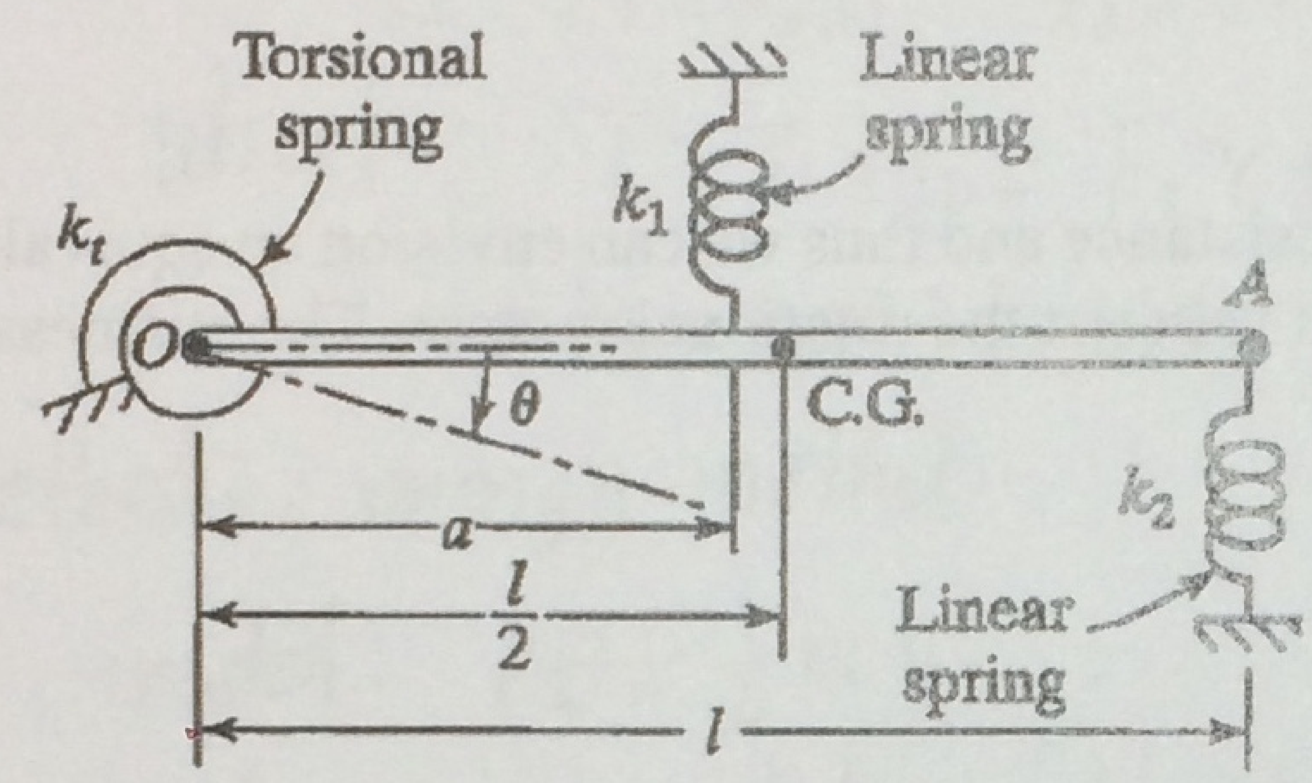
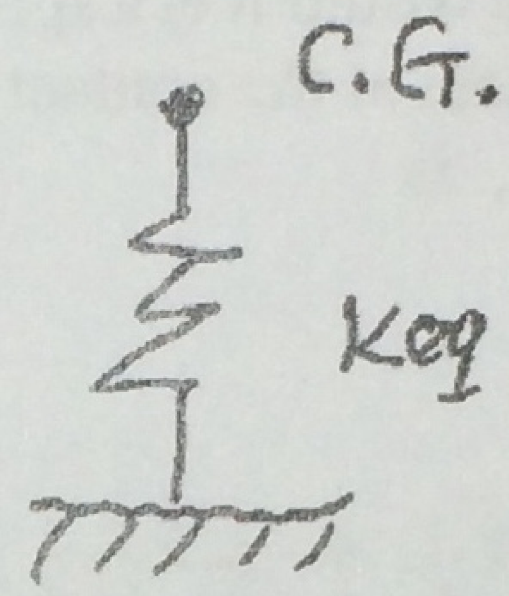
ME 370.1 Fall 2018 Name(s)

Homework 2

Due: *In class*, Friday, September 7

1. (10 pts) For the geometric model given below, the three springs may be replaced by an equivalent linear spring at the C.G. location.





