ME 370.1, Fall 2018

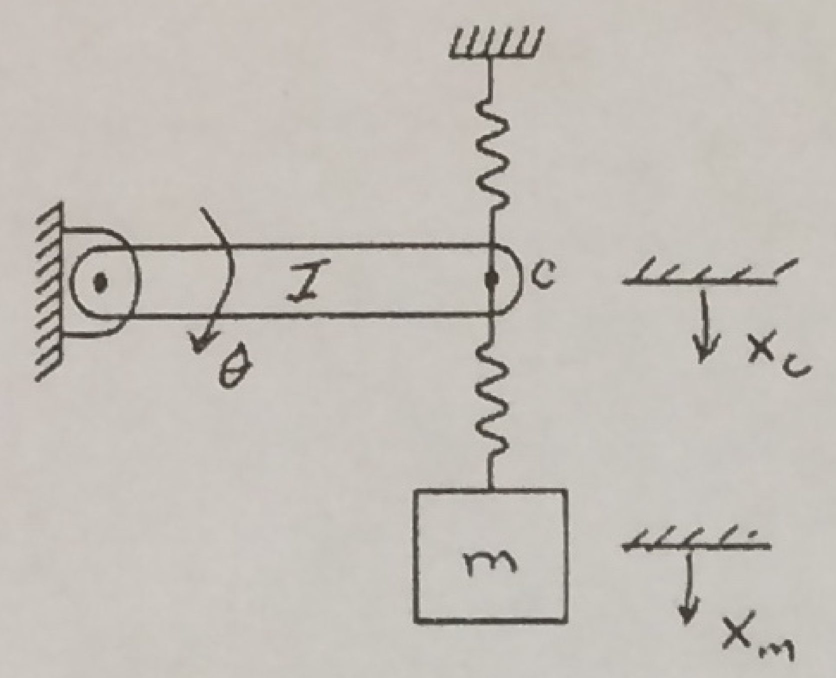
Name(s)

Homework 1

Due: *In class*, Friday August 31

1. (10 pts) Examine each of the geometric models below along with the defined motion variables to picture the motion of the system. Determine the constraints or dependency among various variables. For each system, *a)* list a set of independent motion variables and *b)* define the DOF of the system. Write your answer to the right of each of the model figures.

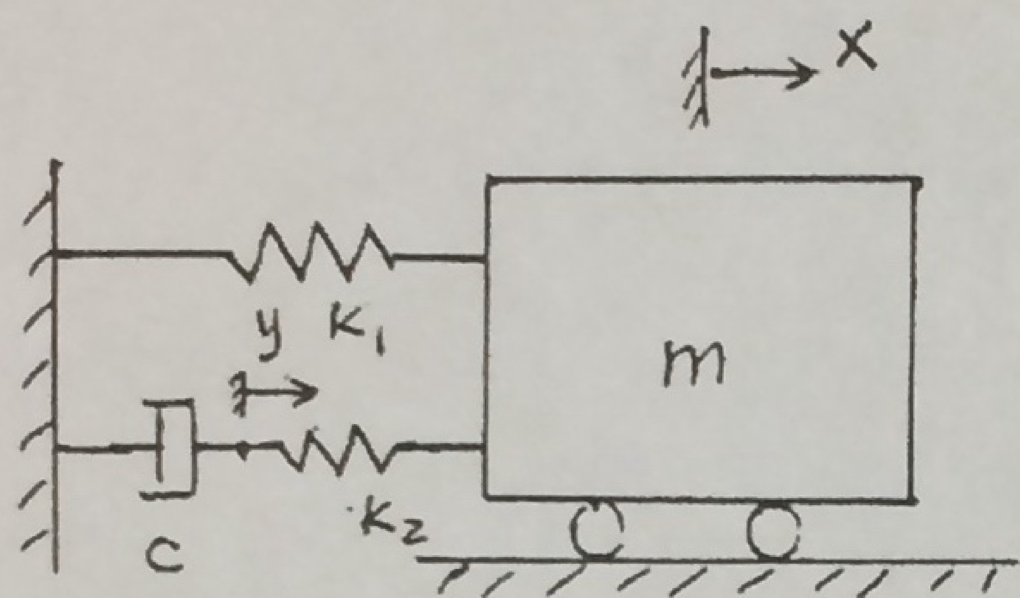
(1)



