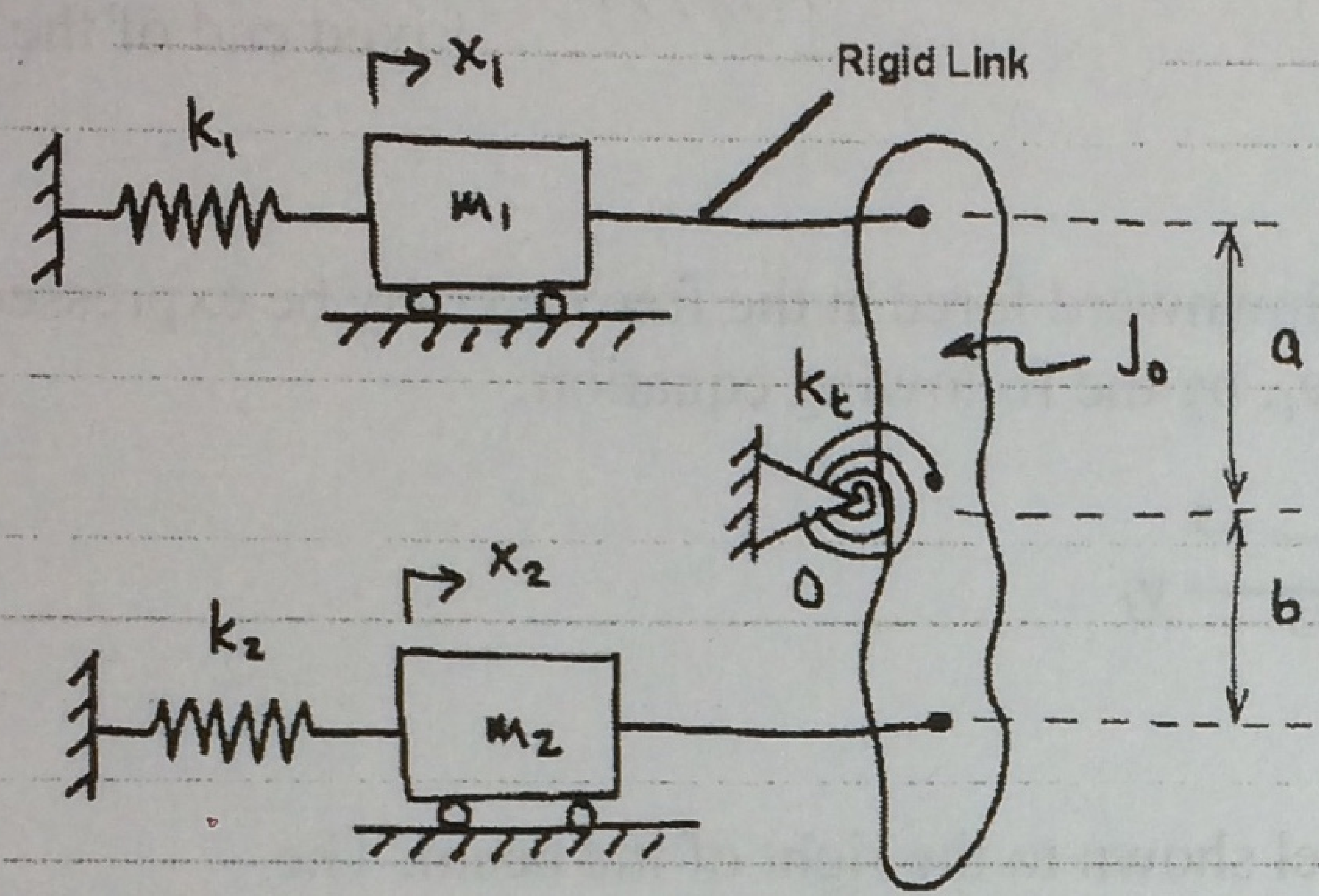
ME 370.1 Fall 2018

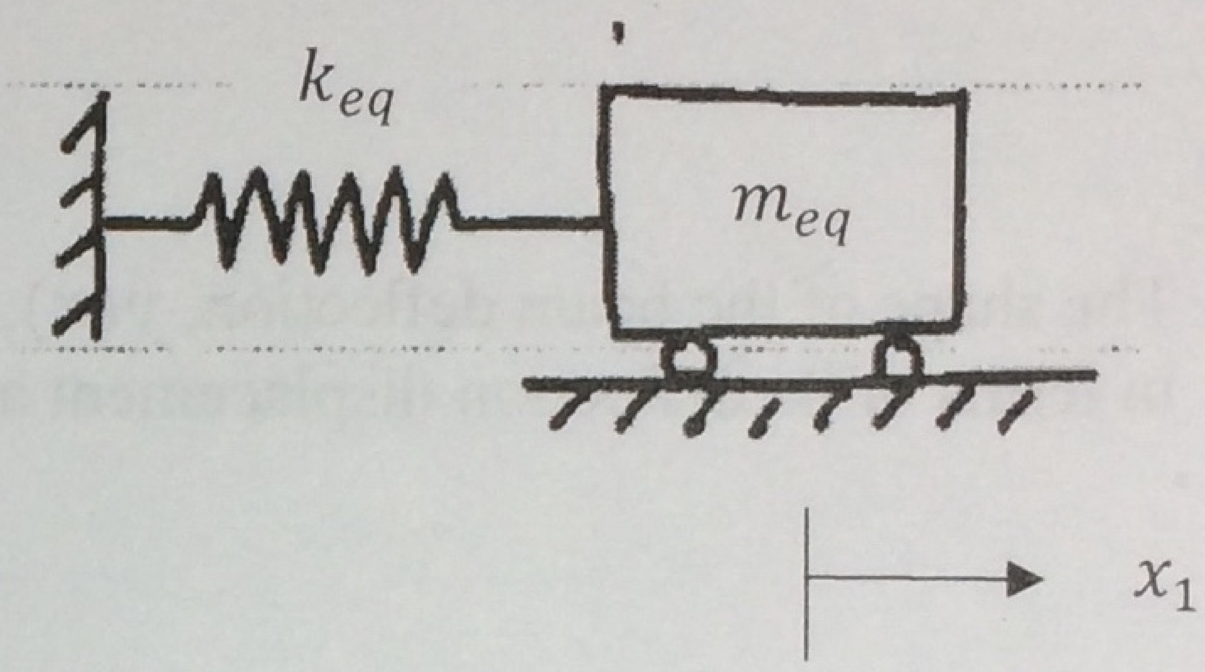
Name(s)

Homework 3

Due: *In class*, Friday 9/14

1. (10 pts) Examine the 1DOF system shown below. It may be reduced to a mass-spring model located at the location of m_1 as shown to the right of the system.





The equivalent mass, m_{eq}, of the model is given by:

a) $m_{eq} = m_1 + J_o + m_2$

b) $m_{eq} = m_1 + \frac{J_o}{a} + \frac{b}{a} m_2$

c) $m_{eq} = m_1 + \frac{J_o}{b} + \frac{a}{b} m_2$

d) $m_{eq} = m_1 + \frac{J_o}{a^2} + \frac{b^2}{a^2} m_2$

e) $m_{eq} = m_1 + \frac{J_o}{b^2} + \frac{a^2}{b^2} m_2$

sol: $KE_m = \frac{1}{2} m_{eq} \dot{x}_1^2$

$KE_s = \frac{1}{2} m_1 \dot{x}_1^2 + \frac{1}{2} m_2 \dot{x}_2^2 + \frac{1}{2} J_o \dot{\theta}^2$