The stiffness of the equivalent spring is

a) $k_{eq} = k_t / l^2 + k_1 / 4 + k_2$

b) $k_{eq} = k_t / l + k_1 / 4 + k_2$

c) $k_{eq} = 4(k_t + a^2 k_1 + l^2 k_2) / l^2$

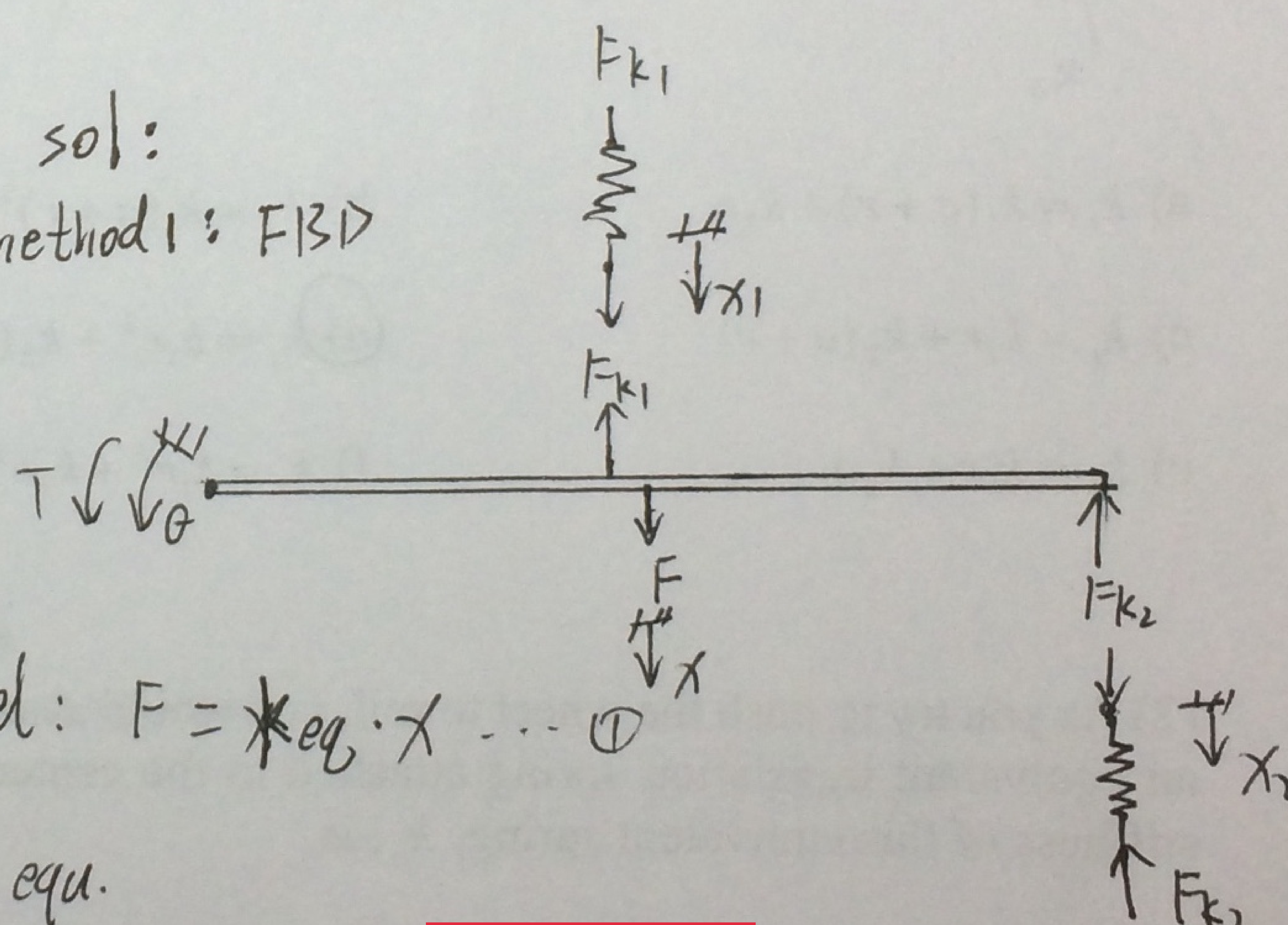
d) $k_{eq} = 4(k_t + a k_1 + l k_2) / l$

e) $k_{eq} = k_t / l + k_1 + k_2$

f) $k_{eq} = k_t / l^2 + k_1 k_2 / (k_1 + k_2)$

sol:

① method 1: FBD



model: $F = k_{eq} \cdot x$ … ①

equ.

k_1: $F_{k_1} = k_1 x_1$, $\boxed{x_1 = \frac{2a}{L} x}$ … ②

k_2: $F_{k_2} = k_2 x_2$, $\boxed{x_2 = \frac{2L}{L} x}$ … ③

k_t: $T = k_t \theta$, $\boxed{\theta = \frac{2x}{L}}$ … ④

Bar: moment balance.