Independent variables: x_c , x_m or x_m, θ

DOF = 2

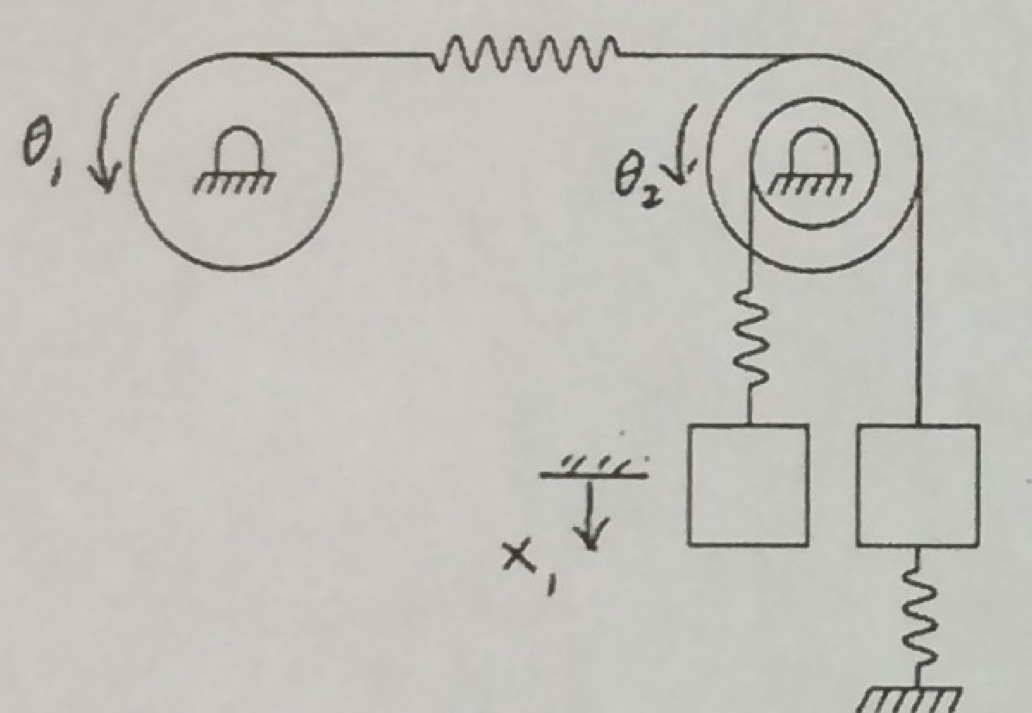
(2)



Independent variables: x , y

DOF = 2

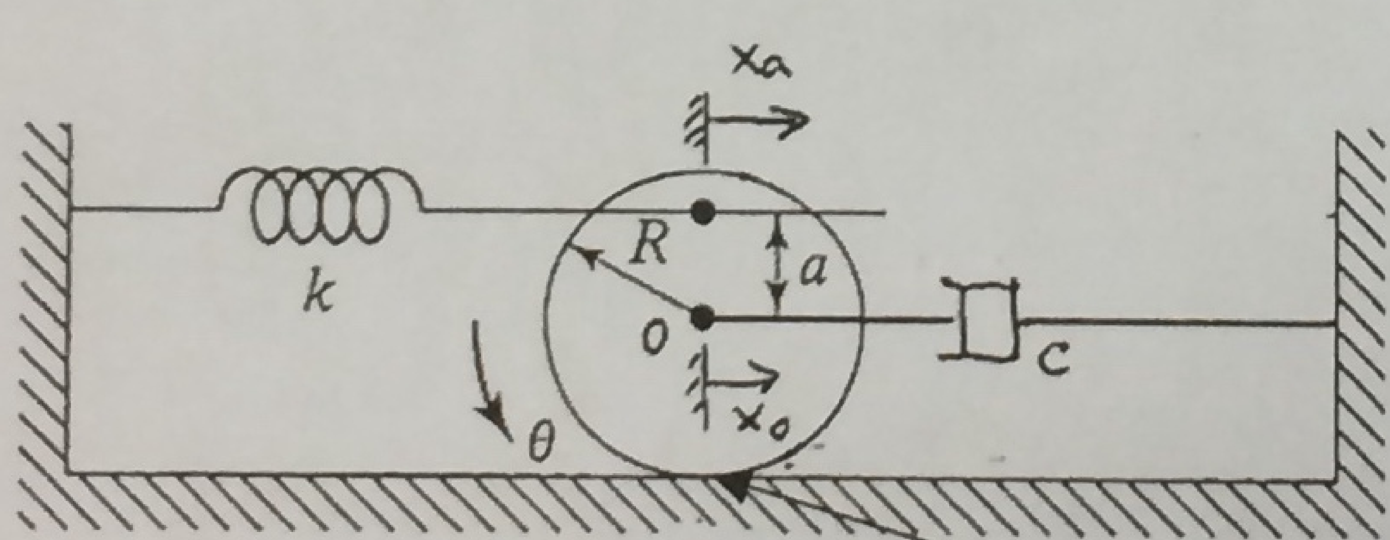
(3)



