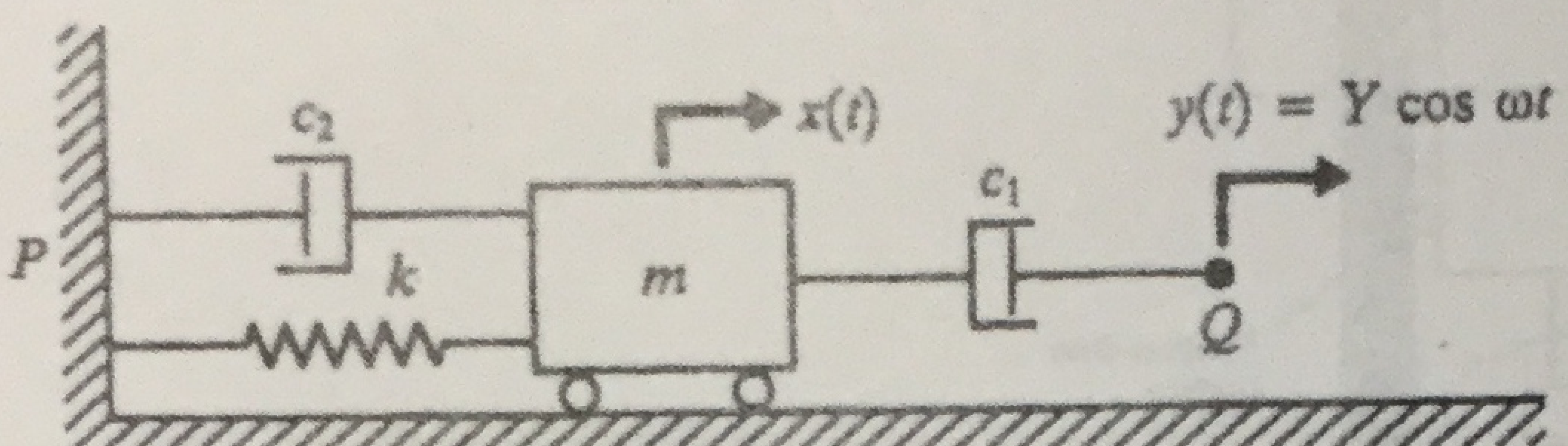
ME 370.1 Fall 2018 Name(s)

Homework 9

Due: *In class*, Friday Oct. 26

1. (32 pts) Examine the geometric model shown below, where $y(t)$ is a motion input applied at the right end of damper c_1. The math model for the system is $m\ddot{x} + (c_1 + c_2)\dot{x} + kx = c_1\omega Y \sin(\omega t + \pi)$



(1) (4 pts) The magnitude of the follow-up vibration $x_p(t) = X\sin(wt + \pi + \psi)$ is

a) $X = \dfrac{c_1 r Y}{m\omega_n}\dfrac{1}{\left[(1-r^2)^2 + (2\xi r)^2\right]^{1/2}}$

b) $X = \dfrac{Y/\omega_n^2}{\left[(1-r^2)^2 + (2\xi r)^2\right]^{1/2}}$

c) $X = \dfrac{c_1 Y}{m\omega_n^2}\dfrac{1}{\left[(1-r^2)^2 + (2\xi r)^2\right]^{1/2}}$

d) $X = \dfrac{r^2 Y}{\left[(1-r^2)^2 + (2\xi r)^2\right]^{1/2}}$

(2) (8 pts) The magnitude of the force $f_Q(t) = F_Q\sin(wt + \alpha)$ on damper c_1 is

a) $F_Q = \left[(1-r^2)^2 + (2\xi r)^2\right]^{1/2} X$

b) $F_Q = \left[(k - m\omega^2)^2 + (c_2\omega)^2\right]^{1/2} X$

c) $F_Q = \left[(1-r^2)^2 + (2\xi r)^2\right]^{1/2} kX$

d) $F_Q = \left[(k - m\omega^2)^2 + (c_2\omega)^2\right]^{1/2} c_1 X$

(hint: this force may be more easily obtained starting with the elemental equation of the mass).