$\boxed{F_{k_1} \cdot a + F_{k_2} \cdot l + T - F \frac{L}{2} = 0}$ … ⑤

substitute ① ② ③ ④ into ⑤ ⇒ $k_{eq} = 4(k_t + a^2 k_1 + l^2 k_2)/L^2$

method 2: energy method

① model $PE = \frac{1}{2} k_{eq} x^2$

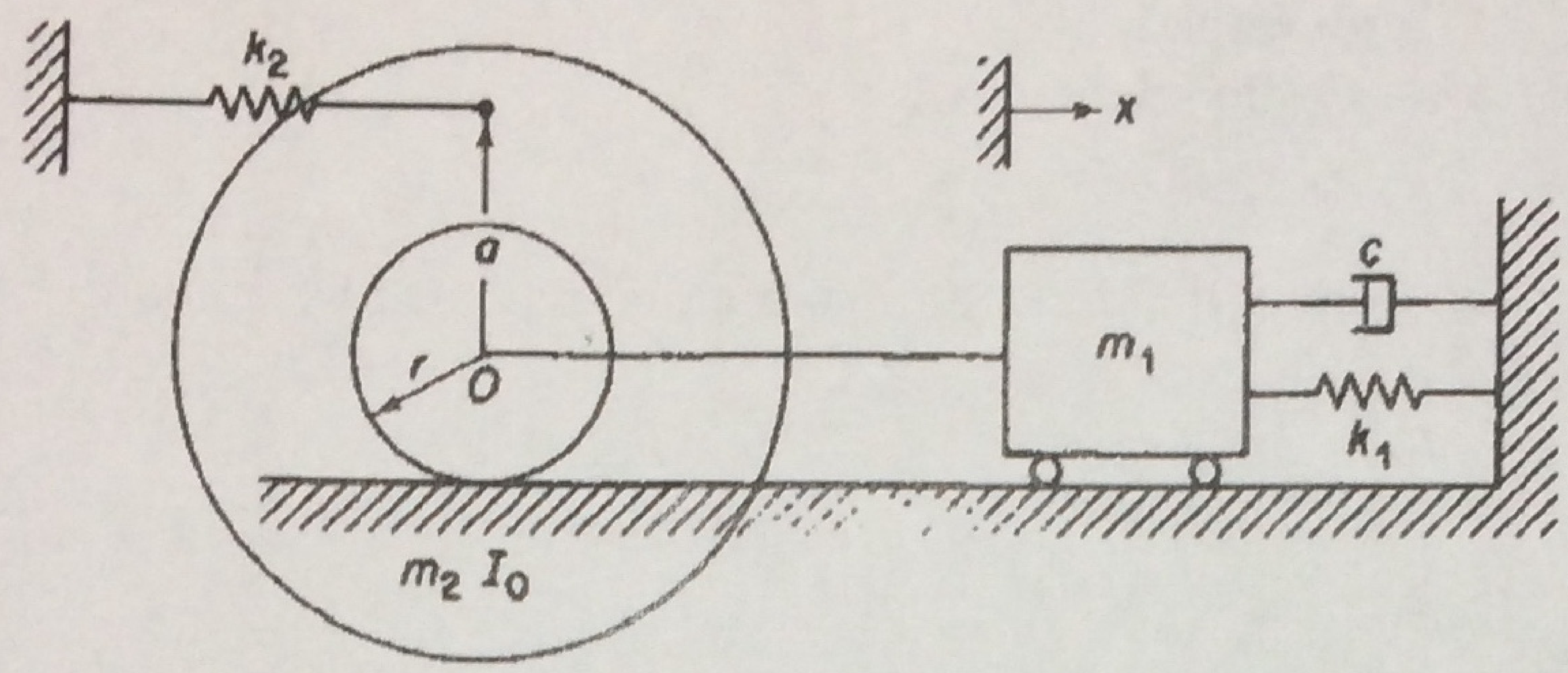
② system

$PE = \frac{1}{2} k_1 x_1^2 + \frac{1}{2} k_2 x_2^2 + \frac{1}{2} k_t \theta^2$

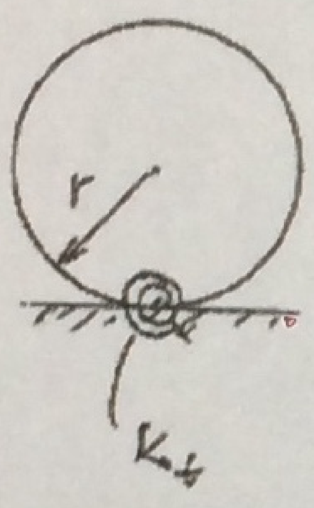
$= \frac{1}{2} k_1 (\frac{2a}{l} x)^2 + \frac{1}{2} k_2 (\frac{2l}{l})^2 x^2 + \frac{1}{2} k_t (\frac{2x}{l})^2$