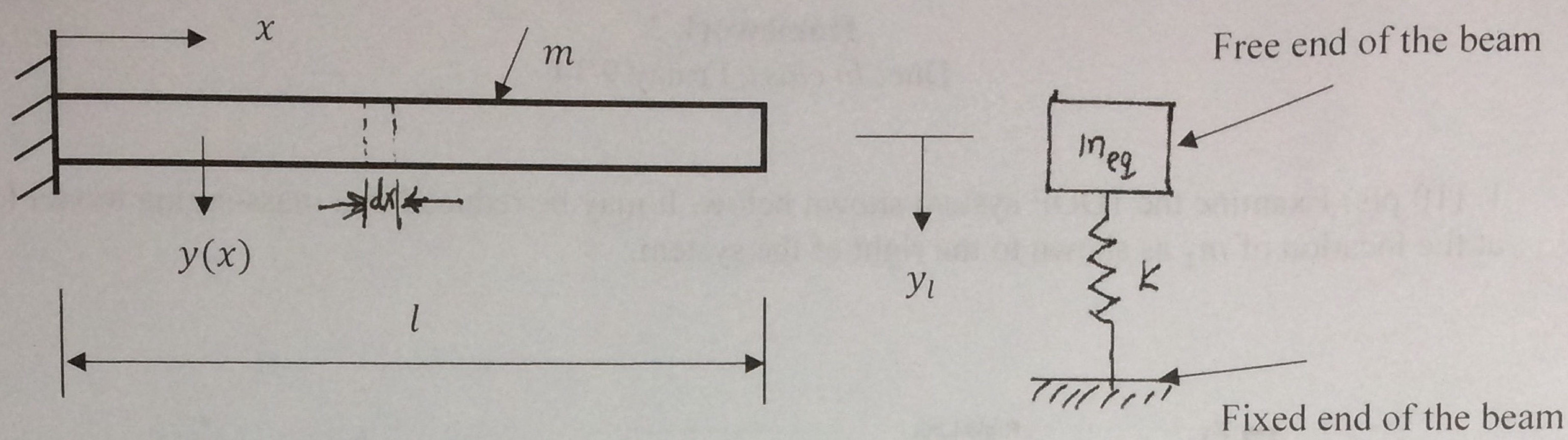
since $\boxed{\dot{x}_1 = a\dot{\theta}, \quad \frac{\dot{x}_1}{a} = \frac{\dot{x}_2}{b}}$

$KE_s = \frac{1}{2} m_1 \dot{x}_1^2 + \frac{1}{2} m_2 \left(\frac{b}{a}\dot{x}_1\right)^2 + \frac{1}{2} J_o \left(\frac{\dot{x}_1}{a}\right)^2$

$= \frac{1}{2}\left(m_1 + \frac{b^2}{a^2} m_2 + \frac{J_o}{a^2}\right)\dot{x}_1^2$

$KE_s = KE_m$, thus $m_{eq} = m_1 + \frac{J_o}{a^2} + \frac{b^2}{a^2} m_2$

2. (10 pts) Consider a uniform-cross-section cantilever beam of mass m and length l shown below:



The shape of the beam deflection, $y(x)$, under a static downward force at the free end may be expressed in terms of its deflection displacement at the free end, y_l, by the following equation:

$$y(x) = \frac{3lx^2 - x^3}{2l^3} y_l$$

The beam may be approximated by a mass-spring model shown to the right of the beam. The equivalent mass, m_{eq}, of the model is given by:

a) $m_{eq} = 0.123m$

b) $m_{eq} = 0.236m$

c) $m_{eq} = 0.344m$

d) $m_{eq} = 0.457m$

e) $m_{eq} = 0.529m$

sol: model: $KE_m = \frac{1}{2} m_{eq} \dot{y}_l^2$

system: the mass of a beam segment of length dx is: $\boxed{dm_x = \frac{m}{l} dx}$

and the velocity profile over the length of the beam is:

$$\dot{y}(x) = \frac{3lx^2 - x^3}{2l^3} \dot{y}_l$$

Then the total KE of the beam is:

$$KE_s = \int_0^l \frac{1}{2} dm_x \cdot (\dot{y}(x))^2$$

$$= \int_0^l \frac{1}{2} \frac{m}{l} \left(\frac{3lx^2 - x^3}{2l^3}\right)^2 \dot{y}_l^2 \, dx$$

$$= \frac{1}{2} \frac{m}{l} \dot{y}_l^2 \cdot \int_0^l \left(\frac{3lx^2 - x^3}{2l^3}\right) dx$$

