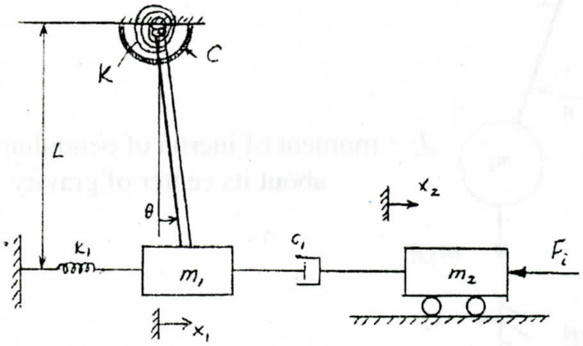
ME 370.1 Fall 2018 Name(s)

Homework 10

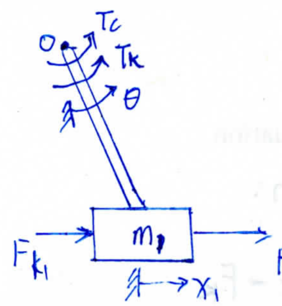
Due: *In class*, Friday, Nov. 9

1. (13 pts) A geometric model of a mechanical system is shown below. A ***torsional*** spring K and a torsional damper C are attached to the bar at the pivot. The inertia effect of the rigid bar is modeled by a lumped mass m_1 at the free end. This mass is connected to the ground through a spring k_1 and to a second mass m_2 through a damper c_1. The mass m_2 is supported on the ground by frictionless bearings and is driven by an input force F_i. All the motions are of small magnitudes.



(1) (3 pts) Draw FBD of the bar including the mass at its free end. Write its elemental equation.

FBD:



elemental equation:

$$m_1 L^2 \ddot{\theta} = T_c + T_K + F_{k_1} L + F_{c_1} L$$

The system is 2 DOF and let the DOF variables be x_1 and x_2.

(2) (5 pts) The damping matrix of the math model is

a) $\begin{bmatrix} C/L + c_1 & -c_1 \\ c_1 & c_1 \end{bmatrix}$ b) $\begin{bmatrix} C/L^2 + c_1 & c_1 \\ c_1 & -c_1 \end{bmatrix}$

c) $\begin{bmatrix} C/L^2 + c_1 & -2c_1 \\ -2c_1 & c_1 \end{bmatrix}$ d) $\begin{bmatrix} C/L^2 + c_1 & -c_1 \\ -c_1 & c_1 \end{bmatrix}$

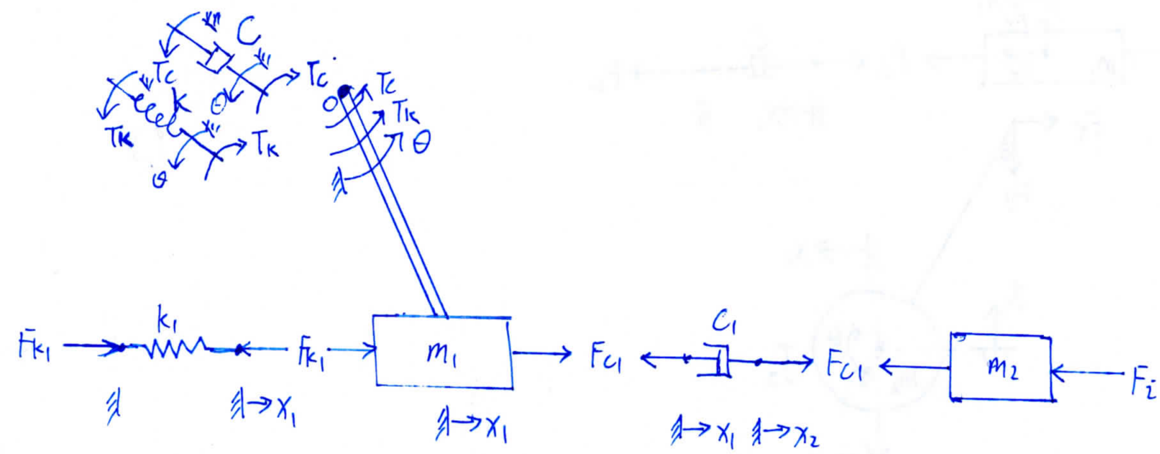
(3) (5 pts) The stiffness matrix is

a) $\begin{bmatrix} k_1 & K/L^2 \\ K/L^2 & 0 \end{bmatrix}$ b) $\begin{bmatrix} k_1 + K/L^2 & 0 \\ 0 & 0 \end{bmatrix}$

c) $\begin{bmatrix} k_1 L + K/L & 0 \\ 0 & 0 \end{bmatrix}$ d) $\begin{bmatrix} k_1 L + K/L & -k_1 \\ -k_1 & k_1 \end{bmatrix}$

