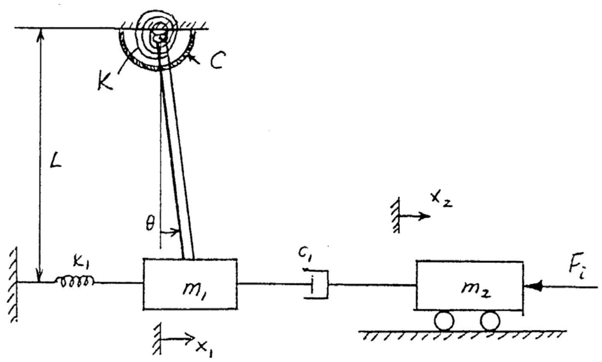
ME 370.1 Fall 2018 Name(s)

Homework 10

Due: *In class*, Friday, Nov. 9

1. (13 pts) A geometric model of a mechanical system is shown below. A ***torsional*** spring K and a torsional damper C are attached to the bar at the pivot. The inertia effect of the rigid bar is modeled by a lumped mass m_1 at the free end. This mass is connected to the ground through a spring k_1 and to a second mass m_2 through a damper c_1. The mass m_2 is supported on the ground by frictionless bearings and is driven by an input force F_i. All the motions are of small magnitudes.



(1) (3 pts) Draw FBD of the bar including the mass at its free end. Write its elemental equation.

The system is 2 DOF and let the DOF variables be x_1 and x_2.

(2) (5 pts) The damping matrix of the math model is

a) $\begin{bmatrix} C/L + c_1 & -c_1 \\ c_1 & c_1 \end{bmatrix}$ b) $\begin{bmatrix} C/L^2 + c_1 & c_1 \\ c_1 & -c_1 \end{bmatrix}$

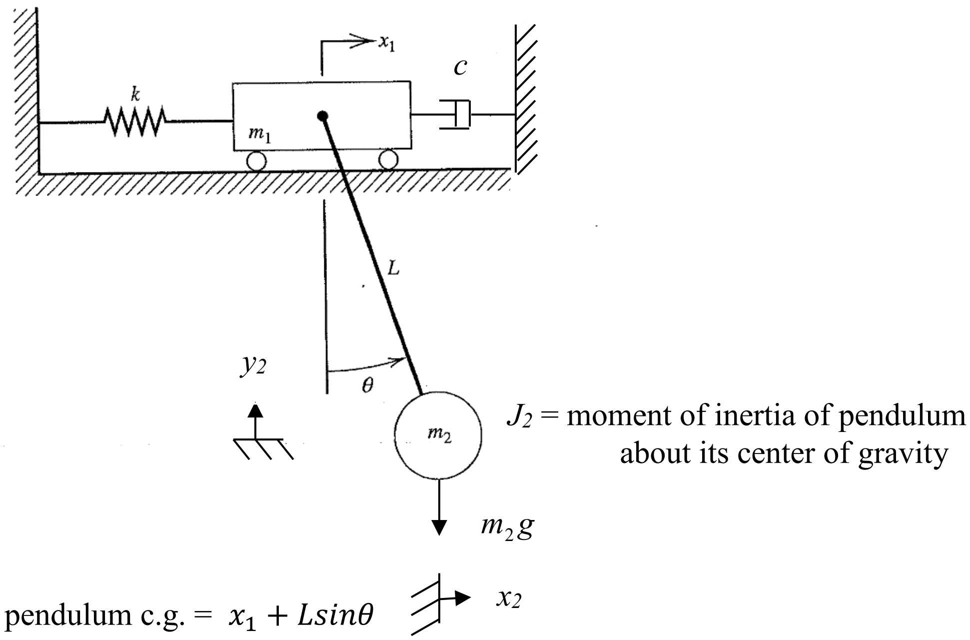
c) $\begin{bmatrix} C/L^2 + c_1 & -2c_1 \\ -2c_1 & c_1 \end{bmatrix}$ d) $\begin{bmatrix} C/L^2 + c_1 & -c_1 \\ -c_1 & c_1 \end{bmatrix}$