$$= \frac{1}{2} \frac{m}{l} \cdot \dot{y}_l^2 \cdot \frac{33l}{140}$$

$$= \frac{1}{2} \frac{33m}{140} \dot{y}_L^2$$

So, $m_{eq} = \frac{33m}{140} \approx 0.236m$

3. (14 pts) Refer to the figure below of three torsional dampers on geared shafts. The gear on shaft 1 has n_1 teeth, the gear on shaft 2 has n_2 teeth, and the gear on shaft 3 has n_3 teeth. Let $\omega_1, \omega_2, \omega_3$ be the angular velocities of the three shafts and J_1, J_2, J_3 be the moments of inertia of the three rotating bodies. The system may be modeled as one inertia and one torsional damper (J_{eq} and C_{eq}) located at the third shaft (ie. shaft with n_3 and c_{t_3}).

sol: (1) since $\boxed{\frac{W_1}{W_2} = \frac{n_2}{n_1} \qquad \frac{W_2}{W_3} = \frac{n_3}{n_2}}$

$$\Rightarrow \frac{W_1}{W_2} \cdot \frac{W_2}{W_3} = \frac{n_2}{n_1} \cdot \frac{n_3}{n_2}$$

$$\Rightarrow \frac{W_1}{W_3} = \frac{n_3}{n_1}$$

$$\Rightarrow W_3 = \left(\frac{n_1}{n_3}\right) W_1$$

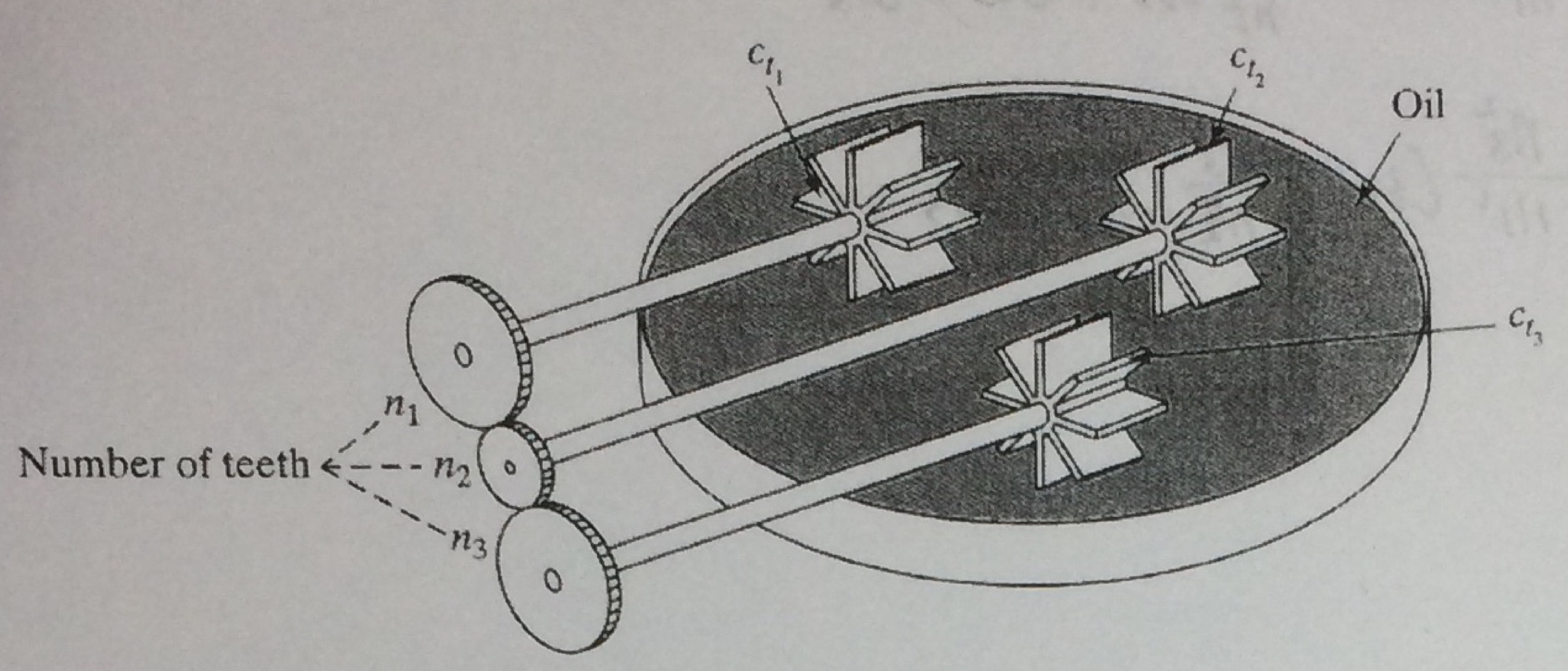


FIGURE 1.82 Dampers located on geared shafts.

(1) (2 pts) The angular velocity of shaft 3 is related to the angular velocity of shaft 1 by

a) $\omega_3 = (n_1 / n_3)\omega_1$ b) $\omega_3 = (n_3 / n_1)\omega_1$

c) $\omega_3 = (n_1 / n_3)^2 \omega_1$ d) $\omega_3 = (n_3 / n_1)^2 \omega_1$

(2) (5 pts) The inertia of the model is

a) $J_{eq} = (J_1 + J_2 + J_3)/3$ b) $J_{eq} = 3(J_1 + J_2 + J_3)$

