-2 below. There are step-by-step procedures that follow that explain the process in detail. Remember that as you change the machine configuration, the axes 1, 2, and 3 attached to each body A, B, C, and D rotate with a body as you change its orientation for each experiment.

Note that the measured sensor counts must be converted to radians in the calculations. Please see Table 1-1 below for encoder gains and divide the experimental values by $k_{ei}$ to obtain motion data in radians. Also, if the calculations give any negative inertia, there is a very strong possibility that one has used an incorrect definition in the inertias of equation (A).

<table>
<thead>
<tr>
<th>Table 1-1: Encoder Gain Measurements</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Axis Number (Encoder Number)</strong></td>
</tr>
<tr>
<td>1</td>
</tr>
<tr>
<td>2</td>
</tr>
<tr>
<td>3</td>
</tr>
<tr>
<td>4</td>
</tr>
</tbody>
</table>

**Experiment 1a: Determination of $J_C$**

In this part of Experiment 1, the inertia of Body C will be found experimentally.

1. Turn on the hardware and software. Start the ECP program. Select Abort Control on the Background Screen to disable any controller that may be still running on the DSP board from a previous user. Turn on power to the Control Box.

2. Setup the mechanism as shown in Figure 1-2a. The Axis 3 and 4 brakes are turned on and off via toggle switches on the Control Box. The Axis
Virtual Brake is engaged via a button on the Executive Program background screen. In this and all subsequent experiments, do not move axes 3 and 4 while their brakes are engaged. This leads to wear of the brakes. Select Zero Position (Command menu) to zero the incremental encoder values at these gimbal positions.

![Figure 1-2. Configurations For Moment of Inertia Tests](image)

3. Prepare to write a simple real-time algorithm to activate Motor #1 as illustrated below:

```plaintext
begin
cancel_effort1 = cmd1_pos/32
end
```

4. Enter Setup Control Algorithm via the Setup menu. Select Edit Algorithm. This opens up the control algorithm editor. Type (or load from a disk file) the above algorithm in the editor window. Then click File on the menu bar and choose Save changes and quit. Click Implement Algorithm thereafter. Talk to TA if the above steps do not work.

5. Set up the input and data acquisition. Go to Trajectory 1 Configuration under the Setup menu and deselect Unidirectional Moves. Enter Impulse and specify a Amplitude of 16000 counts, a Pulse Width of 1000 ms, a Dwell Time of 0 ms, and 2 repetitions (this prepares the controller board to input a 16000 count positive-going step followed immediately by a 16000 count negative-going one.) Click OK, and then enter Setup Data.
Acquisition (Setup menu). Specify Encoder 1 Position and Encoder 3 Position as data to be acquired with a Sample Period of one (1) servo cycle.

6. Run the test. Go to Execute in Command menu and verify that the apparatus is in the configuration of Figure 6.1-2a. Select Normal Data Sampling and Execute Trajectory 1 and then Run. One should see the rotor spin up then slow down while the inner assembly (bodies B, C, and D) rotates about Axis 3. Is the motion of the rotor and that of the inner assembly in the same direction or opposite directions? Select OK once the data has been uploaded.

7a. Plot on screen to see Encoder 1 Velocity and Encoder 3 Velocity in the same plot.

7b. Export the data to save in a file using ‘export raw data’ in the data menu. Use the matlab program, plotdata.m, on the class website to plot the two velocities vs. time (plot key iplot=2). Adjust the ‘istep’ number in the program so that the slope in the plot looks reasonably smooth. You may use Data Curser tool to determine the velocity values at a given time moment.

7c. Fill the data collected and calculated from this experiment in Table 1-2 in Page 8.

The final report is expected to include:

One matlab plot along with titles and labels
- Plot of Encoder 1 Velocity and Encoder 3 Velocity

For all the questions highlighted, the questions should be copied and pasted into the report and answered immediately thereafter.

**Experiment 1b: Determination of $K_a$**

In this part, the value for $K_a$ will be determined experimentally

8. Setup the mechanism as shown in Figure 1-2b. Rotate the yoke relative to the base using a very light touch to position it where there is least friction (some residual friction from the brake may exist in some locations).

9. Reconfigure the input given to the system. Go to Trajectory 1 Configuration under the Setup menu, select Impulse and change the Pulse Width to 2000 ms. Leave all other parameters the same as in Step 5 above.