(3) (5 pts) The magnitude of the force $f_P(t) = F_P\sin(wt + \beta)$ transmitted to the wall at point P is

a) $F_P = \left[k^2 + (c_2\omega)^2\right]^{1/2} X$

b) $F_P = \left[(k - m\omega^2)^2 + (c_2\omega)^2\right]^{1/2} c_2 X$

c) $F_P = \left[(1-r^2)^2 + (2\xi r)^2\right]^{1/2} kX$

d) $F_P = \left[(1-r^2)^2 + (2\xi r)^2\right]^{1/2} X$

(4) (15 pts) Let $m = 10$ kg, $k = 1000$ N/m, $c_1 = 110$ N-s/m, and $c_2 = 25$ N-s/m. Program in matlab to plot X/Y vs. ω/ω_n and F_P/F_Q vs. ω/ω_n for r ranging from 0 to 2. Submit the matlab program and the plots. From the plots, use matlab data curser tool to determine the maximum values of X/Y and F_P/F_Q and the corresponding values of r:

max(X/Y) = 0.8148 corresponding r = 1

max (F_P/F_Q) = 4.152 corresponding r = 0.985

1\. 1.(1) sol: The math model of the system is: $m\ddot{x} + (c_1+c_2)\dot{x} + kx = c_1\omega Y \sin(\omega t+\pi)$

Then magnitude of excitation $E = \frac{c_1 \omega Y}{m}$, and $r = \frac{\omega}{\omega_n}$

Thus $\frac{E}{\omega_n^2} = \frac{c_1 \omega Y}{m\omega_n^2} = \frac{c_1 r Y}{m\omega_n}$

$\Rightarrow$ The magnitude of follow-up vibration:

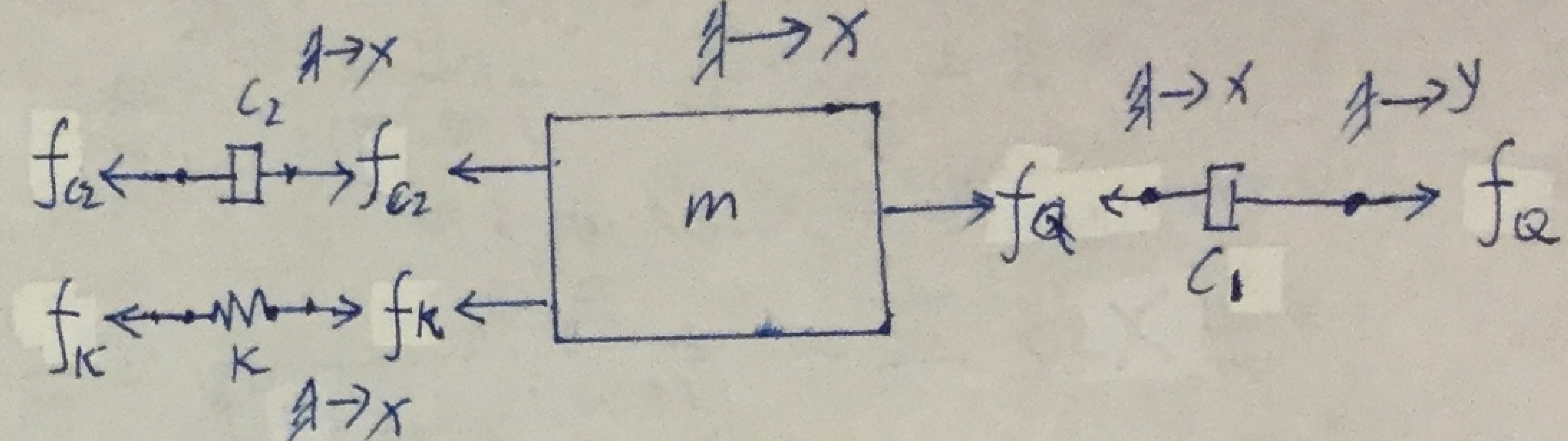
$$X = \frac{E/\omega_n^2}{[(1-r^2)^2+(2\xi r)^2]^{\frac{1}{2}}} = \frac{c_1 r Y}{m\omega_n} \frac{1}{[(1-r^2)^2+(2\xi r)^2]^{\frac{1}{2}}}$$

(2) sol: ele. eq.

m: $m\ddot{x} = f_Q - f_{c_2} - f_k$

c_2: $f_{c_2} = c_2 \dot{x}$

k: $f_k = kx$



$\Rightarrow f_Q = m\ddot{x} + c_2\dot{x} + kx$ --- ①

Here we just focus on the effects of follow-up response and assume $x_h(t)$ has died out.

