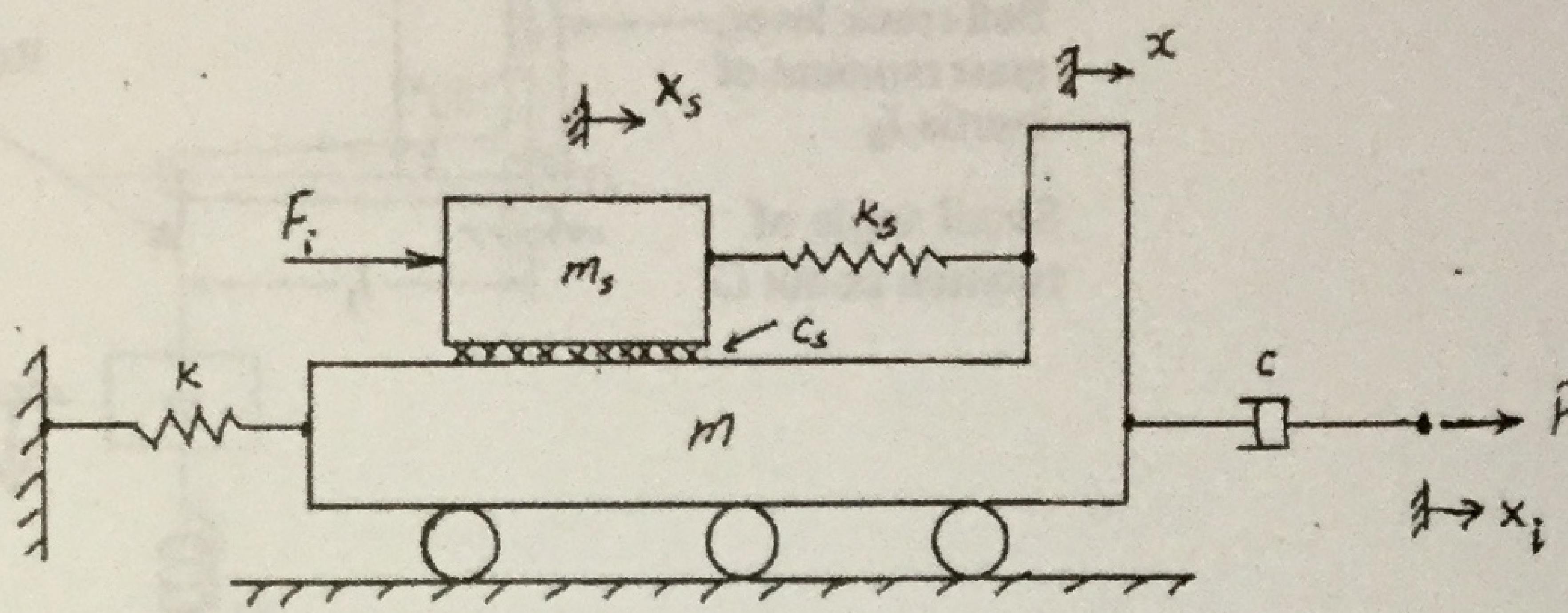
ME 370.1 Fall 2018 Name(s)

