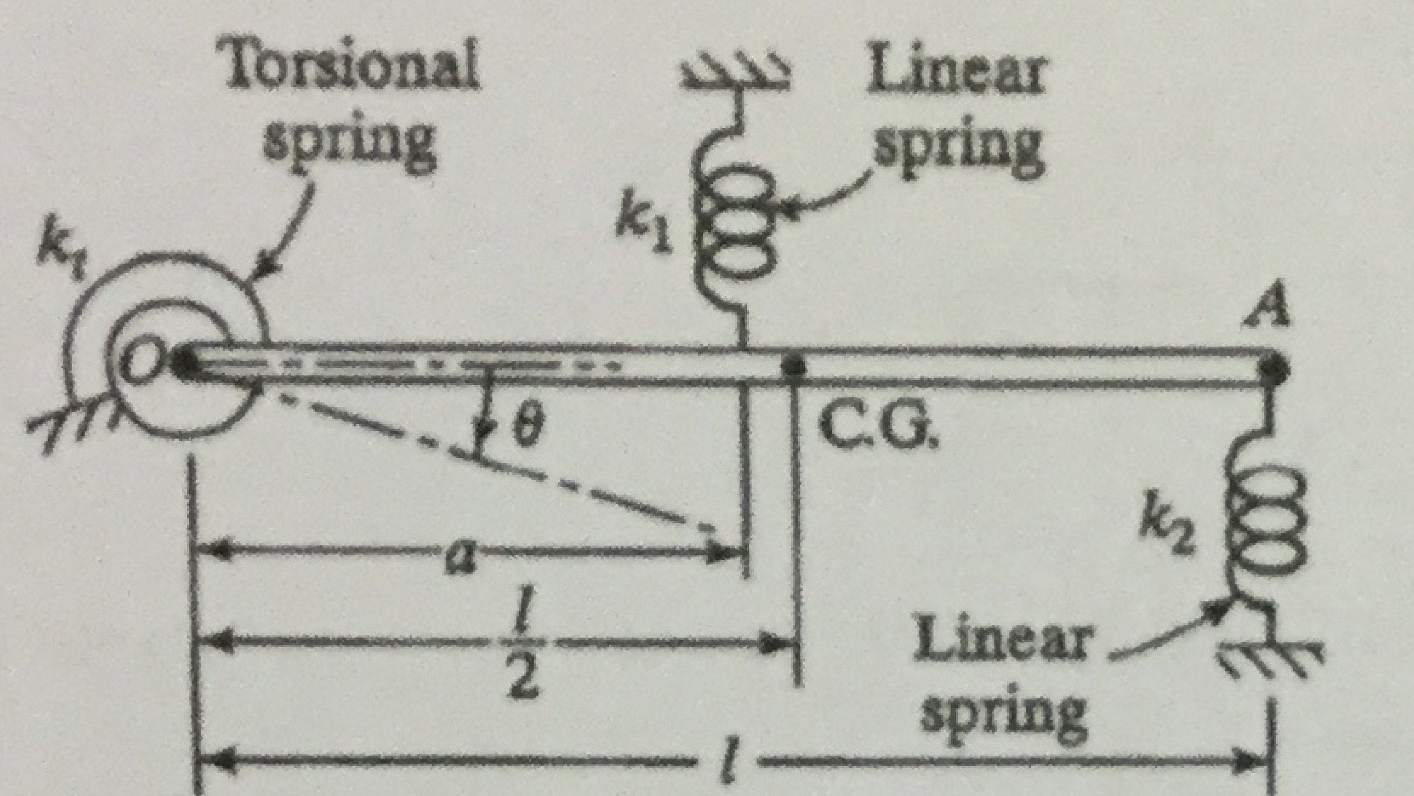
ME 370.1 Fall 18 Name(s)

Homework 5

Due: *In class*, Friday, 9/28

1. (10 pts) For the system shown below, let $l = 1.0$ m , $a = 0.45l$, $k_t = 500$ N-m/rad and $k_1 = k_2 = 1000$ N/m. Also, the mass of the bar is $m = 10$ kg and gravity excluded.