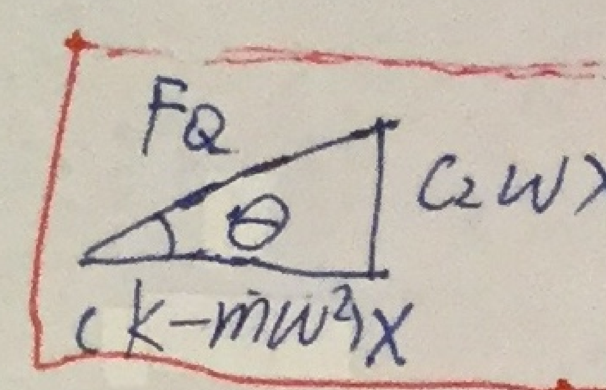
From 1.(1) $x_p = X\sin(\omega t+\pi+\psi)$ --- ②

$\dot{x}_p = X\omega\cos(\omega t+\pi+\psi)$ --- ③

$\ddot{x}_p = -X\omega^2\sin(\omega t+\pi+\psi)$ --- ④

substitute ② ③ ④ into ①, get $f_Q = (k-m\omega^2)X\sin(\omega t+\pi+\psi) + c_2\omega X\cos(\omega t+\pi+\psi)$

i.e. $f_Q = F_Q\left[\frac{(k-m\omega^2)X}{F_Q}\sin(\omega t+\pi+\psi) + \frac{c_2\omega X}{F_Q}\cos(\omega t+\pi+\psi)\right]$



$= F_Q \sin(\omega t+\pi+\psi+\theta)$

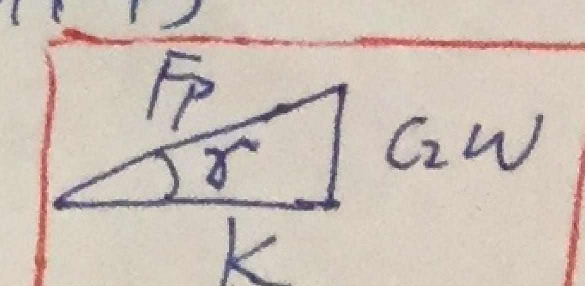
$= F_Q \sin(\omega t+\alpha)$

where $\boxed{F_Q = [(k-m\omega^2)^2+(c_2\omega)^2]^{\frac{1}{2}} X}$, $\alpha = \pi+\psi+\theta$, $\theta = \tan_2^{-1}\frac{c_2\omega}{k-m\omega^2}$

(3) sol: $f_P = f_{c_2} + f_k = c_2\dot{x} + kx$ --- ⑥

Substitute ② ③ into ⑥, get $f_P = c_2\omega X\cos(\omega t+\pi+\psi) + kX\sin(\omega t+\pi+\psi)$

i.e. $f_P = F_P\left[\frac{c_2\omega X}{F_P}\cos(\omega t+\pi+\psi) + \frac{kX}{F_P}\sin(\omega t+\pi+\psi)\right]$



$= F_P\sin(\omega t+\pi+\psi+\gamma)$

$= F_P\sin(\omega t+\beta)$

where $F_P = [k^2+(c_2\omega)^2]^{\frac{1}{2}} X$, $\beta = \pi+\psi+\gamma$, $\gamma = \tan_2^{-1}\frac{c_2\omega}{k}$

(4) sol: $\omega_n = \sqrt{\frac{k}{m}} = \sqrt{\frac{1000}{10}} = 10$, $\xi = \frac{1}{2\omega_n}\cdot\frac{c_1+c_2}{m} = \frac{1}{2\times10}\cdot\frac{110+25}{10} = 0.675$, $\omega = \omega_n r$

① From 1.(1), $\frac{X}{Y} = \frac{c_1 r}{m\omega_n}\frac{1}{[(1-r^2)^2+(2\xi r)^2]^{\frac{1}{2}}}$, plot graph $\frac{X}{Y}\sim r$, shown as Figure 1.