Homework 4

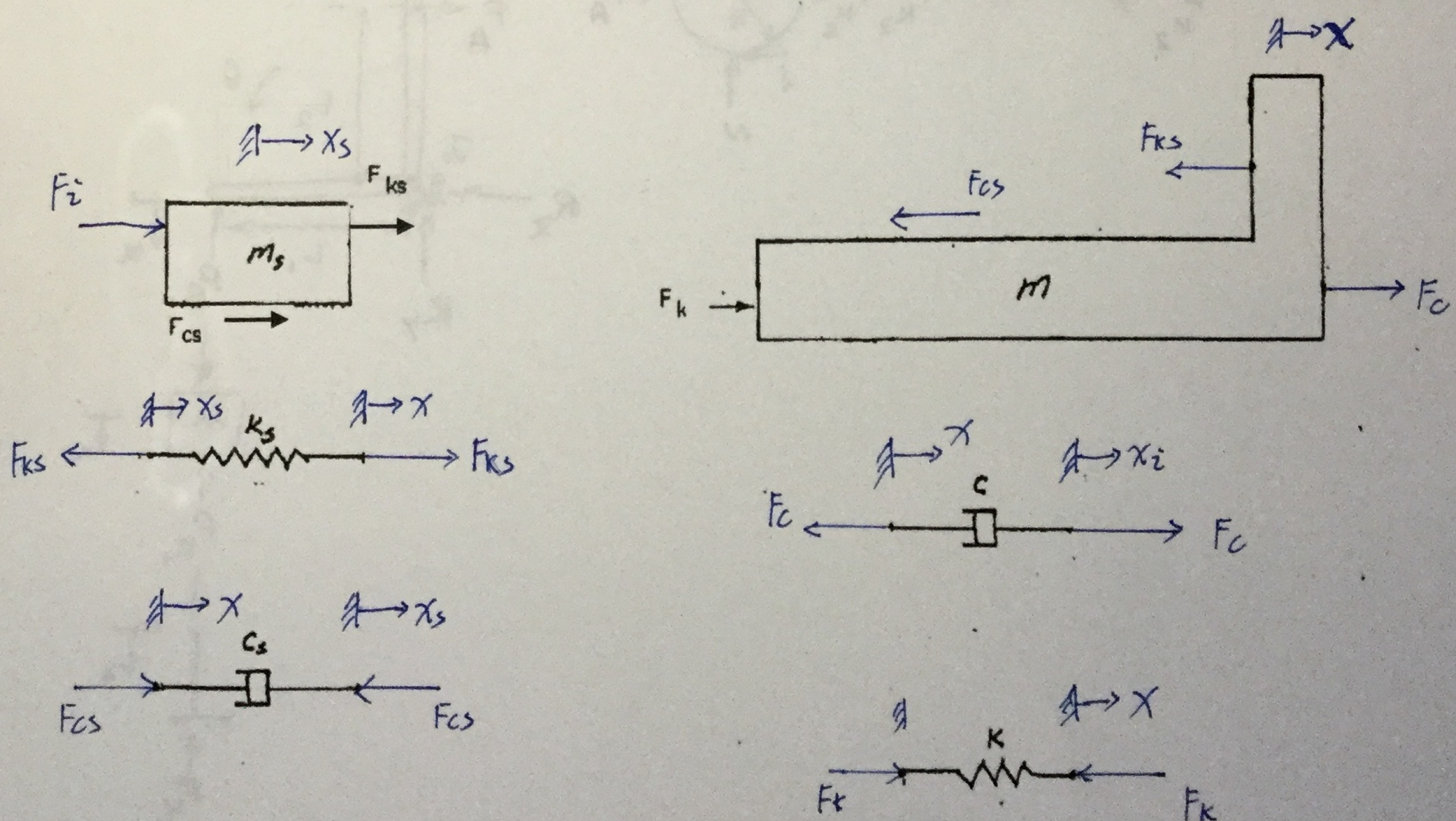
Due: *In class*, Friday, 9/21

1. (12 pts) A 2DOF geometric model of a mechanical system is shown below. The system is driven by two inputs: force F_i on mass m_s and force F_c on the right part of damper c. The mass m_s can slide on the surface of mass m. The sliding surface is lubricated with oil so that the friction may be modeled using a viscous damper of damping coefficient c_s.

