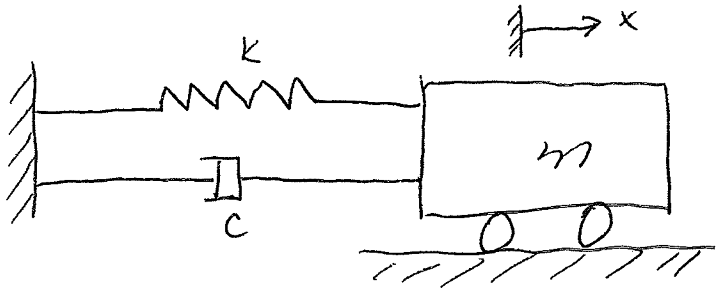
2.2 Free vibration of damped systems

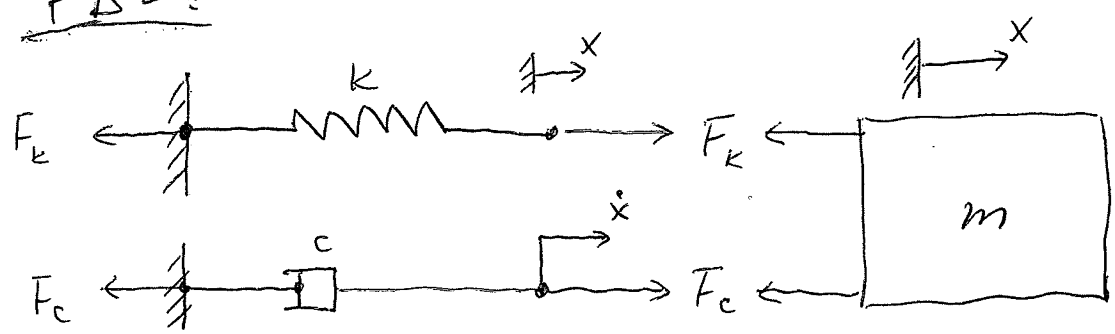
— same steps of work as in Sec. 2.1

① Math model

Example: mass-spring-damper system



FBD:



F. Eq.:

k: $F_k = kx$ ①

c: $F_c = c\dot{x}$ ②

m: $m\ddot{x} = -F_k - F_c$ ③

Gov. eq

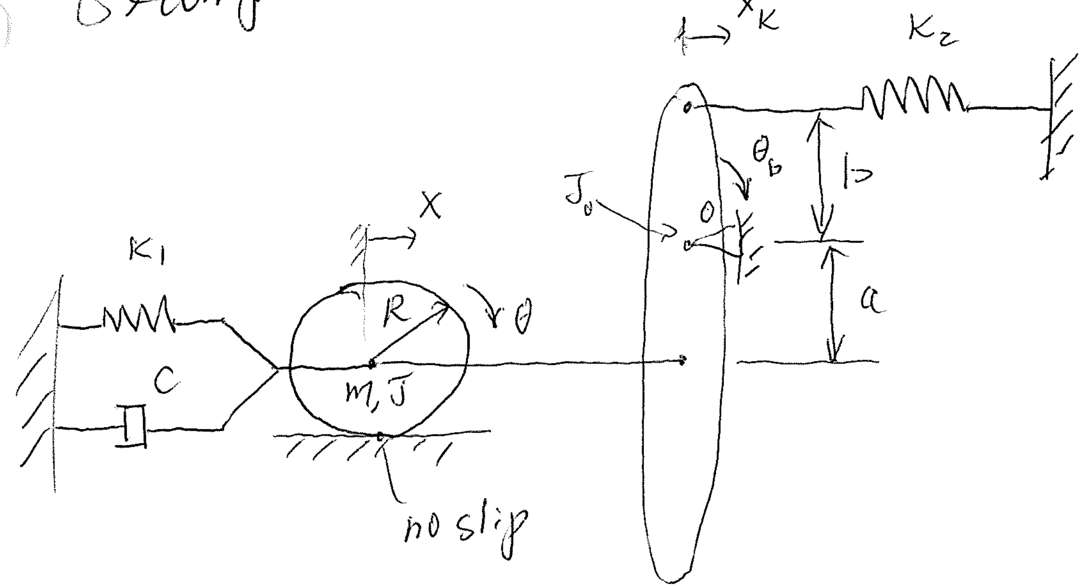
Eq. (2) $\Rightarrow$

$$m\ddot{x} = -kx - c\dot{x}$$

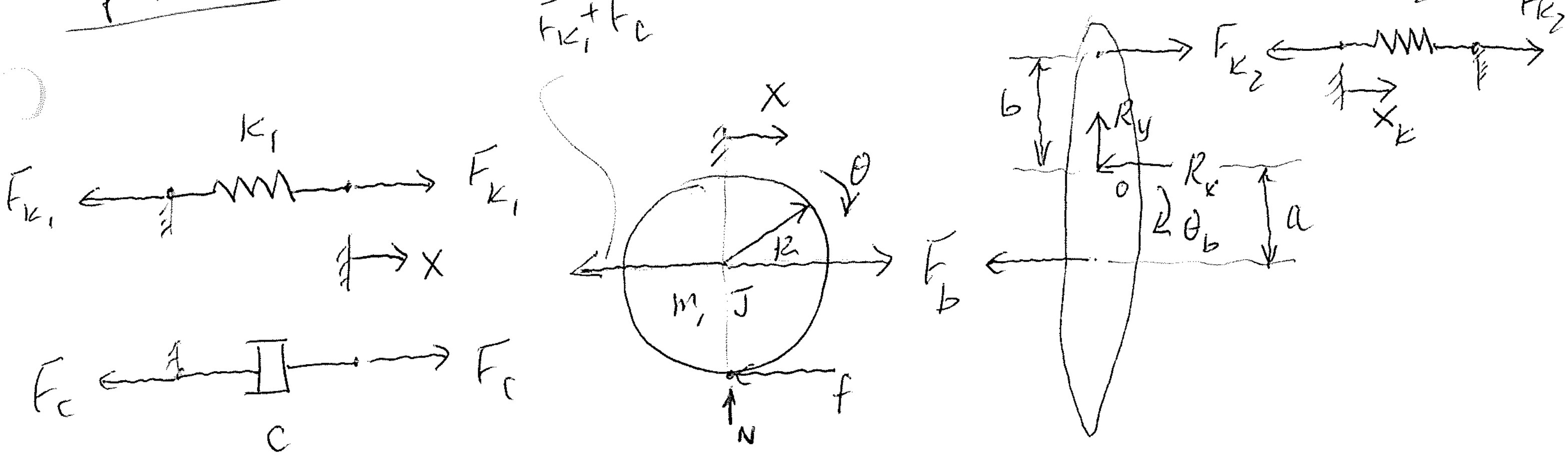
Or $$\boxed{m\ddot{x} + c\dot{x} + kx = 0} \qquad (4)$$

Eq. (4) is the standard form of the math model for free vibration of damped SDOF system.

Example:



FBDs



Ele eqs

k_1 — $F_{k_1} = k_1 x$ (1)

c — $F_c = c\dot{x}$ (2)

m — $m\ddot{x} = F_b - F_{k_1} - F_c - f$ (3)

J — $J\ddot{\theta} = Rf$ (4)

J_0 — $J_0\ddot{\theta}_b = aF_b + bF_{k_2}$ (5)

k_2 — $F_{k_2} = k_2(-x_k)$ (6)

Also,

$x = R\theta$ (7)

$x = -a\theta_b$ (8)

$x_k = b\theta_b$ (9)

Gov. eq (DOF var = θ_b)

Expect: $(\ )\ddot{\theta}_b + (\ )\dot{\theta}_b + (\ )\theta_b = 0$

Eq. (5) $\Rightarrow$

$$J_0 \ddot{\theta}_b = a\left(m\ddot{x} + F_{k_1} + F_c + f\right) + b k_2(-x_k)$$

$$= a\left(m(-a\ddot{\theta}_b) + k_1(-a\theta_b) + c(-a\dot{\theta}_b) + \frac{J}{R}\ddot{\theta}\right) - b^2 k_2 \theta_b$$

$$\ddot{\theta} = \frac{1}{R}\ddot{x} = -\frac{a}{R}\ddot{\theta}_b$$

$\Rightarrow$

$$\boxed{\left(J_0 + ma^2 + \left(\frac{a}{R}\right)^2 J\right)\ddot{\theta}_b + (ca^2)\dot{\theta}_b + (a^2k_1 + b^2k_2)\theta_b = 0}$$

Math model by energy method

For damped free vibration, the time rate of change of energy in the system is equal to the rate of energy dissipated by system damping

$$\boxed{\frac{d}{dt}\left(\sum KE + \sum PE\right) = -\sum \dot{E}_d} \quad \text{(A)}$$

rate of E. diss.

For the previous example,

$$\sum KE = \frac{1}{2} m\dot{x}^2 + \frac{1}{2} J \dot{\theta}^2 + \frac{1}{2} J_0 \dot{\theta}_b^2$$

$$= \frac{1}{2} \underbrace{\left(m a^2 + \left(\frac{a}{R}\right)^2 J + J_0 \right)}_{J_{eq}} \dot{\theta}_b^2$$

$$\sum PE = \frac{1}{2} k_1 x^2 + \frac{1}{2} k_2 x_2^2$$

$$= \frac{1}{2} \underbrace{\left(k_1 a^2 + k_2 b^2 \right)}_{k_{eq}} \theta_b^2$$

$$\sum \dot{E}_d = c\dot{x}^2 = \underbrace{c a^2}_{c_{eq}} \dot{\theta}_b^2$$

Eq (A) $\Rightarrow$

$$J_{eq} \dot{\theta}_b \cdot \ddot{\theta}_b + k_{eq} \theta_b \cdot \dot{\theta}_b = - c_{eq} \dot{\theta}_b^2$$

$$\Rightarrow \boxed{J_{eq} \ddot{\theta}_b + c_{eq} \dot{\theta}_b + k_{eq} \theta_b = 0}$$