Independent variables: θ_1, θ_2, x_1 or θ_1, x_1, x_2

DOF = 3

(4)

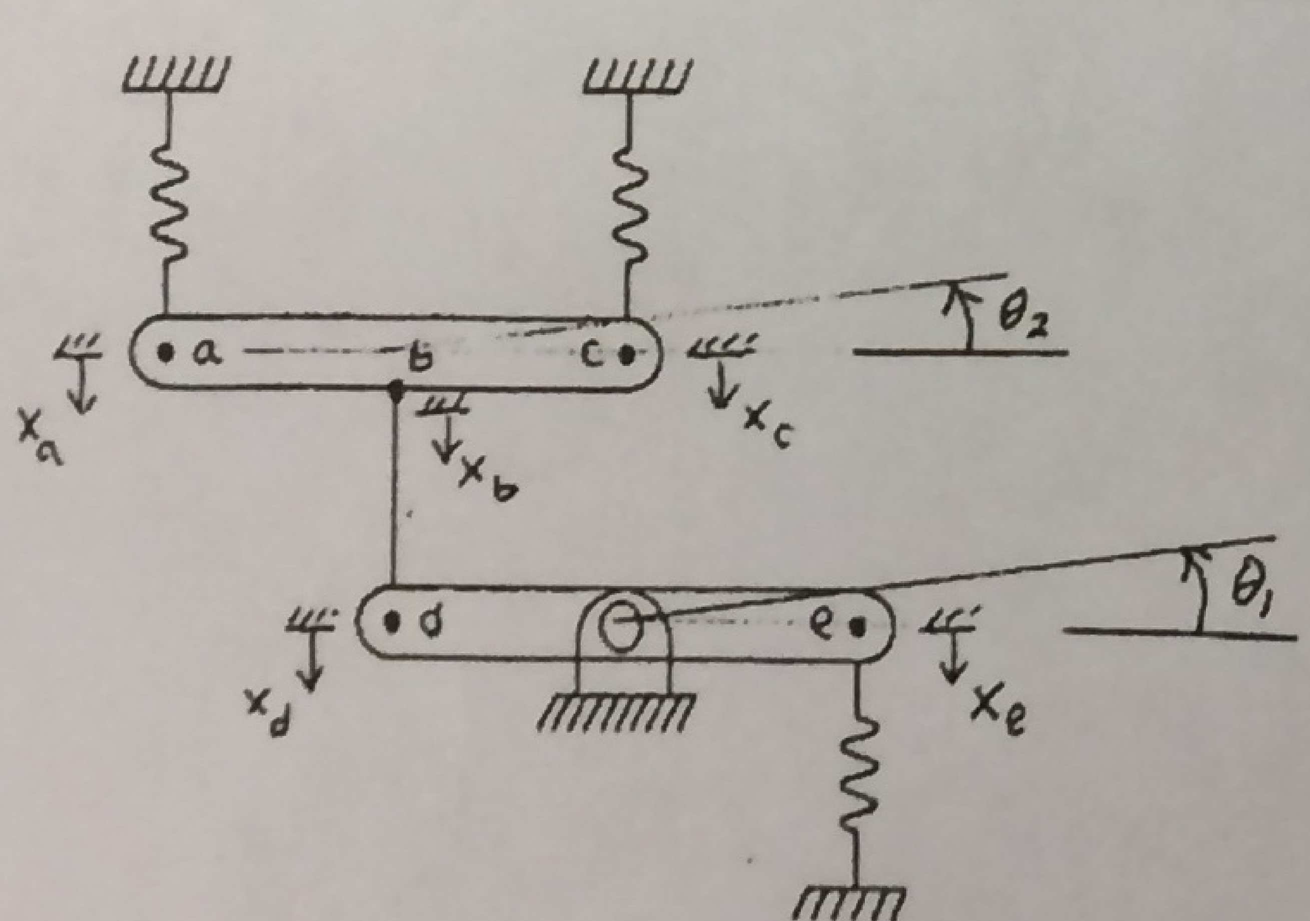


Wheel rolls without slip

Independent variables: θ, or x_a or x_o

DOF = 1

(5)



Independent variables: x_a, x_b

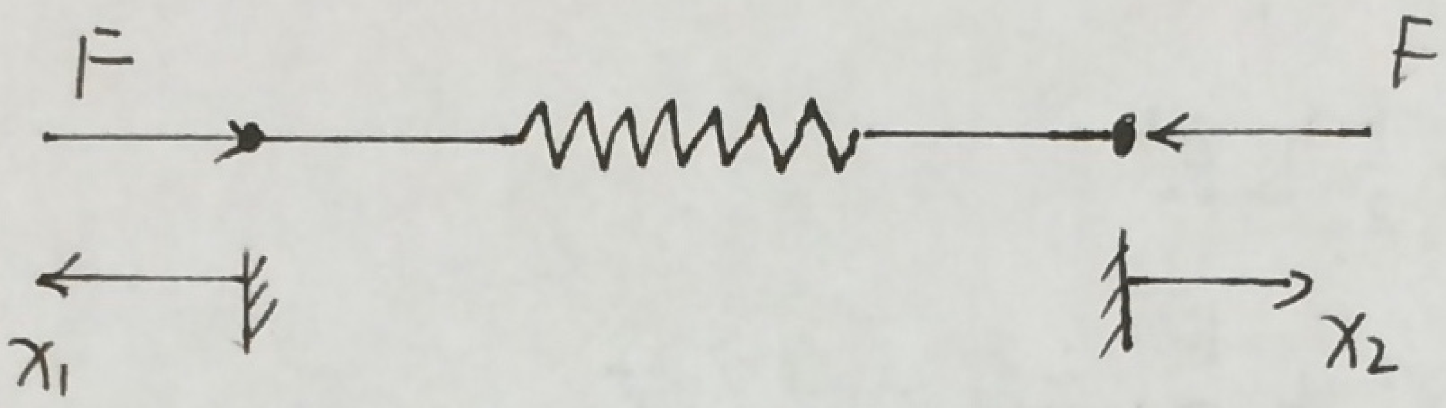
DOF = 2

2. (6 pts) Draw the free-body diagram and write the corresponding elemental equation for a spring element with the following definitions:

Spring moves horizontally. Compression force in the spring is considered positive. Displacement of the left end of the spring, x_1, is considered positive to the left, while that of the right end, x_2, considered positive to the right.

