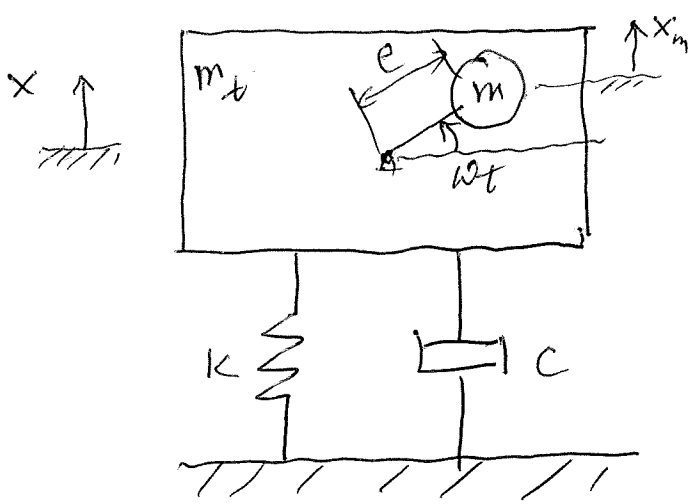
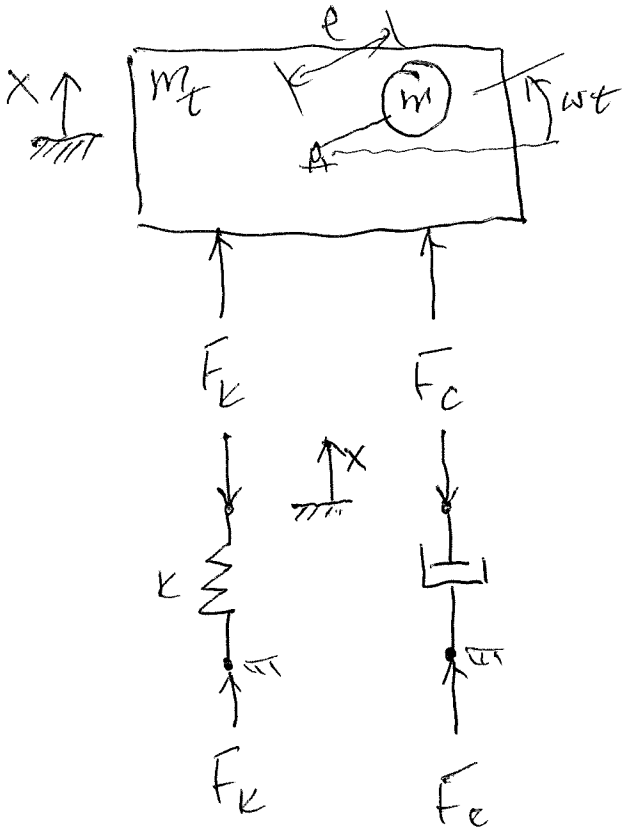
Example: Machine with an unbalanced rotating mass



m_t = total mass

m = rotating mass

FBDs



F/e eqs

K — $F_K = -Kx$ (1)

c — $F_c = -c\dot{x}$ (2)

$(m_t\ w/\ m)$ — $\sum m_i \ddot{x}_c = \sum F_x$

$$\Rightarrow (m_t - m)\ddot{x} + m\ddot{x}_m = F_K + F_c \quad (3)$$

where $x_m = x + e\sin\omega t$

$$\Rightarrow \ddot{x}_m = \ddot{x} + \frac{d^2}{dt^2}(e\sin\omega t)$$

$$= \ddot{x} - e\omega^2 \sin\omega t \quad (4)$$

Gov. eq in x

Expect: $(\ )\ddot{x} + (\ )\dot{x} + (\ )x = RHS$

Eq. (3) $\Rightarrow$ $(m_t\ddot{x} - m\ddot{x}) + (m\ddot{x} - me\omega^2\sin\omega t) = -Kx - c\dot{x}$

Or $$\boxed{m_t\ddot{x} + c\dot{x} + Kx = \underbrace{me\omega^2}_{F}\sin\omega t} \quad (5)$$