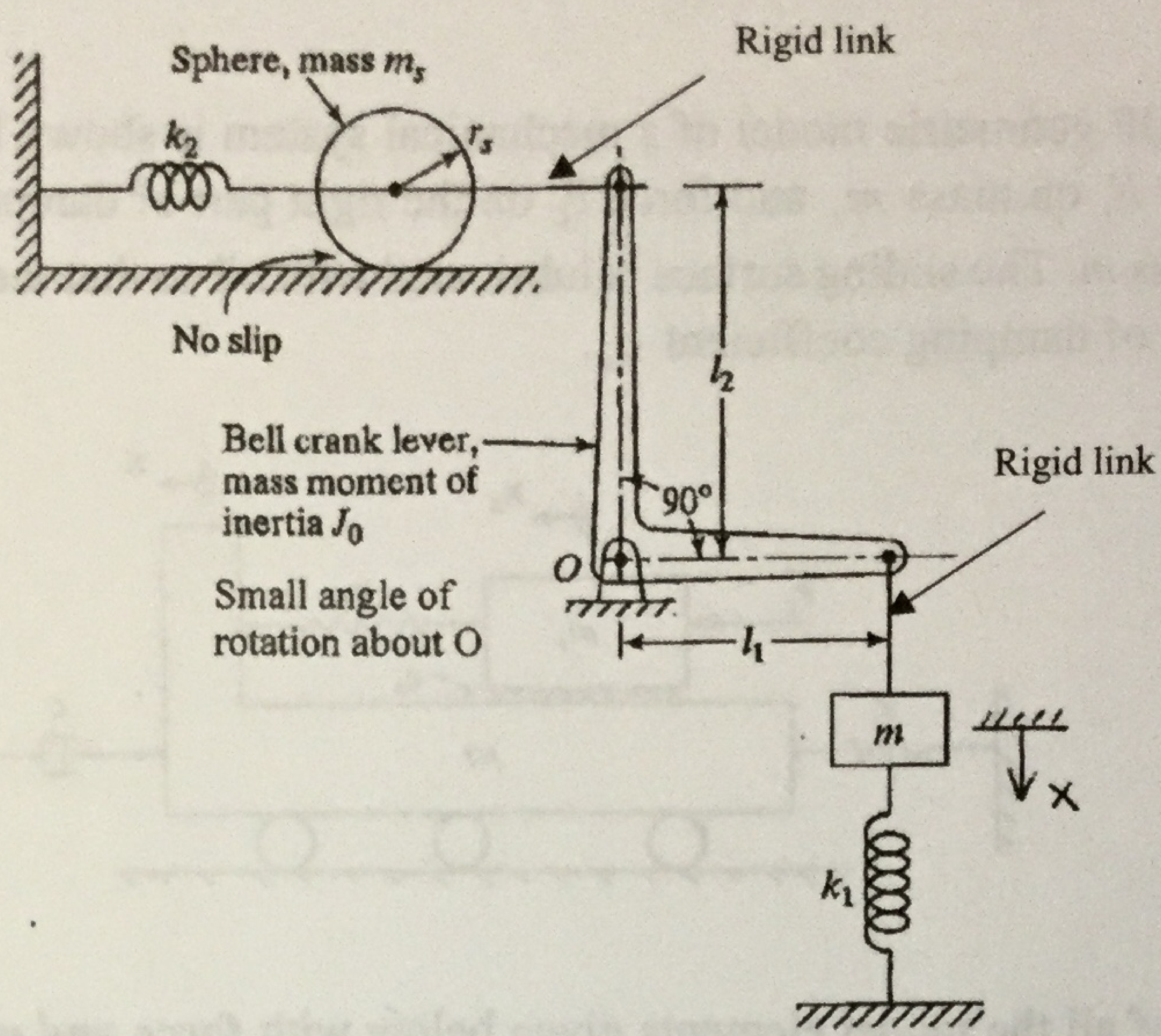


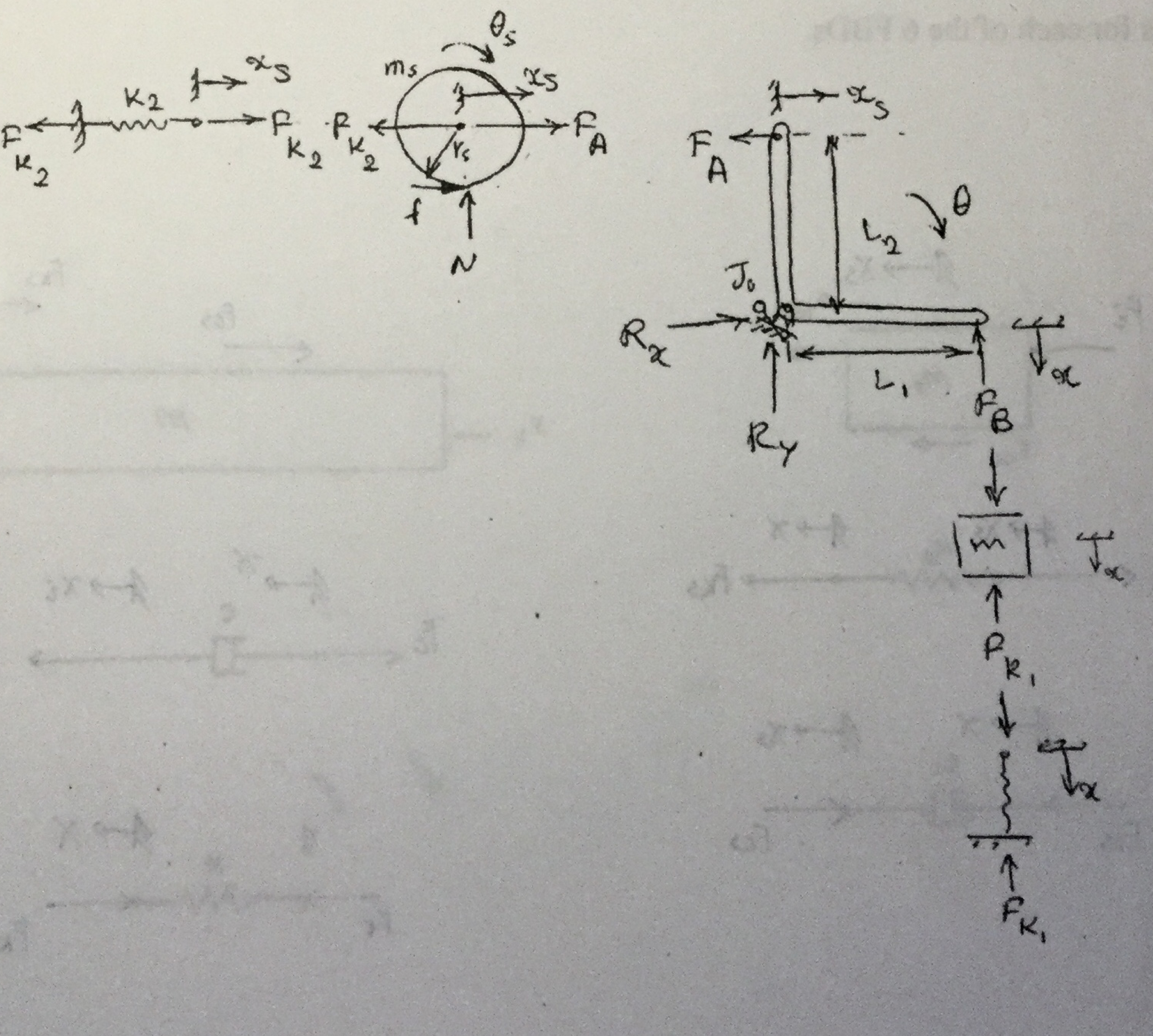
Draw the FBDs of all the model elements given below with ***force and motion variables***. Make sure all FBDs are ***complete*** and ***consistent*** with the forces, F_{cs}, F_{ks} and F_k, labelled on the incomplete FBDs of the two masses. Pay attention to action/reaction pair of forces between two connected elements.

2 pts for each of the 6 FBDs.



2. (15 pts) Examine the SDOF geometric model and the FBDs below, where J_o is the moment of inertia of the crank with respect to the pivot "O".