(1) (3 pts) The moment of inertia of the bar in kg-m² with respect to the pivot O is:

a) $J_o = 3.33$

b) $J_o = 3.97$

c) $J_o = 4.71$

d) $J_o = 5.82$

e) $J_o = 6.65$

sol: The moment of inertia of the bar about its center C is:

$$J_c = \frac{1}{12} m l^2$$

According to parallel axis theorem.

$$J_o = J_c + m d_{oc}^2$$

$$= \frac{1}{12} m l^2 + m\left(\frac{l}{2}\right)^2$$

$$= \frac{1}{3} m l^2$$

Thus, $J_o = \frac{1}{3} \cdot 10\text{kg} \cdot (1\text{m})^2 = 3.33\text{kg}\cdot\text{m}^2$

(2) (7 pts) The natural frequency of the system in rad/s is:

a) $\omega_n = 12.3$

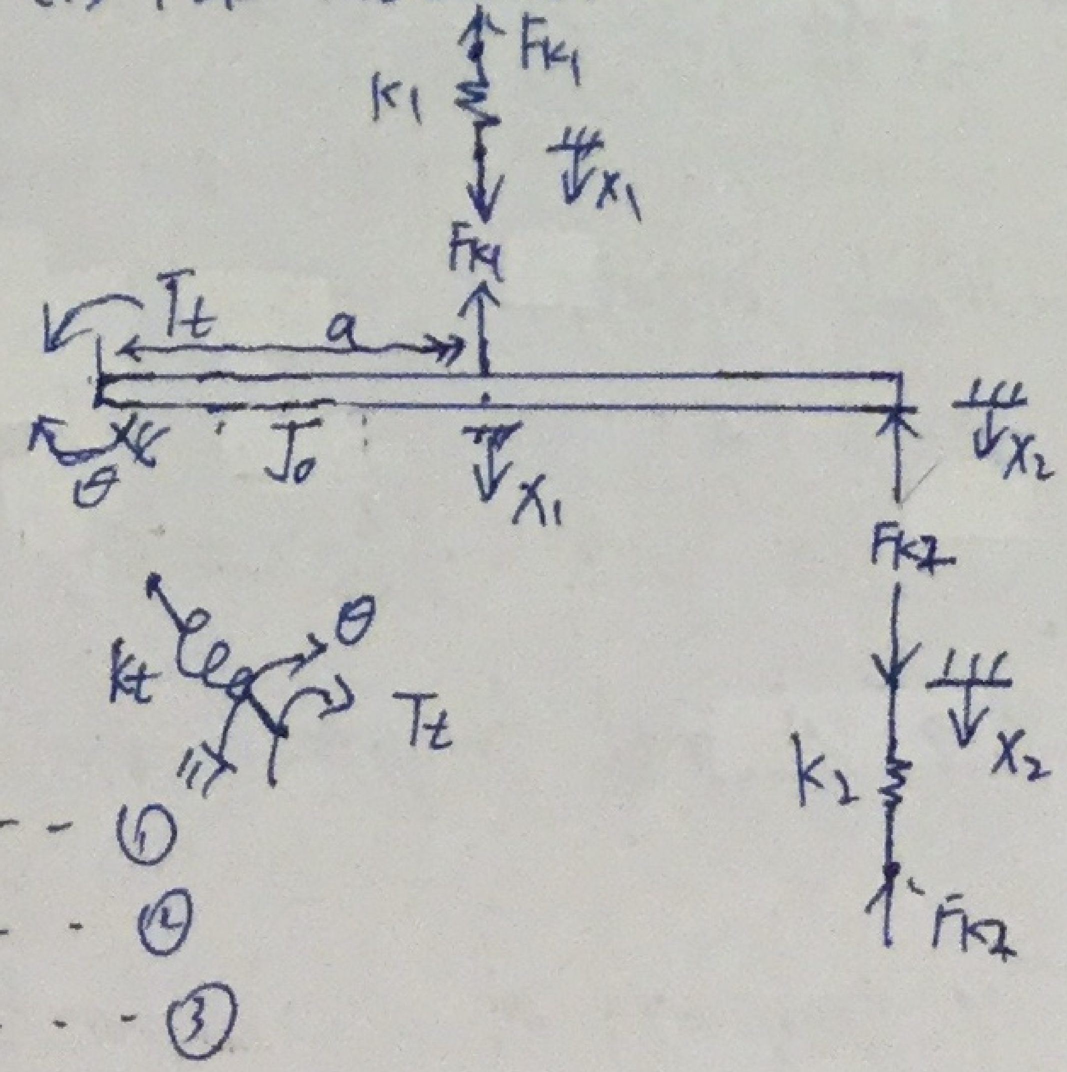
b) $\omega_n = 22.6$

c) $\omega_n = 33.1$

d) $\omega_n = 41.5$

e) $\omega_n = 54.2$

sd: <1> FBD method.



de. eq. k_1: $F_{k1} = k_1 x_1$ -- ①

k_2: $F_{k2} = k_2 x_2$ -- ②

k_t: $T_t = k_t \theta$ --- ③

bar: $J_o \ddot{\theta} = -T_t - F_{k1} a - F_{k2} L$ -- ④

and: $\boxed{\begin{matrix} x_1 = a\theta \\ x_2 = l\theta \end{matrix}}$ -- ⑤ -- ⑥