Problem 1

(2) sol: system FBD:



elemental equations:

$C: \quad T_c = -c\dot{\theta} \quad \cdots ①$

$k: \quad T_k = -k\theta \quad \cdots ②$

$m_1: \quad m_1 L^2 \ddot{\theta} = T_c + T_k + F_{k_1} L + F_{c_1} L \quad \cdots ③$

$k_1: \quad F_{k_1} = -k_1 x_1 \quad \cdots ④$

$c_1: \quad F_{c_1} = c_1(\dot{x}_2 - \dot{x}_1) \quad \cdots ⑤$

$m_2: \quad m_2 \ddot{x}_2 = -F_{c_1} - F_i \quad \cdots ⑥$

Also:

$\theta = \frac{x_1}{L} \quad \cdots ⑦$

substitute ①②④⑤ into ③, get:

$$m_1 L^2 \frac{\ddot{x}_1}{L} = -c\frac{\dot{x}_1}{L} - k\frac{x_1}{L} - k_1 x_1 L + c_1(\dot{x}_2 - \dot{x}_1) L$$

$$\Rightarrow \boxed{m_1 \ddot{x}_1 + \left(\frac{c}{L^2} + c_1\right)\dot{x}_1 - c_1 \dot{x}_2 + \left(\frac{k}{L^2} + k_1\right) x_1 = 0} \quad \cdots ⑧$$

substitute ⑥ into ⑤, get

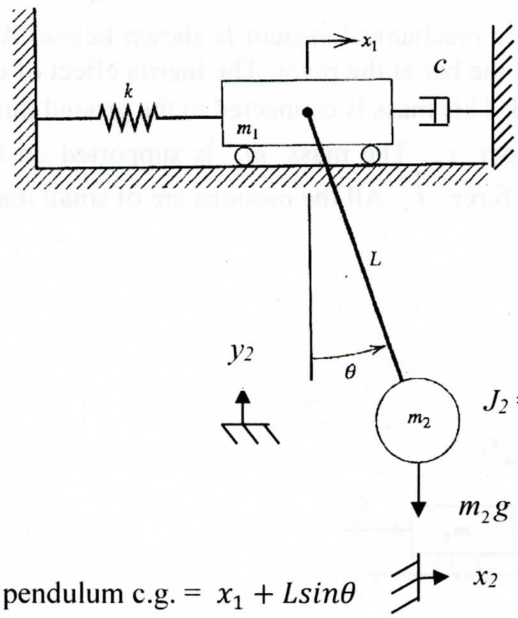
$$m_2 \ddot{x}_2 = -c_1 \dot{x}_2 + c_1 \dot{x}_1 - F_i$$

$$\Rightarrow \boxed{m_2 \ddot{x}_2 + c_2 \dot{x}_2 - c_1 \dot{x}_1 = -F_i} \quad \cdots ⑨$$

math model

$$\underbrace{\begin{bmatrix} m_1 & 0 \\ 0 & m_2 \end{bmatrix}}_{\text{mass matrix}} \begin{Bmatrix} \ddot{x}_1 \\ \ddot{x}_2 \end{Bmatrix} + \underbrace{\begin{bmatrix} c/L^2 + c_1 & -c_1 \\ -c_1 & c_1 \end{bmatrix}}_{\text{damping matrix}} \begin{Bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{Bmatrix} + \underbrace{\begin{bmatrix} k_1 + k/L^2 & 0 \\ 0 & 0 \end{bmatrix}}_{\text{stiffness matrix}} \begin{Bmatrix} x_1 \\ x_2 \end{Bmatrix} = \begin{Bmatrix} 0 \\ -F_i \end{Bmatrix}$$

2. (23 pts) A 2DOF mass-spring-pendulum system is shown below with DOF variables x_1 and θ. The pendulum rod is rigid and massless. The pendulum is pivoted to the moving mass and swings with a small angle, so that $\sin\theta \approx \theta$, $\cos\theta \approx 1.0$ and higher-order terms are negligible, such as $\theta\ddot{\theta} \approx \dot{\theta}^2 \approx 0$ etc.

