ME 370.1 Fall 2018 Name(s)

Homework 12

Due: *In class*, Monday Dec. 3

1. [25 pts] Consider the function given below (where $s = i\omega$):

$$T(s) = \frac{5}{6s^3 + 11s^2 + 6s + 1}$$

(1) (5 pts) Obtain expressions of the amplitude and phase angle of T as functions of ω.

(2) (2 pts) The amplitude of $T(i0.3)$ is

a) $A = 0.721$ b) $A = 2.192$

c) $A = 3.053$ d) $A = 5.334$

(3) (2 pts) The phase angle (in rad) of $T(i0.3)$is

a) $\psi =$ -0.525 b) $\psi =$ -1.565

c) $\psi =$ -2.078 d) $\psi =$ -2.912

(4) (2 pts) The amplitude of $T(i1.0)$ is

a) $A = 0.2$ b) $A = 1.0$

c) $A = 0.5$ d) $A = 2.0$

(5) (2 pts) The phase angle (in rad) of $T(i1.0)$is

a) $\psi =$ -π/3 b) $\psi =$ -π

c) $\psi =$ -π/2 d) $\psi =$ -3π/2

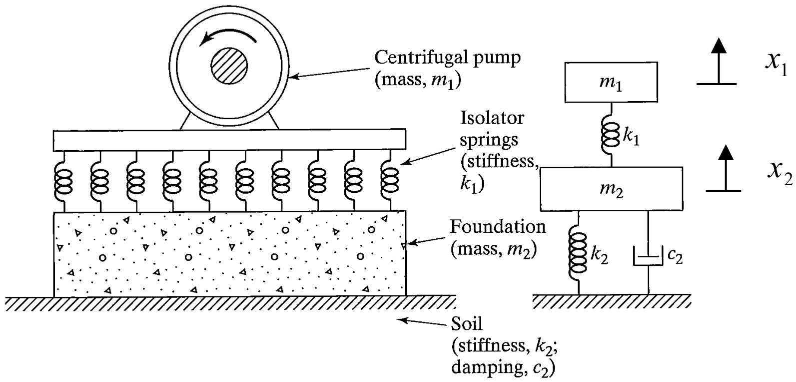
(6) (12 pts) Plot in matlab Im(T) *vs.* Re(T) in the complex plane as ω increases from 0 to 5 rad/s. In your plot, use matlab data curser tool to help obtain:

a) Amplitude of T when the curve crosses the negative real axis, $A|_{\psi=\pi} =$ __________

b) Phase angle of T when the amplitude of T is reduced to unity, $\psi|_{A=1.0} =$ __________ (Determine it using the data curser with $Re^2(T) + Im^2(T) = 1$ and $tan_2^{-1}\frac{Im}{Re}$).

Submit your matlab program along with the plot with data curser marks.

2. [28 pts] A centrifugal pump and its mounting foundation is modeled by a geometrical model shown below. The mass of the rotating part of the pump is m with its center of gravity, e, offset from the axis of rotation. This imbalance, me, generates a rotating centrifugal force, inducing a vertical vibration of the system.



The math model of the system is given by

$$\begin{bmatrix} m_1 & 0 \\ 0 & m_2 \end{bmatrix}\begin{bmatrix} \ddot{x}_1 \\ \ddot{x}_2 \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ 0 & c_2 \end{bmatrix}\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} + \begin{bmatrix} k_1 & -k_1 \\ -k_1 & k_1 + k_2 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} me\omega^2\cos(\omega t + \alpha) \\ 0 \end{bmatrix}$$

Let $m_1 = 300\,\text{kg}$, $m_2 = 4000\,\text{kg}$, $me = 0.1\,\text{kg-m}$, $k = 1{,}000\,\text{kN/m}$, $k_2 = 10{,}000\,\text{kN/m}$, $c_2 = 25{,}000\,\text{N-s/m}$, $\omega = 1200\,\text{rpm}$ and $\alpha = \pi/3$.

(1) (3 pts) The expression describing the stead-state vibration of the pump mass is (where ψ is phase lag behind the physical driving action):

a) $x_1(t) = X_1 \sin(\omega t + \psi_1)$

b) $x_1(t) = X_1 \cos(\omega t + \psi_1)$

c) $x_1(t) = X_1 \sin(\omega t + \alpha + \psi_1)$

d) $x_1(t) = X_1 \cos(\omega t + \alpha + \psi_1)$

e) $x_1(t) = X_1 e^{i(\omega t + \alpha + \psi_1)}$

f) $x_1(t) = X_1 e^{i(\omega t + \psi_1)}$

(2) (3pts) Write down the numerical values of the impedance matrix:

$[Z] =$

(3) (3 pts) The natural frequencies of the system in rad/s are:

$\omega_{n1} =$

$\omega_{n2} =$

(4) (2 pts) The amplitude of vibration of the pump in millimeters is

a) $X_1 = 0.224$ b) $X_1 = 0.324$

c) $X_1 = 0.424$ d) $X_1 = 0.524$

(5) (2 pts) The amplitude of vibration of the foundation in millimeters is

a) $X_2 = 0.008$ b) $X_2 = 0.018$

c) $X_2 = 0.028$ d) $X_2 = 0.038$

(6) (2 pts) The phase lag in rad of $x_1(t)$ from the excitation is

a) $\psi_1 = 0.57$ b) $\psi_1 = -1.57$

c) $\psi_1 = 2.57$ d) $\psi_1 = -3.14$

(7) (2 pts) The phase lag in rad of $x_2(t)$ from the excitation is

b) $\psi_2 = 0.06$ b) $\psi_2 = -0.43$

c) $\psi_2 = 1.57$ d) $\psi_2 = -2.34$

(8) (5 pts) The magnitude of the force in newton transmitted to the soil ground is

a) $F_{to} = 63.2$ b) $F_{to} = 85.2$

c) $F_{to} = 97.2$ d) $F_{to} = 110.2$

(9) (3 pts) If the speed of the pump is reduced to one half of the current speed, the magnitude of the force transmitted to the soil ground would be:

$F_{to} =$

(10) (3 pts) Explain to the point why the force in Step (9) is so much larger than that in Step (8).