Problem 2-(2)

sol: ① FBD method

the ele eqs have been listed in 2.(1).

combine ①②, get: $F_B = m\ddot{x} + F_K = m\ddot{x} + k_1 x$ --- ⑩

combine ⑥⑧, get: $F_{K2} = \frac{k_2 l_2}{l_1} x$ --- ⑪

combine ④⑤, get: $m_s \ddot{x}_s = F_A - F_{K2} - \frac{J_s \ddot{\theta}_s}{r_s}$ --- ⑫

substitute ⑧⑨⑪ into ⑫, get: $m_s \frac{l_2}{l_1} \ddot{x} = F_A - \frac{k_2 l_2}{l_1} x - \frac{J_s}{r_s} \cdot \frac{l_2}{l_1 r_s} \ddot{x}$

$$\Rightarrow F_A = \frac{l_2 m_s}{l_1} \ddot{x} + \frac{l_2 J_s}{l_1 r_s^2} \ddot{x} + \frac{k_2 l_2}{l_1} x \quad \text{---} ⑬$$

substitute ⑦⑩⑬ into ③ get:

$$J_0 \frac{\ddot{x}}{l_1} = -l_2 \left(\frac{l_2 m_s}{l_1} \ddot{x} + \frac{l_2 J_s}{l_1 r_s^2} \ddot{x} + \frac{k_2 l_2}{l_1} x \right) - l_1 (m\ddot{x} + k_1 x)$$

$$\Rightarrow \left(\frac{J_0}{l_1} + \frac{l_2^2 m_s}{l_1} + \frac{l_2^2 J_s}{l_1 r_s^2} + l_1 m \right) \ddot{x} + \left(\frac{k_2 l_2^2}{l_1} + k_1 l_1 \right) x = 0$$

multiple $\frac{1}{l_1}$, $\Rightarrow \left(m + \frac{J_0}{l_1^2} + \frac{l_2^2 (J_s + m_s r_s^2)}{r_s^2 l_1^2} \right) \ddot{x} + \left(k_1 + k_2 \frac{l_2^2}{l_1^2} \right) x = 0$ ⑬

② energy method.

From the problem 3 at Hw 1, we know that

$$\Sigma KE = \frac{1}{2} \left(m + \frac{J_0}{l_1^2} + \frac{l_2^2 (J_s + m_s r_s^2)}{r_s^2 l_1^2} \right) \dot{x}^2 = \frac{1}{2} m_{eq} \dot{x}^2 \quad \text{---} ⓐ$$

$$\Sigma PE = \frac{1}{2} \left(k_1 + \left(\frac{l_2^2}{l_1^2} \right) k_2 \right) x^2 = \frac{1}{2} k_{eq} x^2 \quad \text{---} ⓑ$$

Since for the undamped vibration system, we have

$$\frac{d}{dt} (\Sigma KE + \Sigma PE) = 0 \quad \text{---} ⓒ$$

ⓐⓑⓒ $\Rightarrow \frac{d}{dt} \left(\frac{1}{2} m_{eq} \dot{x}^2 + \frac{1}{2} k_{eq} x^2 \right) = 0$

$$\Rightarrow \frac{1}{2} m_{eq} 2 \dot{x} \ddot{x} + \frac{1}{2} k_{eq} 2 x \dot{x} = 0$$

$$\Rightarrow m_{eq} \ddot{x} + k_{eq} x = 0$$

The governing equation is obtained.

(1) (9 pts) Write a complete set of elemental equations (1 pt each, no partial credit)

a) The ele eq. for k_1 is: $F_{k1} = k_1 X$ --- ①

b) The ele eq. for m is: $m\ddot{x} = F_B - F_{k1}$ --- ②

c) The ele eq. for J_o is: $J_o\ddot{\theta} = -F_A l_2 - F_B l_1$ --- ③

d) The ele eq. for m_s is: $m_s\ddot{x}_s = F_A - F_{k2} + f$ --- ④

e) The ele eq. for J_s is: $J_s\ddot{\theta}_s = -f r_s$ --- ⑤

(J_s is the sphere moment of inertia about the center of sphere)

f) The ele eq. for k_2 is: $F_{k2} = k_2 X_s$ --- ⑥

g) The motion relation between θ and x is: $\theta = \frac{X}{l_1}$ --- ⑦

h) The motion relation between x_s and x is: $x_s = \frac{l_2}{l_1} X$ --- ⑧

i) The motion relation between θ_s and x is: $\theta_s = \frac{X_s}{r_s} = \frac{l_2}{l_1 r_s} X$ --- ⑨