$\max\left(\frac{X}{Y}\right) = 0.8148$, $r_{max} = 1$

② From 1.(2), 1.(3). $\frac{F_P}{F_Q} = \frac{[k^2+(c_2\omega)^2]^{\frac{1}{2}}X}{[(k-m\omega^2)^2+(c_2\omega)^2]^{\frac{1}{2}}X} = \frac{[k^2+(c_2\omega_n r)^2]^{\frac{1}{2}}}{[(k-m\omega_n^2 r^2)^2+(c_2\omega_n r)^2]^{\frac{1}{2}}} = \boxed{\frac{[km+c_2^2r^2]^{\frac{1}{2}}}{[km(1-r^2)^2+c_2^2r^2]^{\frac{1}{2}}}}$

plot $\frac{F_P}{F_Q}\sim r$, shown as Figure 2. $\Rightarrow \max\left(\frac{F_P}{F_Q}\right) = 4.152$, $r_{max} = 0.985$

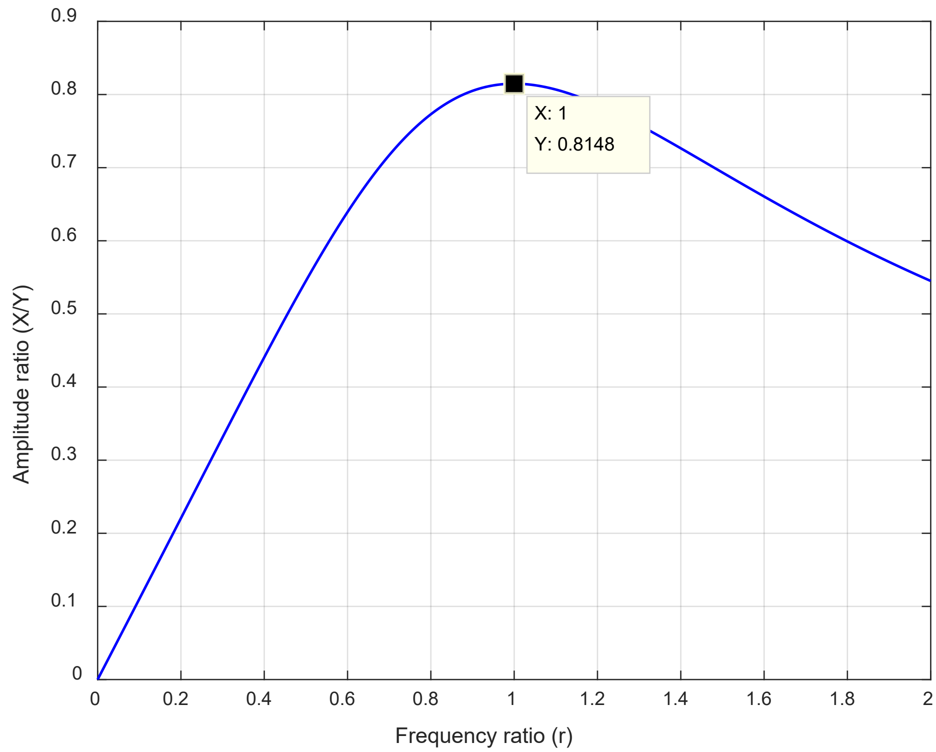


Figure 1 The graph of X/Y vs. r

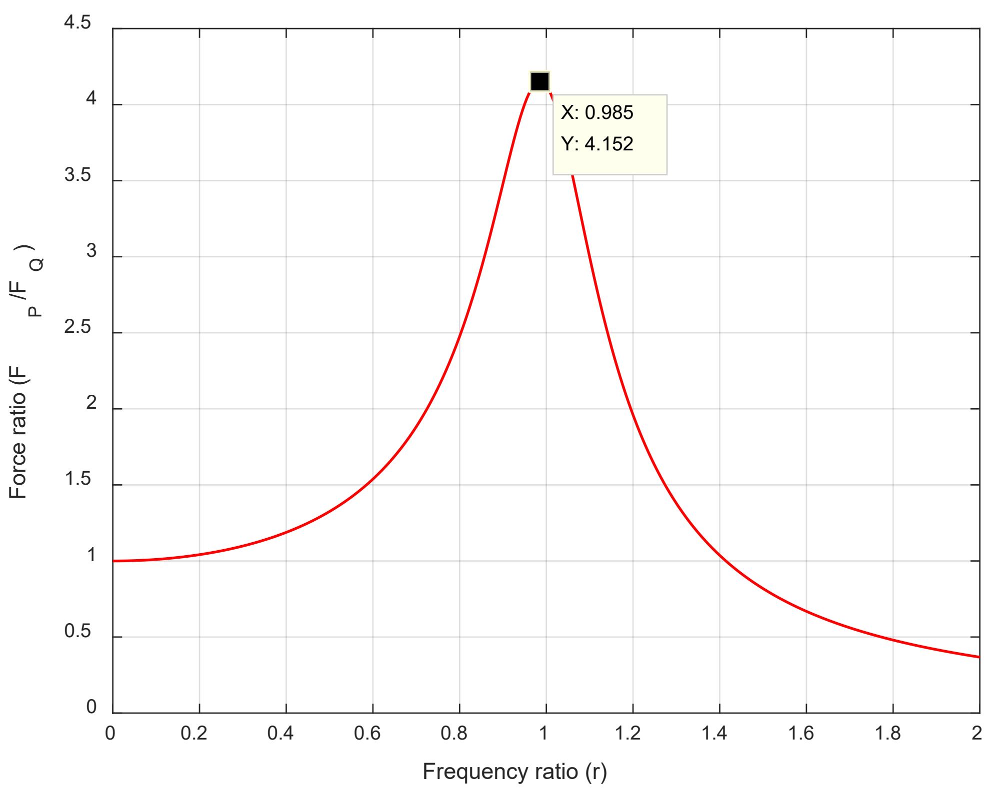
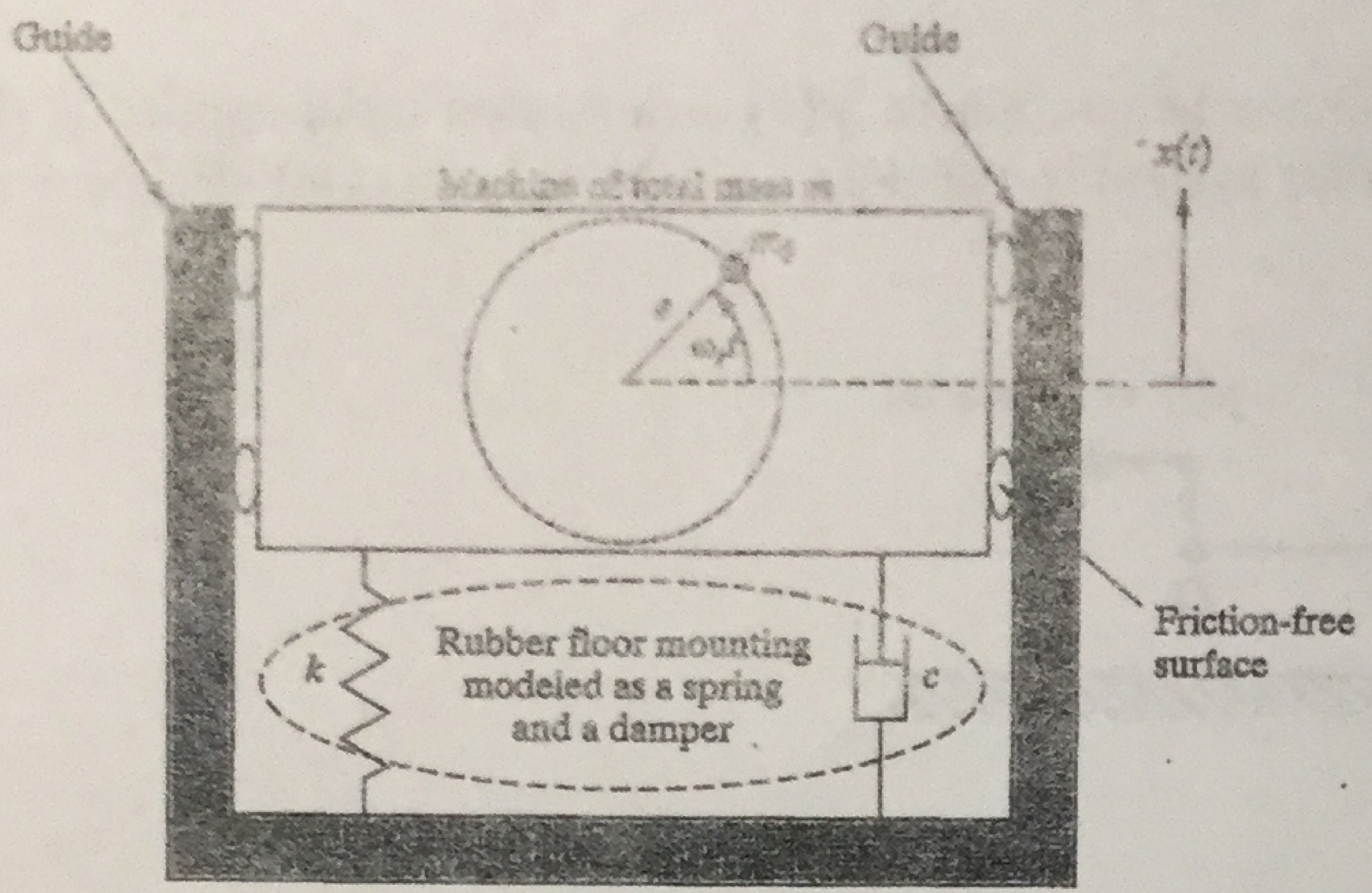