Confirm the correctness of your equation with two numerical examples. The example should show that your elemental equation produces a positive (negative) force when the spring is in compression (elongation) with some x_1 and x_2 combinations. **One example is to stretch the spring, and the other compress it.** Choose *nonzero* x_1 and x_2 values for both examples.

sol: ① FBD:



② elemental equation:

$$F = k(-x_1 - x_2) = -k(x_1 + x_2)$$

③ numerical examples:

a) stretch

$k = 1\,N/m$, $x_1 = 1\,m$, $x_2 = 2\,m$,

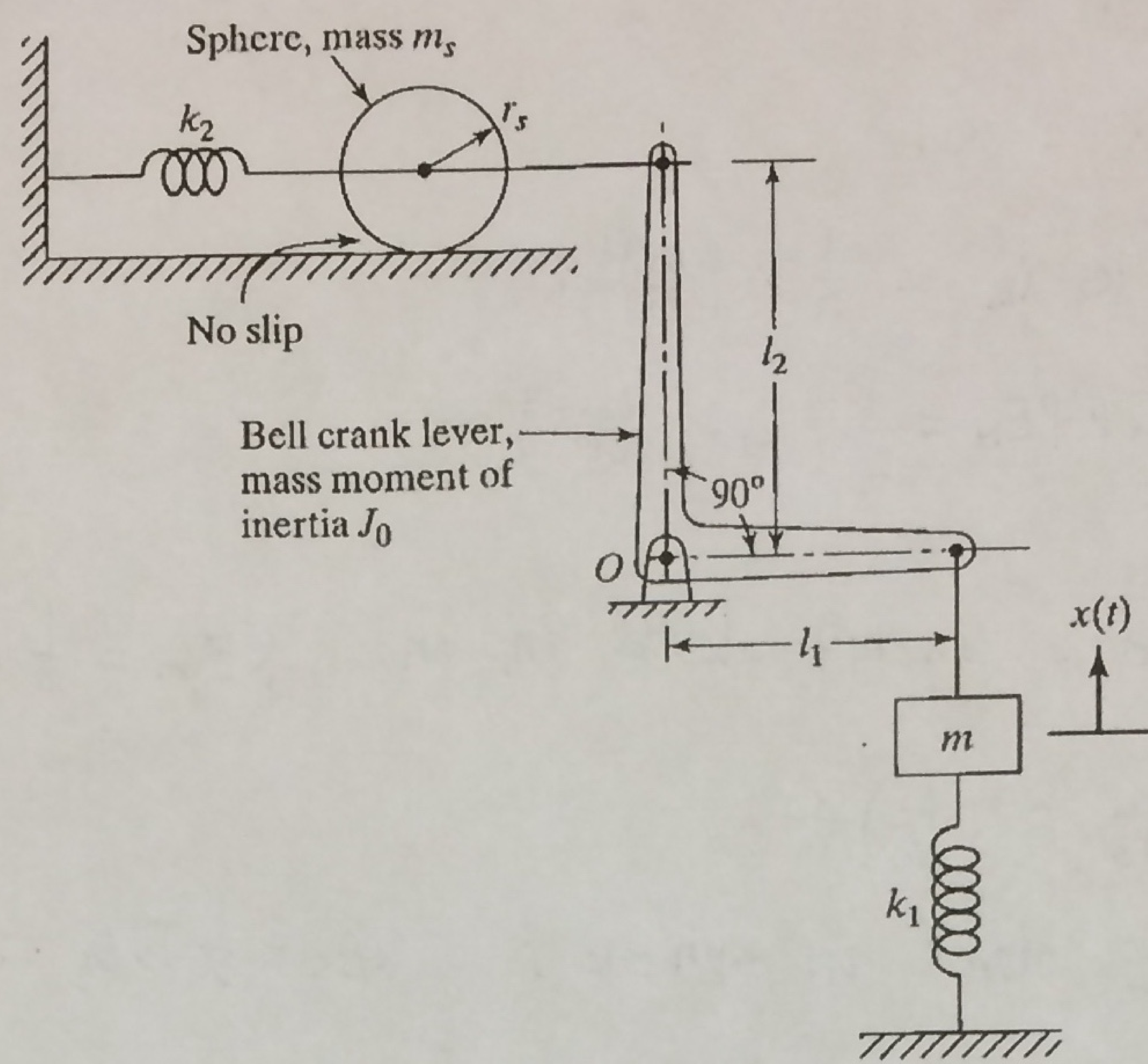
$F = -k(x_1 + x_2) = -1\,N/m \cdot (1m + 2m) = -3N$

b) compress

$k = 1\,N/m$, $x_1 = -1\,m$, $x_2 = -1\,m$,

$F = -k(x_1 + x_2) = -1\,N/m\,(-1m - 1m) = 2N$

3. Examine the 1DOF geometric model shown below. Let $x(t)$ be the degree of freedom variable.



(1) (2 pts) The moment of inertia of the solid sphere with respect to its center of gravity is b

a) $J_s = \frac{1}{4} m_s r_s^2$ b) $J_s = \frac{2}{5} m_s r_s^2$ c) $J_s = \frac{3}{4} m_s r_s^2$