c) $J_{eq} = \left(\frac{n_3}{n_1}J_1 + \frac{n_3}{n_2}J_2 + J_3\right)$ d) $J_{eq} = \left(\frac{n_3}{n_1}J_1^2 + \frac{n_3}{n_2}J_2^2 + J_3^2\right)^{1/2}$

e) $J_{eq} = \left(\frac{n_3^2}{n_1^2}J_1 + \frac{n_3^2}{n_2^2}J_2 + J_3\right)$ f) $J_{eq} = \left(\frac{n_3^2}{n_1^2}J_1^2 + \frac{n_3^2}{n_2^2}J_2^2 + J_3^2\right)^{1/2}$

(3) (7 pts) Use the energy method to determine the damping constant of the model, C_{eq}. Show essential work to receive credits.

3.(2) sol: model: $KE_m = \frac{1}{2} J_{eq} W_3^2$

system: $KE_s = \frac{1}{2} J_1 W_1^2 + \frac{1}{2} J_2 W_2^2 + \frac{1}{2} J_3 W_3^2$

use the result of 3.(1).

we have: $KE_s = \frac{1}{2} J_1 \left(\frac{n_3}{n_1}\right)^2 W_3^2 + \frac{1}{2} J_2 \left(\frac{n_3}{n_2}\right) W_3^2 + \frac{1}{2} J_3 W_3^2$

$$= \frac{1}{2}\left(\frac{n_3^2}{n_1^2} J_1 + \frac{n_3^2}{n_2^2} J_2 + J_3\right) W_3^2$$

$$\Rightarrow J_{eq} = \frac{n_3^2}{n_1^2} J_1 + \frac{n_3^2}{n_2^2} J_2 + J_3$$

3.(3) sol: model: $\dot{E}_d = C_{eq} \cdot \omega_3^2$

system: $\dot{E}_d = C_{t1}\omega_1^2 + C_{t2}\omega_2^2 + C_{t3}\omega_3^2$

$$= C_{t1}\left(\frac{n_3}{n_1}\right)^2 \omega_3^2 + C_{t2}\left(\frac{n_3}{n_2}\right)^2 \omega_3^2 + C_{t3}\omega_3^2$$

$$= \left(\frac{n_3^2}{n_1^2} \cdot C_{t1} + \frac{n_3^2}{n_2^2} C_{t2} + C_{t3}\right) \omega_3^2$$

$$\Rightarrow C_{eq} = \frac{n_3^2}{n_1^2} C_{t1} + \frac{n_3^2}{n_2^2} C_{t2} + C_{t3}$$