From ①②③④⑤⑥, get governing equation:

$$J_o \ddot{\theta} + (k_t + k_1 a^2 + k_2 l^2)\theta = 0$$

$$\Rightarrow W_n = \sqrt{\frac{k_{eq}}{J_{eq}}} = \sqrt{\frac{k_t + k_1 a^2 + k_2 l^2}{J_o}},$$

substitute values into W_n, get $W_n = 22.6$ rad/s

<2> energy method.

$$\Sigma PE = \frac{1}{2} k_t \theta^2 + \frac{1}{2} k_1 x_1^2 + \frac{1}{2} k_2 x_2^2$$

$$= \frac{1}{2} k_t \theta^2 + \frac{1}{2} k_1 (a\theta)^2 + \frac{1}{2} k_2 (l\theta)^2$$

$$= \frac{1}{2}(k_t + a^2 k_1 + l^2 k_2)\theta^2$$

$$\Rightarrow k_{eq} = k_t + a^2 k_1 + l^2 k_2 \quad \text{--- (1)}$$

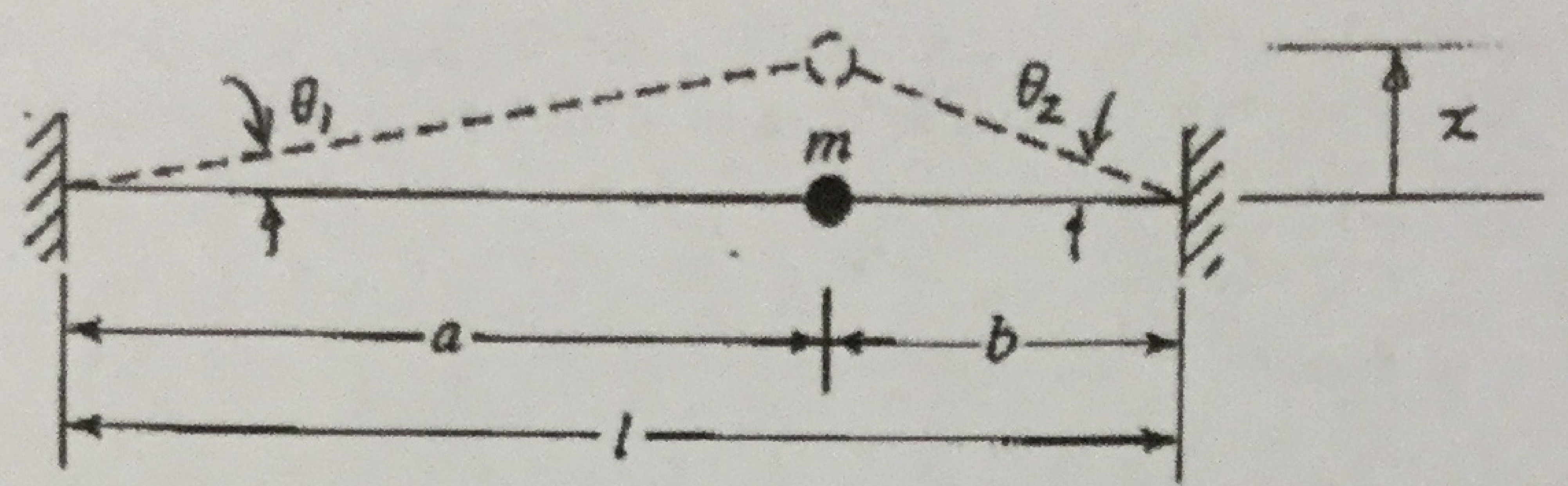
$$\Sigma KE = \frac{1}{2} J_o \dot{\theta}^2$$

$$\Rightarrow J_{eq} = J_o \quad \text{-- (2)}$$

From (1),(2). get $W_n = \sqrt{\frac{k_{eq}}{J_{eq}}}$ -- (3)

substitute values into (1)(2)(3). get $W_n = 22.6$ rad/s.

2. (10pts) A ball of mass is attached to an elastic cord as shown below. The cord is stretched with a tension T. Assume the tension remains the same as the mass is pulled up by a small amount (ie. $\sin\theta = \tan\theta = \theta$). Furthermore, gravity is neglected.