(2) (2 pts) Let the CW angular displacement of the sphere be θ_s and the to-the-right displacement of the center of the sphere be x_s. The relation between x_s and θ_s is: a

a) $x_s = r_s \theta_s$ b) $x_s = 2\pi r_s \theta_s$ c) $x_s = 2 r_s \theta_s$

(3) (4 pts) The sum of potential energies stored in the spring elements is e

a) $PE = \frac{1}{2}(k_1 + k_2)x^2$

b) $PE = \frac{1}{2}\left(k_1 + \left(\frac{l_1}{l_2}\right)k_2\right)x^2$

c) $PE = \frac{1}{2}\left(k_1 + \left(\frac{l_1}{l_2}\right)^2 k_2\right)x^2$

d) $PE = \frac{1}{2}\left(k_1 + \left(\frac{l_2}{l_1}\right)k_2\right)x^2$

e) $PE = \frac{1}{2}\left(k_1 + \left(\frac{l_2}{l_1}\right)^2 k_2\right)x^2$

(4) (6 pts) The sum of kinetic energies stored in the system elements is d

a) $KE = \frac{1}{2}(m_s + m + J_o)\dot{x}^2$

b) $KE = \frac{1}{2}\left(1.2\frac{l_2}{l_1} m_s + m + \frac{1}{l_1} J_o\right)\dot{x}^2$

c) $KE = \frac{1}{2}\left(1.2\left(\frac{l_2}{l_1}\right)^2 m_s + m + \frac{1}{l_1^2} J_o\right)\dot{x}^2$

d) $KE = \frac{1}{2}\left(1.4\left(\frac{l_2}{l_1}\right)^2 m_s + m + \frac{1}{l_1^2} J_o\right)\dot{x}^2$

e) $KE = \frac{1}{2}\left(1.4\frac{l_2}{l_1} m_s + m + \frac{1}{l_1} J_o\right)\dot{x}^2$

3. (3) sol: PE stored in k_1: $PE_1 = \frac{1}{2} k_1 x^2$

PE stored in k_2: $PE_2 = \frac{1}{2} k_2 x_s^2 = \frac{1}{2} k_2 \left(\frac{l_2}{l_1}\right)^2 x^2$

$\therefore$ Total PE stored in the spring elements:

$$PE = PE_1 + PE_2 = \frac{1}{2}\left(k_1 + \left(\frac{l_2}{l_1}\right)^2\right) x^2$$

(4) sol: KE stored in m: $KE_m = \frac{1}{2} m \dot{x}^2$

KE stored in the bell crank lever: $KE_o = \frac{1}{2} J_o \dot{\theta}_o^2 = \frac{1}{2} J_o \left(\frac{\dot{x}}{l_1}\right)^2$

KE stored in the sphere:

$$KE_s = \frac{1}{2} m_s \dot{x}_s^2 + \frac{1}{2} J_s \dot{\theta}_s^2$$

$$= \frac{1}{2} m_s \left(\frac{l_2}{l_1}\right)^2 \dot{x}^2 + \frac{1}{2} \cdot \frac{2}{5} m_s r_s^2 \cdot \left(\frac{\dot{x}_s}{r_s}\right)^2$$

$$= \frac{1}{2} \cdot \frac{7}{5} m_s \left(\frac{l_2}{l_1}\right)^2 \dot{x}^2$$

$\therefore$ Total KE stored in the system is:

$$KE = KE_m + KE_o + KE_s = \frac{1}{2}\left(\frac{7}{5}\left(\frac{l_2}{l_1}\right)^2 m_s + m + \frac{1}{l_1^2} J_o\right) \dot{x}^2$$