Figure 2 The graph of F_P/F_Q vs. r

```
%{
 * Course:ME370
 * Name: Liming Gao
 * Date: October 15, 2018
 *
 * Program Description: problem 1(04).
 %}

clear all
close all

m = 10; %mass
k = 1000; % stiffness
c1 = 110; % damper1
c2 = 25; %damper2
wn = (k/m)^0.5; % natural frequency

c = (c1+c2)/(2*wn*m); %damping ratio
r = 0:0.005:2; % frequency ratio
w = wn*r;

X_Y = (c1*r/(m*wn))./((1-
r.^2).^2+(2*c*r).^2).^0.5; % Amplitude ratio
FP_FQ = ((k*k+c2*c2*w.^2)./ ((k-
m*w.*w).^2+c2*c2*w.^2)).^0.5; %Force
transmissibility
%FP_FQ = ((m*k+c2*c2*r.^2)./ (k*m*(1-
r.*r).^2+c2*c2*r.^2)).^0.5; %Force transmissibility

figure(1)
plot(r,X_Y,'b','LineWidth',1); %plot the curve
xlabel('Frequency ratio (r)')
ylabel('Amplitude ratio (X/Y)')
grid on

figure(2)
plot(r,FP_FQ ,'r','LineWidth',1); %plot the curve
xlabel('Frequency ratio (r)')
ylabel('Force ratio (F_P/F_Q)')
grid on
```

2. (15 pts) A machine with a rotating unbalance, $m_o e$, is mounted on a rubber floor as shown. The total mass of the machine is $m_t = 120$ kg, the rubber-mount stiffness is $k = 8 \times 10^5$ N/m and damping constant $c = 9000$ N-s/m. The centrifugal force generated by the machine unbalance is determined to be $F_o = 600$ N when the machine operates at $\omega = 1500$ rpm.



(1) (2 pts) The machine unbalance in kg-m is

a) $m_o e = 0.024$ b) $m_o e = 0.042$

c) $m_o e = 0.064$ d) $m_o e = 0.092$

(2) (4 pts) The magnitude of machine vibration in millimeter is

a) $X = 0.11$ b) $X = 0.23$

c) $X = 0.45$ d) $X = 0.86$

(3) (4 pts) The magnitude of the force in Newton transmitted to machine foundation is

a) $F_{to} = 102$ b) $F_{to} = 243$

c) $F_{to} = 377$ d) $F_{to} = 458$

(4) (5 pts) To reduce the transmitted force by 25%, the damping constant, c, in N-s/m of the rubber floor needs to be at least

a) larger than 10410

b) larger than 12540

c) smaller than 4558

b) smaller than 6758

2. (1) sol: centrifugal force $F_0 = m_0 e \omega^2$

and $\omega = 1500(rpm) = \frac{1500 \cdot 2\pi}{60}(rad/s) = 50\pi(rad/s)$

$\Rightarrow m_0 e = \frac{F_0}{\omega^2} = \frac{600}{(50\pi)^2} \approx 0.0243(kg \cdot m)$

(2) sol: math model: $m_t \ddot{x} + c\dot{x} + kx = m_0 e \omega^2 \sin\omega t$

$\Rightarrow \ddot{x} + 2\xi\omega_n \dot{x} + \omega_n^2 x = E\sin\omega t$,

where $E = \frac{m_0 e \omega^2}{m} = \frac{F_0}{m} = 5$

$\omega_n = \sqrt{\frac{k}{m}} = \sqrt{\frac{8\times10^5}{120}} = 100\sqrt{\frac{2}{3}} \approx 81.6497$, $r = \frac{\omega}{\omega_n} = \frac{\pi}{2}\sqrt{\frac{3}{2}} \approx 1.9238$