(2) (6 pts) The governing equation of the system in x variable is

a) $\left(m + \frac{J_o}{l_1^2} + \frac{J_s + m_s r_s^2}{r_s^2}\right)\ddot{x} + (k_1 + k_2)x = 0$

b) $\left(m + \frac{J_o}{l_1^2} + \frac{l_2^2}{l_1^2}\frac{J_s + m_s r_s^2}{r_s^2}\right)\ddot{x} + \left(k_1 + \frac{l_2^2}{l_1^2}k_2\right)x = 0$

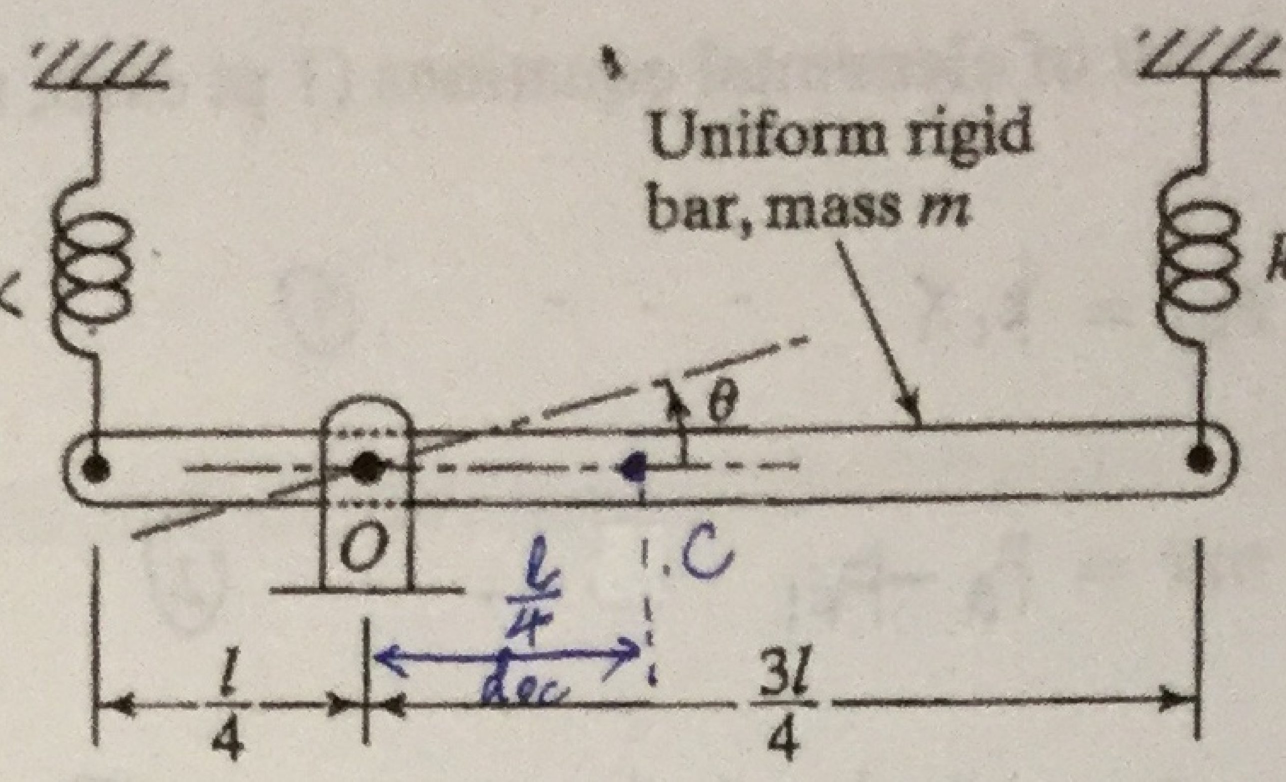
c) $\left(m + \frac{J_o}{l_1^2} + \frac{l_2^2}{l_1^2}\frac{J_s + m_s r_s^2}{r_s^2}\right)\ddot{x} + (k_1 + k_2)x = 0$