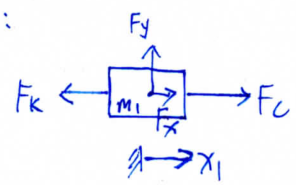


J_2 = moment of inertia of pendulum about its center of gravity

x_2 = x-position of the pendulum c.g. = $x_1 + L\sin\theta$
y_2 = y-position of the pendulum c.g. = $L - L\cos\theta$

(1) (2 pts) Draw the FBD for the mass and writes its elemental equation

FBD:

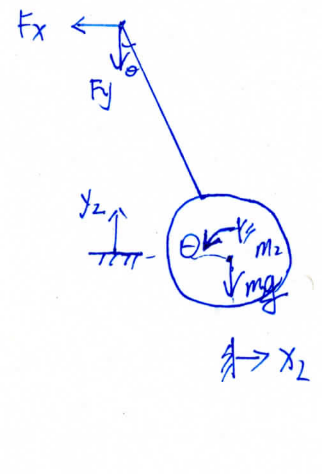


elemental equation:

$$m_1\ddot{x}_1 = F_c + F_x - F_K$$

(2) (5 pts) Draw the FBD for the pendulum and write its elemental equations (it has three ele. eqs and the rotational elemental eq. cannot be written with respect to a moving pivot)

FBD:



elemental equation:

$$m_2\ddot{x}_2 = -F_x$$

$$m_2\ddot{y}_2 = -m_2 g - F_y$$

$$J_2\ddot{\theta} = F_x L\cos\theta + F_y L\sin\theta \quad \text{<θ small angle>}$$

$$= F_x L + F_y L\theta$$

(3) (3 pts) With the small angle approximation, $\ddot{x}_2$ is related to the DOF variables by

a) $\ddot{x}_2 = \ddot{x}_1$

b) $\ddot{x}_2 = \ddot{x}_1 + L\ddot{\theta}$

c) $\ddot{x}_2 = \ddot{x}_1 - L\ddot{\theta}$

d) $\ddot{x}_2 = L\ddot{\theta}$

$x_2 = x_1 + L\sin\theta$

$\dot{x}_2 = \dot{x}_1 + L\dot{\theta}\cos\theta = \dot{x}_1 + L\dot{\theta}$

$\ddot{x}_2 = \ddot{x}_1 + L\ddot{\theta}$

(4) (3 pts) With the small angle approximation, $\ddot{y}_2$ is given by

a) $\ddot{y}_2 = 0$

b) $\ddot{y}_2 = \ddot{x}_2 + L\ddot{\theta}$

c) $\ddot{y}_2 = \ddot{x}_2 - L\ddot{\theta}$

d) $\ddot{y}_2 = L\ddot{\theta}$

$y_2 = L - L\cos\theta$

$\dot{y}_2 = 0 - (-L\dot{\theta}\sin\theta) = L\dot{\theta}\theta$

$\ddot{y}_2 = L(\ddot{\theta}\theta + \dot{\theta}\dot{\theta}) = 0$

(5) (5 pts) One of the two governing equations in the math model is

a) $m_1\ddot{x}_1 + c\dot{x}_1 + kx_1 = 0$

b) $(m_1 + m_2)\ddot{x}_1 + J_2\ddot{\theta} + c\dot{x}_1 + kx_1 = 0$

c) $(m_1 + m_2)\ddot{x}_1 + Lm_2\ddot{\theta} + c\dot{x}_1 + kx_1 = 0$

d) $(m_1 + m_2)\ddot{x}_1 + c\dot{x}_1 + kx_1 + m_2g\theta = 0$

e) $(m_1 + m_2)\ddot{x}_1 + c\dot{x}_1 + kx_1 = 0$

f) $(m_1 + m_2)\ddot{x}_1 + Lm_2\ddot{\theta} + c\dot{x}_1 + kx_1 + m_2g\theta = 0$