$\xi = \frac{1}{2\omega_n} \cdot \frac{c}{m} = \frac{1}{2\times81.6497} \cdot \frac{9000}{120} = \frac{3\sqrt{3}}{8\sqrt{2}} \approx 0.4593$

vibration $x_p = X\sin(\omega t + \psi)$

magnitude $X = \frac{E/\omega_n^2}{[(1-r^2)^2 + (2\xi r)^2]^{\frac{1}{2}}} = \frac{5/(100\sqrt{\frac{2}{3}})^2}{[(1-1.9238^2)^2 + (2 \cdot 0.4593 \cdot 1.9238)^2]^{\frac{1}{2}}} \approx 2.32\times10^{-4}(m)$

$= 0.232(mm)$

(3) sol: $\boxed{F_{to} = (k^2 + c^2\omega^2)^{\frac{1}{2}} X}$ ← (Details can be found in course note example " Mount design of rotating machine" 10/17/2018

$= \sqrt{(8\times10^5)^2 + (9000 \cdot 50\pi)^2} \cdot 2.32\times10^{-4}$

$\approx 376.85\ (N) \approx 377\ (N)$

Or: $\boxed{F_{to} = \frac{F_0[1 + (2\xi r)^2]^{\frac{1}{2}}}{[(1-r^2)^2 + (2\xi r)^2]^{\frac{1}{2}}}} = \frac{600[1 + (2\times0.4593\times1.9238)^2]^{\frac{1}{2}}}{[(1-1.9238^2)^2 + (2\times0.4593\times1.9238)^2]^{\frac{1}{2}}} \approx 377.45\ (N) \approx 377\ (N)$

(4) sol: $F_{to}' = (1 - 25\%)F_{to} = 0.75 F_{to}$

$\Rightarrow F_{to}' = \frac{F_0[1 + (2\xi' r)^2]^{\frac{1}{2}}}{[(1-r^2)^2 + (2\xi' r)^2]^{\frac{1}{2}}} = 0.75 \cdot 377$, where ξ' represents new damping ratio

i.e. $\frac{600[1 + (2 \cdot 1.9238 \cdot \xi')^2]^{\frac{1}{2}}}{[(1-r^2)^2 + (2 \cdot 1.9238 \cdot \xi')^2]^{\frac{1}{2}}} = 0.75 \cdot 377$

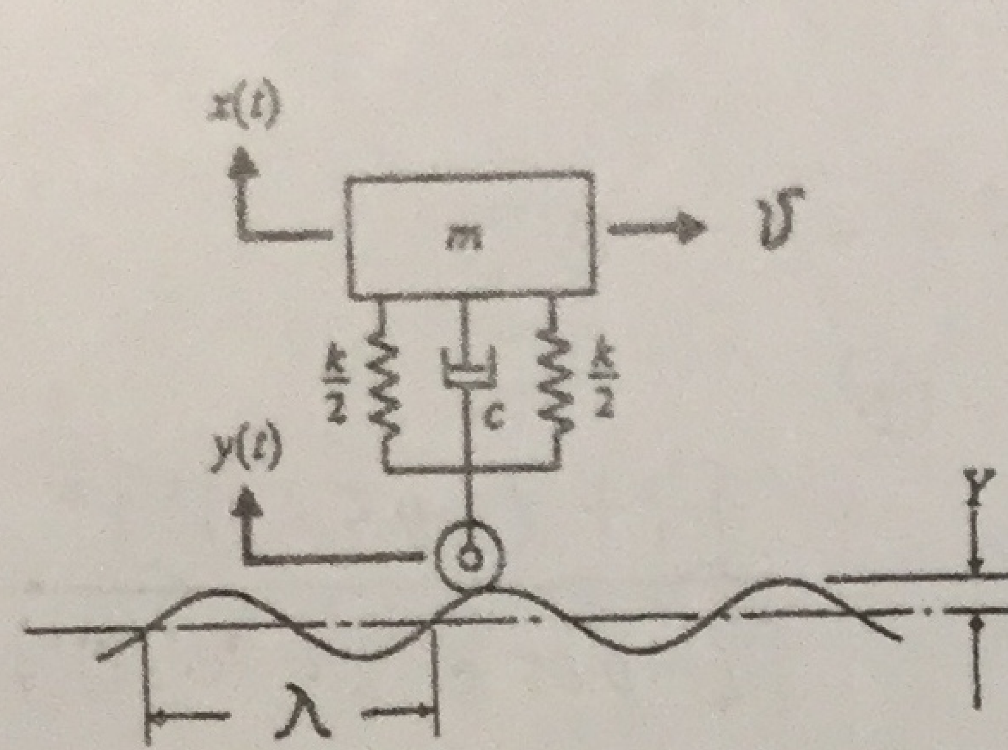
$\Rightarrow \frac{1 + 4 \cdot 1.9238^2 \cdot \xi'^2}{7.2954 + 4 \cdot 1.9238^2 \cdot \xi'^2} = 0.2226$