d) $\left(m + \frac{J_o}{l_1^2} + \frac{J_s + m_s r_s^2}{r_s^2}\right)\ddot{x} + \left(\frac{l_2^2}{l_1^2}k_1 + k_2\right)x = 0$

e) $\left(m + \frac{J_o}{l_1^2} + \frac{l_2^2}{l_1^2}\frac{J_s + m_s r_s^2}{r_s^2}\right)\ddot{x} + \left(\frac{l_2^2}{l_1^2}k_1 + k_2\right)x = 0$

f) $\left(m + \frac{J_o}{l_1^2} + \frac{J_s + m_s r_s^2}{r_s^2}\right)\ddot{x} + \left(k_1 + \frac{l_2^2}{l_1^2}k_2\right)x = 0$

3. (10 pts) Examine the SDOF geometric model below:



(1) (3 pts) The moment of inertia of the bar about the pivot point O is

a) $J_o = \frac{1}{12}ml^2$

b) $J_o = \frac{7}{48}ml^2$

c) $J_o = \frac{5}{24}ml^2$

d) $J_o = \frac{1}{3}ml^2$

e) $J_o = ml^2$

sol: The moment of inertia of the bar about its center C is:

$$J_C = \frac{1}{12} m l^2$$

According to parallel axis theorem,

$$J_o = J_C + m d_{oc}^2$$
$$= \frac{1}{12} m l^2 + m\left(\frac{l}{4}\right)^2$$
$$= \frac{7}{48} m l^2$$