(6) (5 pts) The other governing equation is

a) $(J_2 + L^2m_2)\ddot{\theta} + m_2gL\theta = 0$

b) $Lm_2\ddot{x}_1 + J_2\ddot{\theta} + m_2gL\theta = 0$

c) $J_2\ddot{\theta} + m_2gL\theta = 0$

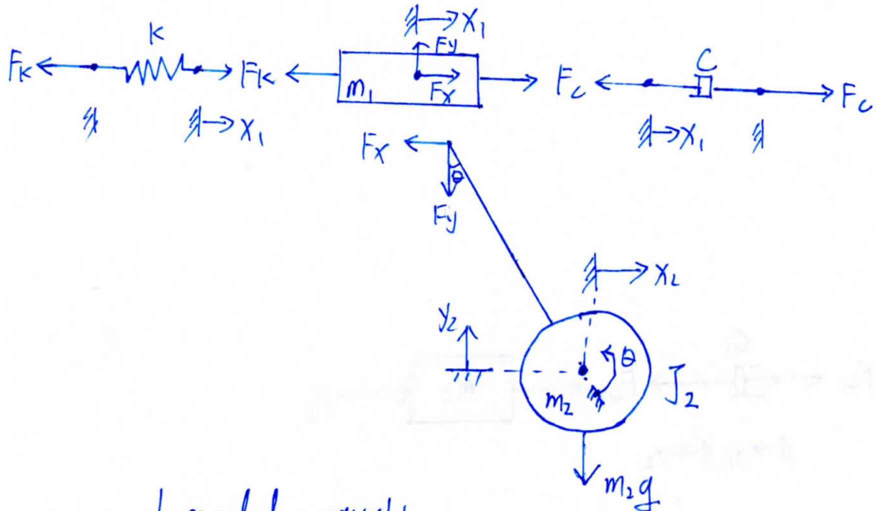
d) $Lm_2\ddot{x}_1 + (J_2 + L^2m_2)\ddot{\theta} + m_2gL\theta = 0$

e) $(J_2 + L^2m_2)\ddot{\theta} + cL^2\dot{\theta} + (kL^2 + m_2gL)\theta = 0$

f) $Lm_2\ddot{x}_1 + (J_2 + L^2m_2)\ddot{\theta} + cL^2\dot{\theta} + (kL^2 + m_2gL)\theta = 0$

problem 2

(5)(6) sol: system FBD:



elemental equation:

k: $F_K = kx_1$ --- ①

c: $F_c = -c\dot{x}_1$ -- ②

m_1: $m_1\ddot{x}_1 = F_c + F_x - F_K$ --- ③

m_2: $m_2\ddot{x}_2 = -F_x$ -- ④

$m_2\ddot{y}_2 = -m_2 g - F_y$ --- ⑤

$J_2\ddot{\theta} = F_x L + F_y L\theta$ --- ⑥

we have

$$\begin{cases} x_2 = x_1 + L\theta \\ y_2 = L - L = 0 \end{cases} \qquad \begin{cases} \dot{x}_2 = \dot{x}_1 + L\dot{\theta} \\ \dot{y}_2 = 0 \;\; \text{-- ⑧} \end{cases} \qquad \begin{cases} \ddot{x}_2 = \ddot{x}_1 + L\ddot{\theta} \;\; \text{-- ⑦} \\ \ddot{y}_2 = 0 \end{cases}$$

substitute ①②④⑦ into ③, get first governing equation:

$$m_1\ddot{x}_1 = -c\dot{x}_1 - kx_1 - m_2(\ddot{x}_1 + L\ddot{\theta})$$

i.e. $\boxed{(m_1+m_2)\ddot{x}_1 + Lm_2\ddot{\theta} + c_1\dot{x}_1 + kx_1 = 0}$

From ⑤⑧, get $F_y = -m_2 g$ --- ⑨

substitute ④⑦⑨ into ⑥, get second governing equation:

$$J_2\ddot{\theta} = -m_2(\ddot{x}_1 + L\ddot{\theta})L - m_2 gL\theta$$

i.e. $\boxed{Lm_2\ddot{x}_1 + (J_2 + L^2 m_2)\ddot{\theta} + m_2 gL\theta = 0}$