Name(s)

## Homework 4

Due: In class, Friday, 9/21

1. (12 pts) A 2DOF geometric model of a mechanical system is shown below. The system is driven by two inputs: force  $F_i$  on mass  $m_s$  and force  $F_c$  on the right part of damper c. The mass  $m_s$  can slide on the surface of mass m. The sliding surface is lubricated with oil so that the friction may be modeled using a viscous damper of damping coefficient  $c_s$ .



Draw the FBDs of all the model elements given below with *force and motion variables*. Make sure all FBDs are *complete* and *consistent* with the forces,  $F_{cs}$ ,  $F_{ks}$  and  $F_k$ , labelled on the incomplete FBDs of the two masses. Pay attention to action/reaction pair of forces between two connected elements.

2 pts for each of the 6 FBDs.



2. (15 pts) Examine the SDOF geometric model and the FBDs below, where  $J_o$  is the moment of inertia of the crank with respect to the pivot "O".





(1) (9 pts) Write a complete set of elemental equations (1 pt each, no partial credit)

- a) The ele eq. for  $k_1$  is:  $F_{k1} =$
- b) The ele eq. for m is:  $m\ddot{x} =$
- c) The ele eq. for  $J_o$  is:  $J_o\ddot{\theta} =$
- d) The ele eq. for  $m_s$  is:  $m_s \ddot{x}_s =$
- e) The ele eq. for  $J_s$  is:  $J_s \ddot{\theta}_s =$

 $(J_s \text{ is the sphere moment of inertia about the center of sphere})$ 

- f) The ele eq. for  $k_2$  is:  $F_{k2} =$
- g) The motion relation between  $\theta$  and x is:  $\theta =$
- h) The motion relation between  $x_s$  and x is:  $x_s =$
- i) The motion relation between  $\theta_s$  and x is:  $\theta_s =$
- (2) (6 pts) The governing equation of the system in x variable is

a) 
$$\left(m + \frac{J_o}{l_1^2} + \frac{J_s + m_s r_s^2}{r_s^2}\right) \ddot{x} + (k_1 + k_2)x = 0$$
  
b)  $\left(m + \frac{J_o}{l_1^2} + \frac{l_2^2}{l_1^2} \frac{J_s + m_s r_s^2}{r_s^2}\right) \ddot{x} + \left(k_1 + \frac{l_2^2}{l_1^2} k_2\right) x = 0$   
c)  $\left(m + \frac{J_o}{l_1^2} + \frac{l_2^2}{l_1^2} \frac{J_s + m_s r_s^2}{r_s^2}\right) \ddot{x} + (k_1 + k_2)x = 0$   
d)  $\left(m + \frac{J_o}{l_1^2} + \frac{J_s + m_s r_s^2}{r_s^2}\right) \ddot{x} + \left(\frac{l_2^2}{l_1^2} k_1 + k_2\right) x = 0$   
e)  $\left(m + \frac{J_o}{l_1^2} + \frac{l_2^2}{l_1^2} \frac{J_s + m_s r_s^2}{r_s^2}\right) \ddot{x} + \left(\frac{l_2^2}{l_1^2} k_1 + k_2\right) x = 0$   
f)  $\left(m + \frac{J_o}{l_1^2} + \frac{J_s + m_s r_s^2}{r_s^2}\right) \ddot{x} + \left(k_1 + \frac{l_2^2}{l_1^2} k_2\right) x = 0$ 

3. (10 pts) Examine the SDOF geometric model below:



(1) (3 pts) The moment of inertia of the bar about the pivot point O is

- a)  $J_o = \frac{1}{12}ml^2$ b)  $J_o = \frac{7}{48}ml^2$ c)  $J_o = \frac{5}{24}ml^2$ d)  $J_o = \frac{1}{3}ml^2$ e)  $J_o = ml^2$
- (2) (7 pts) The governing equation of the system is
- a)  $7m\ddot{\theta} + 24k\theta = 0$
- b)  $7m\ddot{\theta} + 33k\theta = 0$
- c)  $7m\ddot{\theta} + 48k\theta = 0$
- d)  $5m\ddot{\theta} + 24k\theta = 0$
- e)  $5m\ddot{\theta} + 33k\theta = 0$
- f)  $5m\ddot{\theta} + 48k\theta = 0$