$\Rightarrow k_{eq} = 4(k_t + a^2 k_1 + l^2 k_2)/l^2$

2. (10 pts) Examine the geometric model below. The wheel rolls without slip and thus the contact point is the instantaneous center of rotation of the body (i.e. as if the wheel is pined at the point and rotates about it).



(1) As you try to roll the wheel, you would feel a spring resistance and thus we can envision an equivalent torsional spring attached to the wheel at the contact point to resist the rotational motion. The stiffness of the equivalent torsional spring, k_t, is



a) $k_t = k_1(a+r) + k_2 r$

b) $k_t = k_1(a+r)^2 + k_2 r^2$

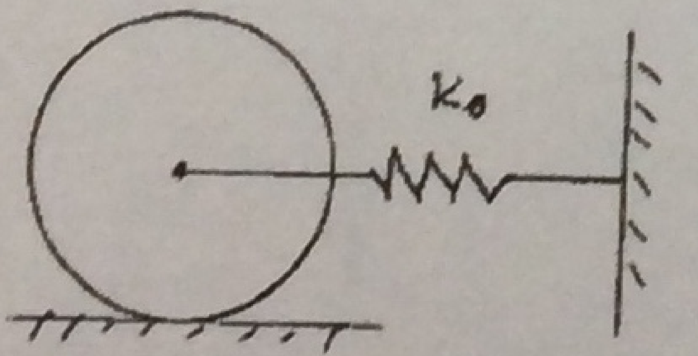
c) $k_t = k_1 r + k_2(a+r)$

d) $k_t = k_1 r^2 + k_2(a+r)^2$

e) $k_t = k_1 r + k_2 a$

f) $k_t = k_1 r^2 + k_2 a^2$

(2) As you try to push the wheel to roll, you would also feel a spring resistance and thus we can envision an equivalent translation spring attached to the center of the wheel to resist its forward motion. The stiffness of this equivalent spring, k_o, is



a) $k_o = k_1(1 + a/r) + k_2$

b) $k_o = k_1 + k_2(1 + a/r)$

c) $k_o = k_1(1 + a/r)^2 + k_2$

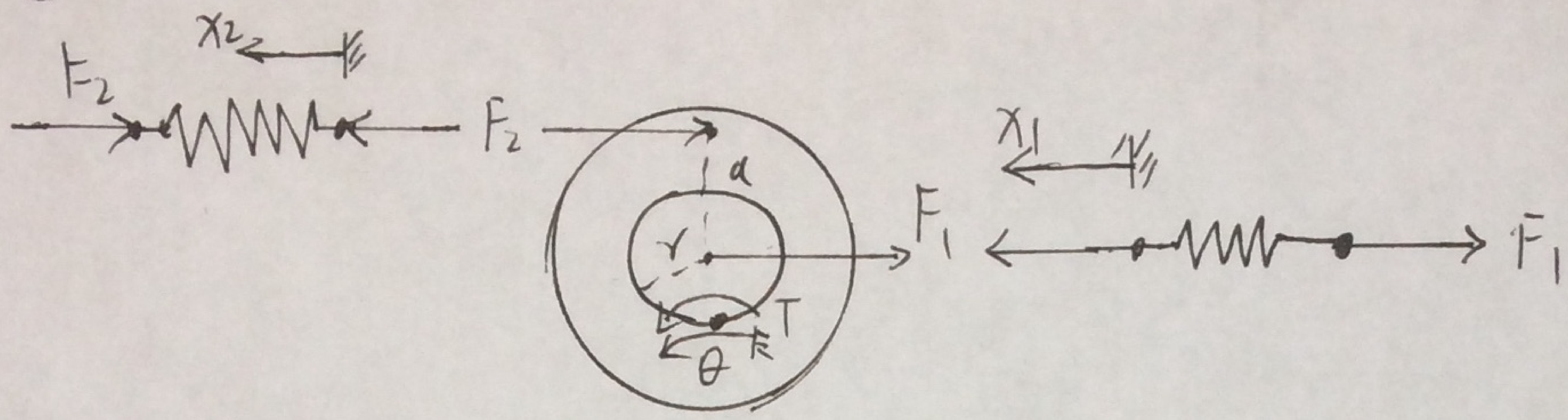
d) $k_o = k_1 + k_2(1 + a/r)^2$

e) $k_o = k_1 + k_2(a/r)$

f) $k_o = k_1 + k_2(a/r)^2$

2. (1) sol:

① method 1, FBD



model: $T = k_t\theta$

ele. equ. $k_1: F_1 = k_1 x_1$, $\boxed{x_1 = r\theta}$

$k_2: F_2 = k_2 x_2$, $\boxed{x_2 = (r+a)\theta}$

wheel: $T = T_1 + T_2 = F_1 r + F_2(a+r) = k_1 r^2\theta + k_2(r+a)^2\theta$

$\Rightarrow k_t = k_1 r^2 + k_2(r+\theta)^2$

② method 2, energy method

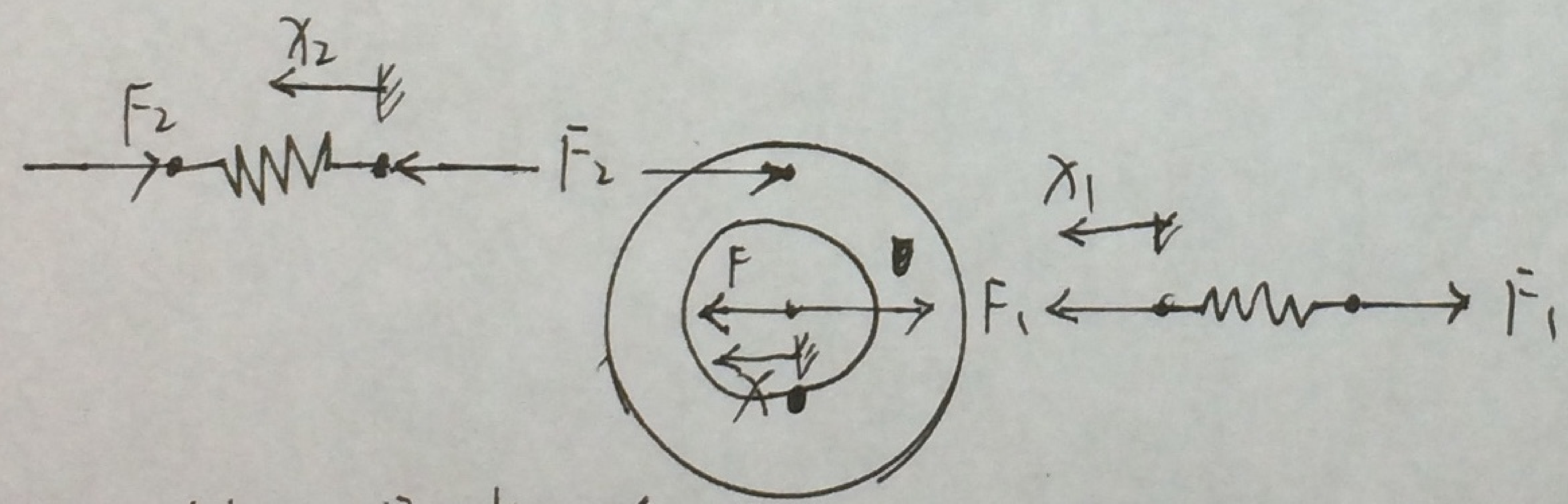
model: $PE = \frac{1}{2}k_t\theta^2$

system: $PE = \frac{1}{2}k_1 x_1^2 + \frac{1}{2}k_2 x_2^2 = \frac{1}{2}k_1 r^2\theta^2 + \frac{1}{2}k_2(r+a)^2\theta^2$

$\Rightarrow k_t = k_1 r^2 + k_2(r+\theta)^2$

(2) sol:

① FBD method.



model: $F = k_{eq}\cdot x$

system: $k_1: F_1 = k_1 x_1$, $\boxed{x_1 = x}$

$k_2: F_2 = k_2 x_2$, $\boxed{x_2 = \frac{a+r}{r}x}$

wheel: $F_1 r + F_2(a+r) - Fr = 0 \Rightarrow k_{eq} = k_1 + k_2(1+\frac{a}{r})^2.$