(2) (7 pts) The governing equation of the system is

a) $7m\ddot{\theta} + 24k\theta = 0$

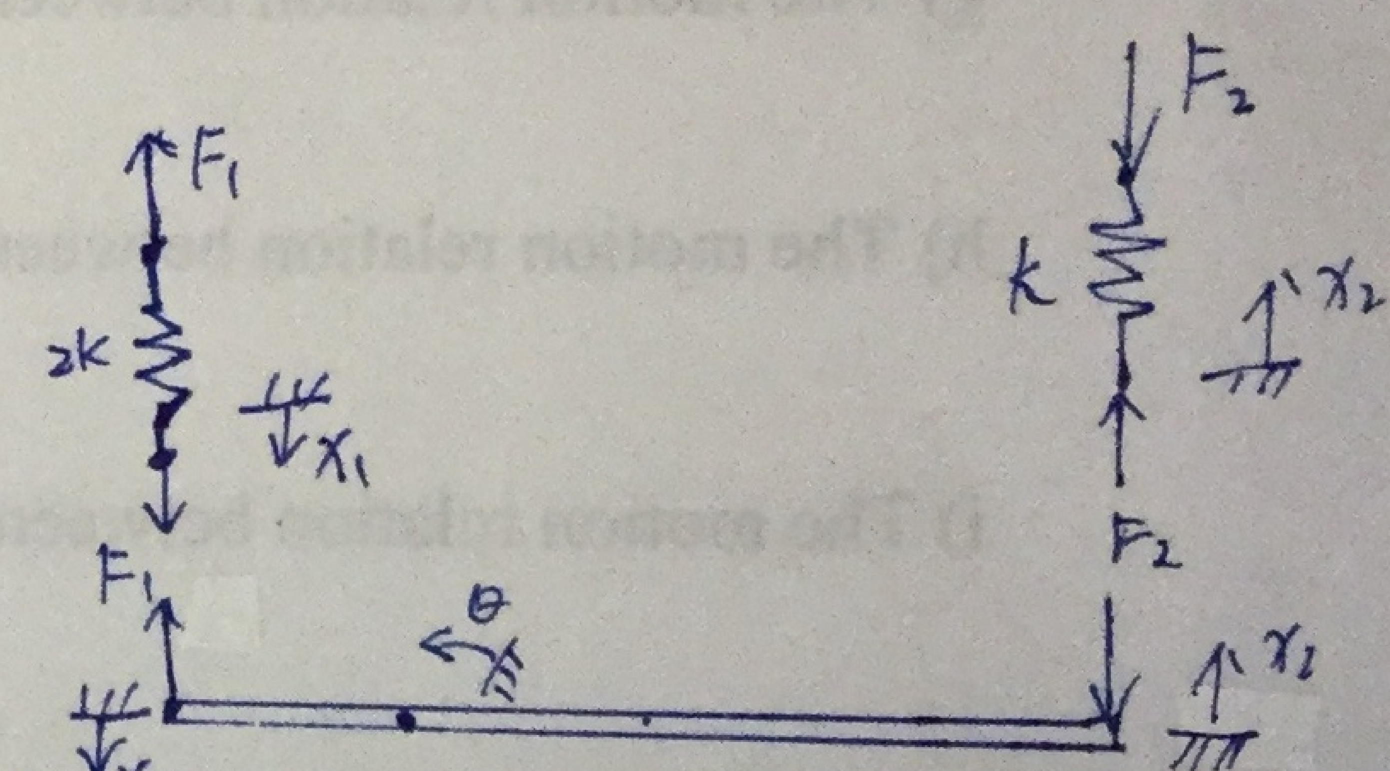
b) $7m\ddot{\theta} + 33k\theta = 0$

c) $7m\ddot{\theta} + 48k\theta = 0$

d) $5m\ddot{\theta} + 24k\theta = 0$

e) $5m\ddot{\theta} + 33k\theta = 0$

f) $5m\ddot{\theta} + 48k\theta = 0$



sol: ① FBD method

ele eq.

2k: $F_1 = 2k x_1$ --- ①

k: $F_2 = k x_2$ --- ②

bar: $F_1 \cdot \frac{l}{4} + F_2 \cdot \frac{3}{4} l = -J_o \ddot{\theta}$ --- ③

motion relation: $x_1 = \frac{l}{4}\theta$ --- ④

$x_2 = \frac{3l}{4}\theta$ --- ⑤

From ①④, we get $F_1 = 2k \cdot \frac{l}{4}\theta = \frac{kl}{2}\theta$ -- ⑥

From ②⑤, we get $F_2 = k\frac{3l}{4}\theta$ --- ⑦

substitute ⑥⑦ into ③, get $\frac{kl^2}{8}\theta + \frac{9kl^2}{16}\theta = -J_o\ddot{\theta}$

using the result of 3.(1) $\Rightarrow \frac{11}{16} kl^2 \theta = -\frac{7}{48} ml^2 \ddot{\theta}$

$\Rightarrow 7m\ddot{\theta} + 33k\theta = 0$

② energy method.

$$\Sigma PE = \frac{1}{2} 2k x_1^2 + \frac{1}{2} k x_2^2$$
$$= k\left(\frac{l}{4}\theta\right)^2 + \frac{1}{2}k\left(\frac{3}{4}l\right)^2\theta^2$$
$$= \frac{11}{32} k\theta^2$$

$$\Sigma KE = \frac{1}{2} J_o \dot{\theta}^2 = \frac{7}{96} ml^2 \dot{\theta}^2$$

since $\frac{d}{dt}(\Sigma PE + \Sigma KE) = 0$

$\Rightarrow \frac{11}{32} k \, 2\theta\dot{\theta} + \frac{7}{96} ml^2 \, 2\dot{\theta}\ddot{\theta} = 0 \quad \Rightarrow 7m\ddot{\theta} + 33k\theta = 0$