$\Rightarrow \xi'^2 = \frac{0.2226 \cdot 7.2954 - 1}{4 \cdot 1.9238^2 (1 - 0.2226)} = 0.05422 \Rightarrow \xi' = 0.2328$

Thus $c' = 2\omega_n m \cdot \xi' = 2 \cdot 81.6497 \cdot 120 \cdot 0.2328 = 4561.9$

since $r > \sqrt{2}$, and $\xi' \propto c'$, and according to Force transmissibility chart for rotating machine in class note, c should be smaller than c' to get a smaller F_{to}.

3. (10 pts) A vehicle is traveling through a road segment of wavy pavement and develops an uncomfortable bouncing vibration of magnitude $X = 0.1$ m. The pavement may be described by a sine wave of amplitude $Y = 0.1$ m and wavelength $\lambda = 3.0$ m. The bouncing vibration of the vehicle may be meaningfully modeled by the SDOF model below



The governing equation was derived in class to be

$$m\ddot{x} + c\dot{x} + kx = Y\sqrt{k^2 + c^2\omega^2}\,\sin\omega t$$

Let $m = 500$ kg, $c = 10000$ N-s/m and $k = 2\times10^5$ N/m.

(1) (5 pts) The traveling speed of the vehicle in *mph* is

a) $v = 15$

b) $v = 20$

c) $v = 25$

d) $v = 30$

(2) (5 pts) To cut down the bounce to one quarter of the original value, the vehicle needs to at least travel

a) faster than 45 mph

b) faster than 90 mph

c) slower than 10 mph

d) slower than 12 mph

3. (1) sol: Refer to the course note example "vehicle suspension design" 10/19/2018

The motion transmissibility $TR = \frac{X}{Y} = \frac{[1+(2\xi r)^2]^{\frac{1}{2}}}{[(1-r^2)^2+(2\xi r)^2]^{\frac{1}{2}}}$ --- ①

since $X = 0.1$, $Y = 0.1$, i.e. $TR = \frac{X}{Y} = \frac{0.1}{0.1} = 1$

Then $r = \sqrt{2}$

And $\begin{cases} \omega = r \cdot \omega_n = r\sqrt{\frac{k}{m}} = \sqrt{2} \cdot \sqrt{\frac{2\times 10^5}{500}} = 20\sqrt{2} \\ \boxed{\omega = \frac{2\pi v}{\lambda}} \end{cases}$

$\Rightarrow v = \frac{\lambda \omega}{2\pi} = \frac{3 \cdot 20\sqrt{2}}{2\pi} \approx 13.5\ m/s = 13.5 \cdot 2.237\ (mph) = 30.2\ (mph)$.

(2) sol: Cutting down the bounce to one quarter of the original value means that:

$X' = \frac{1}{4} X = 0.25 \cdot 0.1$, $Y = 0.1$ --- ②

and $\xi = \frac{1}{2\omega_n} \cdot \frac{c}{m} = \frac{1}{2 \cdot 20} \cdot \frac{10000}{500} = 0.5$ --- ③

substitute ② ③ into ①, get

$$\frac{0.25 \cdot 0.1}{0.1} = \frac{[1+(2 \cdot 0.5 \cdot r')^2]^{\frac{1}{2}}}{[(1-r'^2)^2+(2 \cdot 0.5 \cdot r')^2]^{\frac{1}{2}}}$$

i.e. $\left(\frac{1}{4}\right)^2 = \frac{1+r'^2}{(1-r'^2)^2+r'^2}$

$\Rightarrow r'^4 - 17r'^2 - 15 = 0$

solve the equation, get $r'^2 \approx 17.84$, i.e. $r' = 4.224$

$\Rightarrow v' = \frac{\lambda \omega}{2\pi} = \frac{\lambda r' \cdot \omega_n}{2\pi} = \frac{3 \cdot 4.224 \cdot 20}{2\pi} = 40.336\ (m/s) = 90.23\ (mph)$

Since $r \propto v$, and according to TR chart for Vehicle suspension design, the velocity of the vehicle should be greater than v' to give a larger r and thus a smaller bounce.