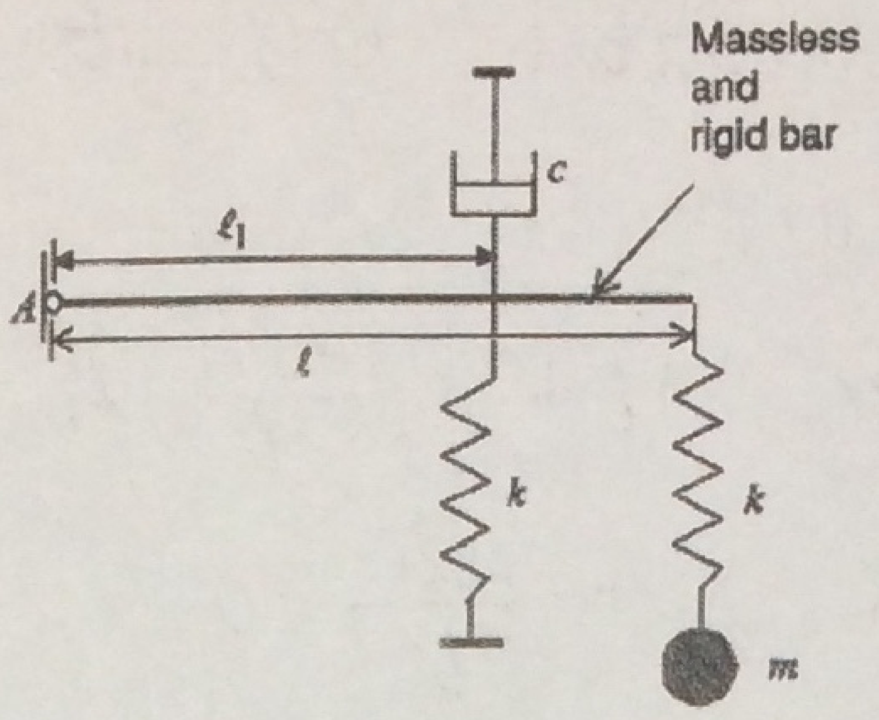
② energy method

model: $PE = \frac{1}{2}k_0 x^2$

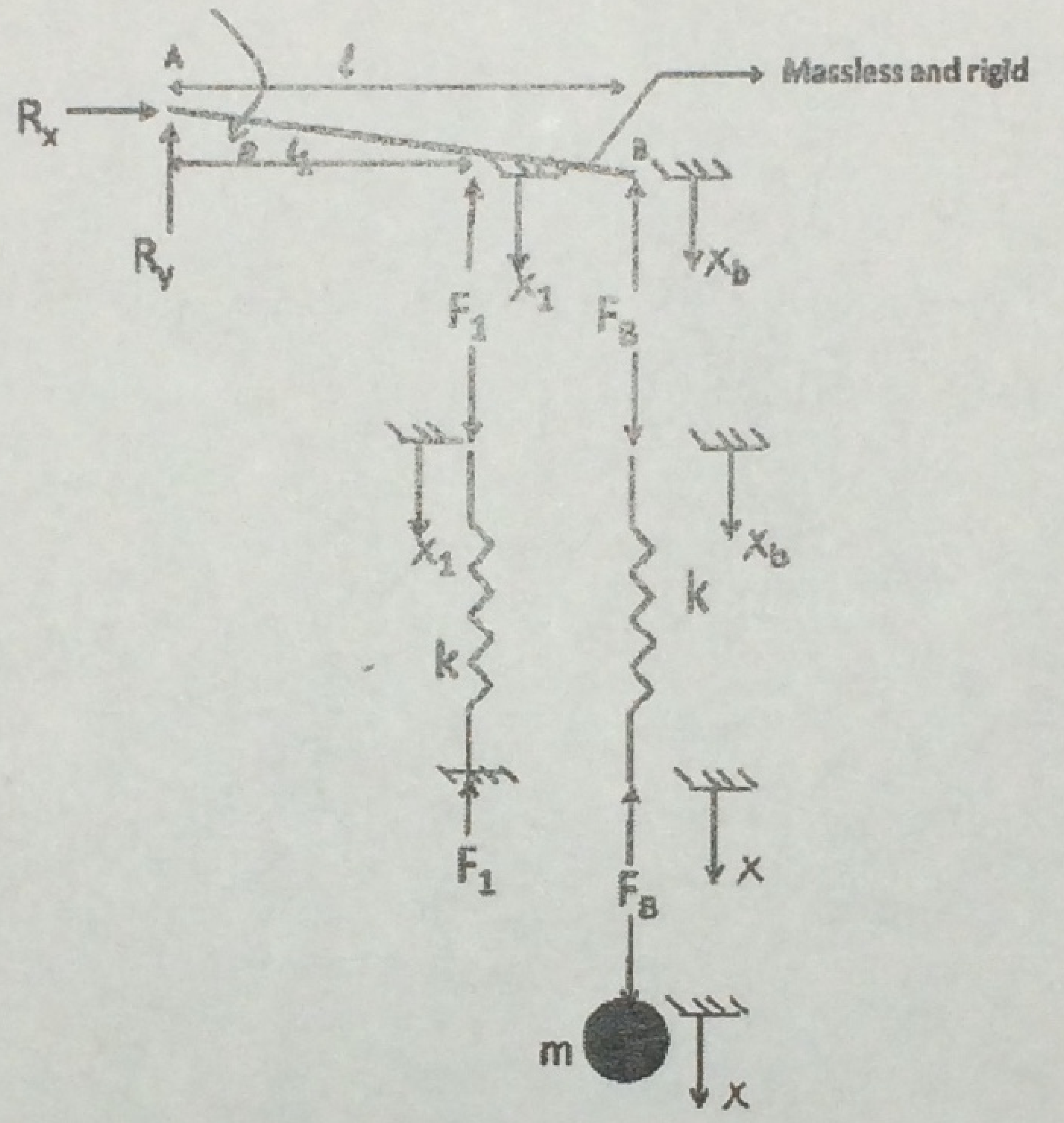
system: $PE = \frac{1}{2}k_1 x_1^2 + \frac{1}{2}k_2 x_1^2 = \frac{1}{2}k_1 x^2 + \frac{1}{2}k_2(\frac{a+r}{r})^2 x^2$