(3) (5 pts) The stiffness matrix is

a) $\begin{bmatrix} k_1 & K/L^2 \\ K/L^2 & 0 \end{bmatrix}$ b) $\begin{bmatrix} k_1 + K/L^2 & 0 \\ 0 & 0 \end{bmatrix}$

c) $\begin{bmatrix} k_1 L + K/L & 0 \\ 0 & 0 \end{bmatrix}$ d) $\begin{bmatrix} k_1 L + K/L & -k_1 \\ -k_1 & k_1 \end{bmatrix}$

2. (23 pts) A 2DOF mass-spring-pendulum system is shown below with DOF variables x_1 and θ. The pendulum rod is rigid and massless. The pendulum is pivoted to the moving mass and swings with a small angle, so that $\sin\theta \approx \theta$, $\cos\theta \approx 1.0$ and higher-order terms are negligible, such as $\theta\ddot{\theta} \approx \dot{\theta}^2 \approx 0$ etc.



x_2 = x-position of the pendulum c.g. = $x_1 + L\sin\theta$
y_2 = y-position of the pendulum c.g. = $L - L\cos\theta$

(1) (2 pts) Draw the FBD for the mass and writes its elemental equation

(2) (5 pts) Draw the FBD for the pendulum and write its elemental equations (it has three ele. eqs and the rotational elemental eq. cannot be written with respect to a moving pivot)

(3) (3 pts) With the small angle approximation, $\ddot{x}_2$ is related to the DOF variables by

a) $\ddot{x}_2 = \ddot{x}_1$

b) $\ddot{x}_2 = \ddot{x}_1 + L\ddot{\theta}$

c) $\ddot{x}_2 = \ddot{x}_1 - L\ddot{\theta}$

d) $\ddot{x}_2 = L\ddot{\theta}$

(4) (3 pts) With the small angle approximation, $\ddot{y}_2$ is given by

a) $\ddot{y}_2 = 0$

b) $\ddot{y}_2 = \ddot{x}_2 + L\ddot{\theta}$

c) $\ddot{y}_2 = \ddot{x}_2 - L\ddot{\theta}$

d) $\ddot{y}_2 = L\ddot{\theta}$

(5) (5 pts) One of the two governing equations in the math model is

a) $m_1\ddot{x}_1 + c\dot{x}_1 + kx_1 = 0$

b) $(m_1 + m_2)\ddot{x}_1 + J_2\ddot{\theta} + c\dot{x}_1 + kx_1 = 0$

c) $(m_1 + m_2)\ddot{x}_1 + Lm_2\ddot{\theta} + c\dot{x}_1 + kx_1 = 0$

d) $(m_1 + m_2)\ddot{x}_1 + c\dot{x}_1 + kx_1 + m_2 g\theta = 0$

e) $(m_1 + m_2)\ddot{x}_1 + c\dot{x}_1 + kx_1 = 0$

f) $(m_1 + m_2)\ddot{x}_1 + Lm_2\ddot{\theta} + c\dot{x}_1 + kx_1 + m_2 g\theta = 0$

(6) (5 pts) The other governing equation is

a) $(J_2 + L^2 m_2)\ddot{\theta} + m_2 gL\theta = 0$

b) $Lm_2\ddot{x}_1 + J_2\ddot{\theta} + m_2 gL\theta = 0$

c) $J_2\ddot{\theta} + m_2 gL\theta = 0$

d) $Lm_2\ddot{x}_1 + (J_2 + L^2 m_2)\ddot{\theta} + m_2 gL\theta = 0$

e) $(J_2 + L^2 m_2)\ddot{\theta} + cL^2\dot{\theta} + (kL^2 + m_2 gL)\theta = 0$

f) $Lm_2\ddot{x}_1 + (J_2 + L^2 m_2)\ddot{\theta} + cL^2\dot{\theta} + (kL^2 + m_2 gL)\theta = 0$