10. Change Data Acquisition to acquire Encoder 1 Position and Encoder 4 Position data. Repeat Step 6 to execute the input to the system. One should see the rotor spin and the remaining assembly rotates in the opposite direction about the base (Encoder 4).
11. Plot to see **Encoder 1 and Encoder 4 velocity** in the same plot. Explore the data to matlab to generate the plot for data analysis. Fill the data collected and calculated from this experiment in **Table 1-2** in Page 8.

**The final report is expected to include:**

One matlab plot along with titles and labels
- Plot of Encoder 1 Velocity and Encoder 4 Velocity

**Experiment 1c: Determination of \( I_c \)**

In the final part of experiment 1, the final parameter will be determined.

12. Setup the mechanism as shown in Figure 1-2c. Set the yoke position as described in Step 8.

13. Edit the previous control algorithm to output `cmd1_pos` to `control_effort2`. (i.e. `control_effort2 = cmd1_pos/32`). Select *Save Changes and Quit* to exit the editor. Then click *Implement Algorithm* to implement this action.

14. Go to *Trajectory 1 Configuration*, select *Impulse* and change the *Pulse Width* to **150** ms. Leave all other parameters the same as in Step 5 above.

15. Change *Data Acquisition* to acquire **Encoder 2 Position** and **Encoder 4 Position** data. Repeat Step 6 to run the test. One should see the inner gimbal ring rotate relative to the outer ring and the remaining assembly rotate about the base (Encoder 4). The inner ring will likely contact the limit switch at the end of the maneuver causing the Control Box to power down. This is normal. If it occurs before completing a satisfactory test, simply re-power the Control Box. It is also possible that the limit switches are contacted during the initial (first 400 ms) portion of the maneuver. If this occurs, reduce the *Pulse Width* duration in Step 15 (to say **100** ms) and repeat the procedure.

17. Plot Encoder 2 and Encoder 4 velocity data. Explore the data to matlab for data analysis. Fill the data collected and calculated from this experiment in **Table 1-2** in Page 8.

**The final report is expected to include:**

One matlab plot along with title and labels
- Plot of Encoder 2 Velocity and Encoder 4 Velocity
Table 1-2 Data collected and calculated for Experiment 1

<table>
<thead>
<tr>
<th>Exp 1a ( \left( J_c \right) )</th>
<th>( J_1 )</th>
<th>Encoder gain for ( J_1 )</th>
<th>( \omega_{10} ) (in counts)</th>
<th>( \omega_{10} ) (in rad/s)</th>
<th>( \omega_{1f} ) (in counts)</th>
<th>( \omega_{1f} ) (in rad/s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>( J_2 )</td>
<td>Encoder gain for ( J_2 )</td>
<td>( \omega_{20} ) (in counts)</td>
<td>( \omega_{20} ) (in rad/s)</td>
<td>( \omega_{2f} ) (in counts)</td>
<td>( \omega_{2f} ) (in rad/s)</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Exp 1b ( \left( K_a \right) )</th>
<th>( J_1 )</th>
<th>Encoder gain for ( J_1 )</th>
<th>( \omega_{10} ) (in counts)</th>
<th>( \omega_{10} ) (in rad/s)</th>
<th>( \omega_{1f} ) (in counts)</th>
<th>( \omega_{1f} ) (in rad/s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>( J_2 )</td>
<td>Encoder gain for ( J_2 )</td>
<td>( \omega_{20} ) (in counts)</td>
<td>( \omega_{20} ) (in rad/s)</td>
<td>( \omega_{2f} ) (in counts)</td>
<td>( \omega_{2f} ) (in rad/s)</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Exp 1c ( \left( L_c \right) )</th>
<th>( J_1 )</th>
<th>Encoder gain for ( J_1 )</th>
<th>( \omega_{10} ) (in counts)</th>
<th>( \omega_{10} ) (in rad/s)</th>
<th>( \omega_{1f} ) (in counts)</th>
<th>( \omega_{1f} ) (in rad/s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>( J_2 )</td>
<td>Encoder gain for ( J_2 )</td>
<td>( \omega_{20} ) (in counts)</td>
<td>( \omega_{20} ) (in rad/s)</td>
<td>( \omega_{2f} ) (in counts)</td>
<td>( \omega_{2f} ) (in rad/s)</td>
<td></td>
</tr>
</tbody>
</table>
After all three parts are completed with three plots obtained in the experiments and Table 1-2 filled, perform calculations to complete Table 1-3 below. Be sure to solve Eq. (A) with symbols before plugging in numbers. This may be best done in MATLAB or excel to prevent algebraic errors. You need to identify what $J_1$ and $J_2$ are in Table 1-2 for each of the three experiments in relation to what you need to calculate in Table 1-3. Show calculations to complete Table 1-3.