$\Rightarrow k_0 = k_1 + k_2(\frac{a+r}{r})^2 = k_1 + k_2(1+\frac{a}{r})^2$

3. (18 pts) Refer to the system shown below where the massless bar is pivoted to frame at point A:



Neglect the damper and consider small rotation motion of the bar about point A. Let the free end of the bar be point B and define its downward displacement to be x_b. Also, define the downward displacement of the mass to be x. A set of FBDs for the system elements are drawn below:



(1) (4 pts) Examine the FBDs noting the definitions of various motion and force variables. Then write down the elemental equations for the two springs and the mass; you may neglect gravity. Also, write the rotational element equation for the massless bar about point A.

Just write down the elemental eqs. in original form, no simplifications no substitutions etc.

The elemental eq. for the spring at location l_1 is:

$F_1 = kx_1$, or $F_1 = kx_1 = k(x_b l_1 / l)$ --- ①

The elemental eq. for the spring at location l is:

$F_B = k(x_b - x)$ -- ②

The elemental eq. for the mass is:

$F_B = m\ddot{x}$

The rotational elemental eq. for the bar about pivot A is:

$F_B l + F_1 l_1 = 0$ -- ③

(2) (8 pts) The motion relation between x_b and x is:

a) $x_b=\left(\frac{l}{l_1}\right)^2 x$

b) $x_b=\left(\frac{l}{l_1}\right) x$

c) $x_b=\left(\frac{l^2}{l_1^2+l^2}\right) x$

d) $x_b=\left(\frac{l}{l_1+l}\right) x$

e) $x_b=\left(\frac{l_1^2}{l_1^2+l^2}\right) x$

f) $x_b=\left(\frac{l_1}{l_1+l}\right) x$

sol: From (1), substitute ①② into ③,

get $k(x_b-x)l + k\left(\frac{l_1}{l}x_b\right)l_1 = 0$

$\Rightarrow x_b - x + \frac{x_b l_1^2}{l^2} = 0$

$\Rightarrow x_b\left(1+\frac{l_1^2}{l^2}\right) = x$

$\Rightarrow x_b\left(\frac{l^2+l_1^2}{l^2}\right) = x$

$\Rightarrow x_b = \left(\frac{l^2}{l_1^2+l^2}\right)x$

(3) (6 pts) Envision a single spring, k_{eq}, with one end connected to the mass and the other end attached to a ceiling directly above it as illustrated below:

(3) sol: ① FBD method

model: ~~...~~

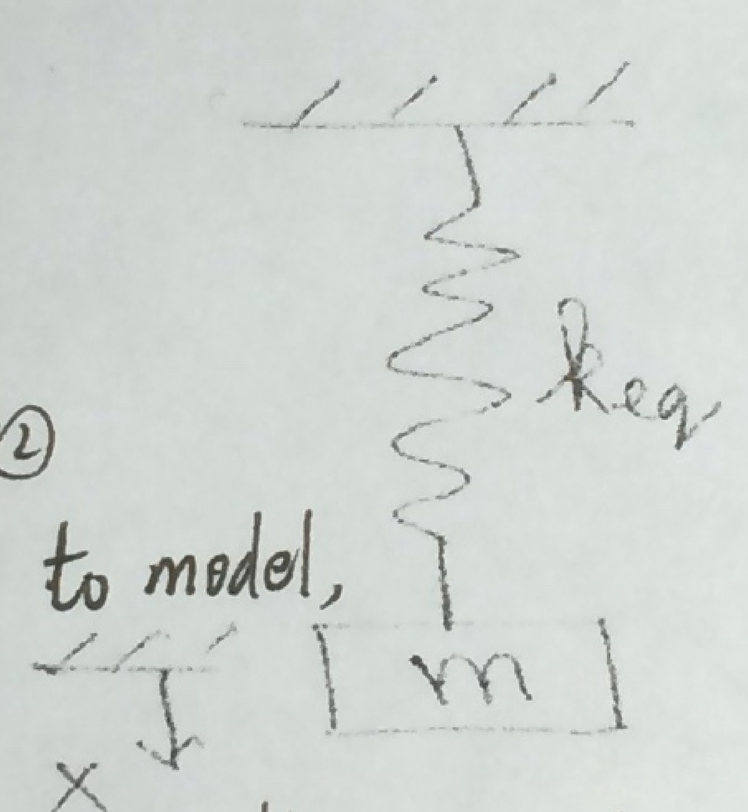
$F = k_{eq}(-x)$ ··· ①

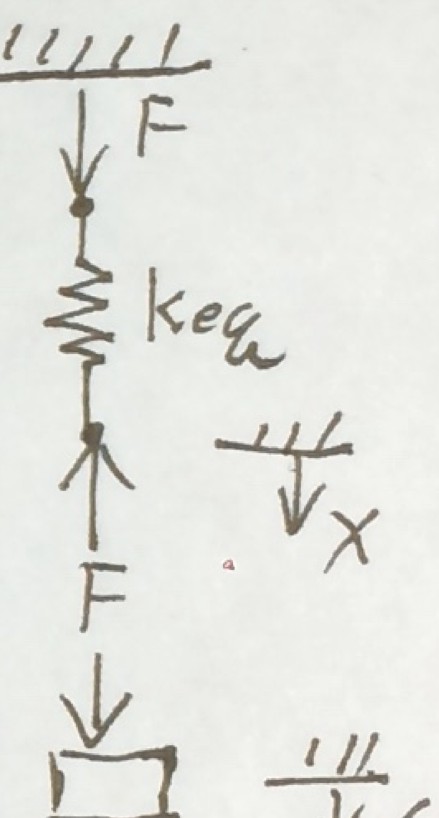
System: $F_B = k(k_b - x)$ ··· ②

since system is equivalent to model,

$\boxed{F = F_B}$

i.e. $k_{eq}(-x) = k(k_b - x)$, using the result of (2), $\boxed{k_{eq} = \frac{l_1^2}{l_1^2+l^2}k}$





To make this mass-spring model equivalent to the original system (without damper), the equivalent spring stiffness must be equal to:

a) $k_{eq}=\left(\frac{l}{l_1}\right) k$

b) $k_{eq}=\left(\frac{l}{l_1}\right)^2 k$

c) $k_{eq}=\left(\frac{l}{l_1+l}\right) k$

d) $k_{eq}=\left(\frac{l^2}{l_1^2+l^2}\right) k$

e) $k_{eq}=\left(\frac{l_1}{l_1+l}\right) k$

f) $k_{eq}=\left(\frac{l_1^2}{l_1^2+l^2}\right) k$

② energy method.

model: $PE_m = \frac{1}{2}k_{eq}x^2$ ···

system: $PE_s = \frac{1}{2}k(x_1)^2 + \frac{1}{2}k(x_b - x)^2$

by (1) and (2), $x_1 = \frac{l_1}{l}x_b = \left(\frac{l_1}{l}\right)\left(\frac{l^2}{l_1^2+l^2}\right)x = \left(\frac{l_1 l}{l_1^2+l^2}\right)x$

thus, $PE_s = \frac{1}{2}k\left[\left(\frac{l_1 l}{l_1^2+l^2}\right)x\right]^2 + \frac{1}{2}k\left[\frac{l^2}{l_1^2+l^2}x - x\right]^2$

$= \frac{1}{2}k\left[\frac{l_1^2 l^2}{(l_1^2+l^2)^2}\right]x^2 + \frac{1}{2}k\left(\frac{l^2}{l_1^2+l^2} - \frac{l_1^2+l^2}{l_1^2+l^2}\right)^2 x^2$

$= \frac{1}{2}k\left[\frac{l_1^2 l^2}{(l_1^2+l^2)^2} + \frac{(-l_1^2)^2}{(l_1^2+l^2)^2}\right]x^2$

$= \frac{1}{2}k\left[\frac{l_1^2(l^2+l_1^2)}{(l_1^2+l^2)^2}\right]x^2$

$= \frac{1}{2}k\left(\frac{l_1^2}{l_1^2+l^2}\right)x^2$

compare PE_m and PE_s, since $PE_m = PE_s$, get $k_{eq} = \left(\frac{l_1^2}{l_1^2+l^2}\right)k$