**Table 1-3. Moment of Inertia Data**

<table>
<thead>
<tr>
<th>Body</th>
<th>Inertia Element</th>
<th>Value (kg-m²)</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>$K_A$</td>
<td></td>
</tr>
<tr>
<td>B</td>
<td>$I_B$</td>
<td>0.0119</td>
</tr>
<tr>
<td></td>
<td>$J_B$</td>
<td>0.0178</td>
</tr>
<tr>
<td></td>
<td>$K_B$</td>
<td>0.0297</td>
</tr>
<tr>
<td>C</td>
<td>$I_C$</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$J_C$</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$K_C$</td>
<td>0.0188</td>
</tr>
<tr>
<td>D</td>
<td>$I_D$</td>
<td>0.0148</td>
</tr>
<tr>
<td></td>
<td>$J_D$</td>
<td>0.0273</td>
</tr>
</tbody>
</table>

Experiment 2 skipped. Go to 3.
Experiment 3: Gyroscopic Dynamics: Nutation and Precession

This sequence of experiments illustrates the three-dimensional dynamics and control of a gyroscope. The 3D dynamics is governed by Equation 2-2 below. Study this equation and the description thereafter.

\[ T = \omega \times H \]  
(Equation 2-2)

Explanation of Equation 2-2:
The symbol \( H \) is the angular momentum vector of the spinning wheel, \( \omega \) is the angular velocity vector associated with the change of direction of \( H \), and \( T \) is the applied torque vector necessary to change the direction of \( H \). The three vectors are in three perpendicular direction in our experiments to follow.

Important: Abort the control used in previous step before continuing on.

Experiment 3a: Nutation: Frequency & Mode Shapes

Follow the procedure to study the nutation motion of the system. When plotting position and velocity on the same figure, place position variables on the left axis and velocity variables on the right axis.

Figure 3-1. Configuration For All Tests In This Section

Procedure

1. Setup the mechanism as shown in Figure 3-1.
2. Similar to Step 4 in the previous experiment, enter the following control algorithm via the Setup menu to activate Motor #2. Click Implement Algorithm thereafter.

```plaintext
begin
  control_effort2 = cmd1_pos/32
end
```

3. Go to Trajectory 1 Configuration. Enter Impulse and specify an Amplitude of 16000 counts, a Pulse Width of 50 ms, a Dwell Time of 4000 ms, and 1 repetition (this prepares the controller board to input a 16000 count positive-going impulse followed immediately by 4 seconds of zero input during which data is collected).

4. Setup Data Acquisition. Select Encoder 2 Position and Encoder 4 Position as data to be acquired with a Sample Period of 2 servo cycles.

5. Set a sampling period of \( T_s = 0.00442 \) seconds.

6. Initialize Rotor Speed to 200 RPM. Zero out the encoder positions using Utility menu. Select Normal Data Sampling, execute Trajectory 1 and run.

7. Plot the Encoder 2 and Encoder 4 Position vs. time. Note the frequency of the oscillations and the relative amplitude and phase of the Encoder 4 response verses the Encoder 2 response. Explore the data to matlab.

8. Turn off the rotor. Disable the rotor speed loop in Command menu. To more rapidly decelerate the rotor, turn off the Control Box. Wait for the rotor to stop (if the Control Box is off, turn it back on at this point).

9. Repeat Steps 6 and 7 for a rotor speed of 400 RPM and then 800 RPM.

The final report is expected to include:

Three matlab plots along with titles and labels to show which plot corresponds to which situation.
- A plot of Encoder 2 and Encoder 4 position at 200 RPM
- A plot of Encoder 2 and Encoder 4 position at 400 RPM
- A plot of Encoder 2 and Encoder 4 position at 800 RPM

Comment on the frequency of oscillations and phase between Encoder 2 and Encoder 4 motions. What are the trends and relationships between the three different rotor speeds?
Experiment 3b: Precession

Follow the procedure below to study the precession motion of the gyroscope:

10. Repeat Steps 1 through 6 of Experiment 3a except in Step 3 setup the Impulse trajectory for an Amplitude of 6000 counts, a Pulse Width of 5000 ms, a Dwell Time of 0 ms, and 1 repetition (this prepares the controller board to input a 6000 count constant input for 8 seconds). The first maneuver should be at 200 RPM and should result in an initial transient series of attenuating nutation oscillations followed by a steady state response. Plot the position and velocity data for Encoders 2 and 4; put position data on the left axis and velocity data on the right axis. The plot key in matlab program, plotdata.m, is iplot=4.

11. Repeat Step 10 for the 400 and 800 RPM cases. Note the change in steady state velocity for Encoder 4 with rotor speed.

The final report is expected to include:

Three matlab plots along with titles and labels to show which plot corresponds to which situation.
- A plot of Encoder #2 and Encoder #4 position and velocity at 200 RPM
- A plot of Encoder #2 and Encoder #4 position and velocity at 400 RPM
- A plot of Encoder #2 and Encoder #4 position and velocity at 800 RPM

Comment on the change in steady state velocity for Encoder #4 as a function of the rotor speed.
Experiment 3c: Nutation Damping

First, abort the control that was used in previous experiment. Input via the Setup menu the following control algorithm (a sample program is saved in the PC at C:\Program Files (x86)\ECP Systems_MV\mv\Experiment3c.alg):

```
;**************DEFINE USER VARIABLES**********************
#define Ts q1
#define kv q2
#define kdd q3
#define enc2_last q4
;****************INITIALIZE VARIABLES********************
Ts = 0.00884
kv = 0.005
kdd = kv/Ts
;********************REAL TIME CODE**********************
begin
;CONTROL LAW: output torque = demand – gimbal2 rate feedback
control_effort2 = cmd1_pos/32 – kdd*(enc2_pos – enc2_last)
;UPDATE VARIABLES
enc2_last = enc2_pos
end
```

Procedure

12. The algorithm above adds rate feedback damping at Axis 2. That is, it adds a term $u_{2damp}$ of the form

$$u_{2damp} = -k_v q_2 s$$

where $k_v$ is the rate feedback gain, $q_2$ is the position measurement, and $s$ is the derivative operator. Thus, the control effort is just the velocity times the rate feedback gain, $k_v$, which is to be entered into the control algorithm above.

13. Set $k_v = 0.005$ for the first experiment, and then click implement the algorithm.

14. Similar to Step 10 above to set up the Impulse trajectory for an Amplitude of 6000 counts, a Pulse Width of 5000 ms, a Dwell Time of 0 ms, and 1 repetition. Set a sampling period of $T_s = 0.00884$ s (note: not 0.000884). Run the experiment with a rotor speed at 200 RPM. Plot velocities of Encoders 2 and 4 in the same vertical axis (plot key iplot=-2).

15. Repeat Step 14 with a rotor speed of 400 RPM. Is there a reduction observed in the nutation amplitude?
16. Repeat Steps 13 and 14 for a couple of increasing values of $k_v$: 0.02 and 0.08. **How are the nutation oscillations affected by increased rate feedback gain?**

The final report is expected to include:
Six plots along with titles and labels to show which plot corresponds to which situation
- Plot of Velocities 2&4 with $k_v = 0.005$ and Rotor Speed of 200 RPM
- Plot of Velocities 2&4 with $k_v = 0.005$ and Rotor Speed of 400 RPM
- Plot of Velocities 2&4 with $k_v = 0.020$ and Rotor Speed of 200 RPM
- Plot of Velocities 2&4 with $k_v = 0.020$ and Rotor Speed of 400 RPM
- Plot of Velocities 2&4 with $k_v = 0.080$ and Rotor Speed of 200 RPM
- Plot of Velocities 2&4 with $k_v = 0.080$ and Rotor Speed of 400 RPM

**How are the nutation oscillations affected by increased rate feedback gain?**
**How does an increase in Rotor Speed affect the amplitude of the oscillations?**
**Identify the case which generates the plot with a best damped nutation response.**

For all the questions highlighted, the questions should be copied and pasted into the student’s lab report and answered immediately thereafter.