(1) (3 pts) The magnitude ***and direction*** of the vertical force, F_m, on the mass by the cord tension as a function of the displacement of the mass, x, is given by:

a) $F_m = -\left(\frac{Tl}{ab}\right)x$

b) $F_m = -\left(\frac{2Tl}{ab}\right)x$

c) $F_m = -\left(\frac{0.7Tl}{ab}\right)x$

d) $F_m = \left(\frac{Tl}{ab}\right)x$

e) $F_m = \left(\frac{2Tl}{ab}\right)x$

f) $F_m = \left(\frac{0.7Tl}{ab}\right)x$

sol: FBD:

the magnitude of F_m:

$$F_m = T\sin\theta_1 + T\sin\theta_2$$
$$\doteq T\tan\theta_1 + T\tan\theta_2$$
$$= T\frac{x}{a} + T\frac{x}{b}$$
$$= T\left(\frac{a+b}{ab}\right)x$$
$$= \left(\frac{Tl}{ab}\right)x$$

since the direction of F_m is opposite to the direction of x

$$F_m = \left(\frac{Tl}{ab}\right)(-x) = -\left(\frac{Tl}{ab}\right)x$$

(2) (7 pts) Let $l = 1.0$ m, $a = 0.7l$, $b = 0.3l$, $m = 0.1$ kg and $T = 100$ N. If the ball is pulled up by an amount of $x_o = 1.0$ cm and then let go, the number of times the ball will bounce up and down in one second is equal to:

a) $N = 11$

b) $N = 21$

c) $N = 31$

d) $N = 41$

e) $N = 51$

sol: element equation:

m: $F_m = m\ddot{x}$

$$\Rightarrow m\ddot{x} + \frac{Tl}{ab}x = 0$$

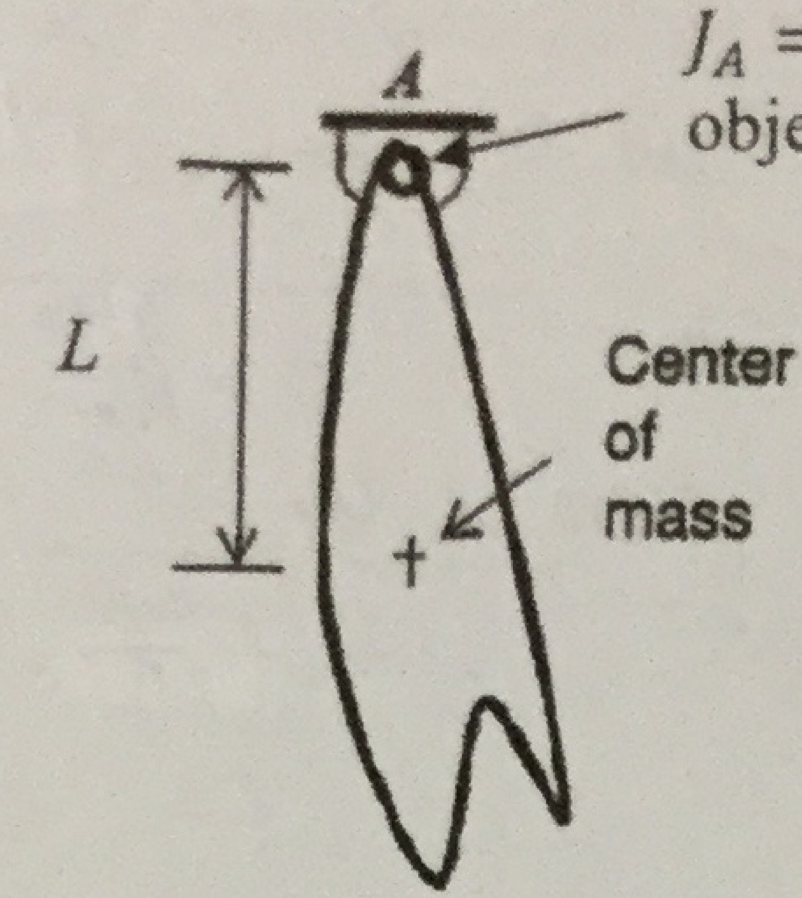
Then natural frequency is: $\omega_n = \sqrt{\frac{Tl}{abm}}$ (rad/second)

The frequency of oscillation is: $f = \frac{\omega_n}{2\pi}$ (cycles/second)

Take the values into f,

$$f = \frac{1}{2\pi}\sqrt{\frac{Tl}{abm}} = \frac{1}{2\pi}\sqrt{\frac{100N \cdot 1m}{0.7m \cdot 0.3m \cdot 0.1kg}} \approx 11$$

3. (25 pts) A complex-shaped object of mass m is shown below. When it is suspended like a pendulum at point A, it swings 60 times in one minute.



J_A = moment of inertia of the object about pt. A.

(1) (2pts) The natural frequency of the system in rad/s is

a) $\omega_n = \pi/2$ b) $\omega_n = \pi$

c) $\omega_n = 2\pi$ d) $\omega_n = 4\pi$

e) $\omega_n = 1.0$

sol: Since the pendulum swings 60 times per minute, i.e. 1 time per second

$f = \frac{W_n}{2\pi} = 1$

$\Rightarrow W_n = 2\pi$ --- ①

(2) (4 pts) The governing equation for the pendulum in free swing motion is

a) $J_A\ddot{\theta} + mgL\theta = 0$

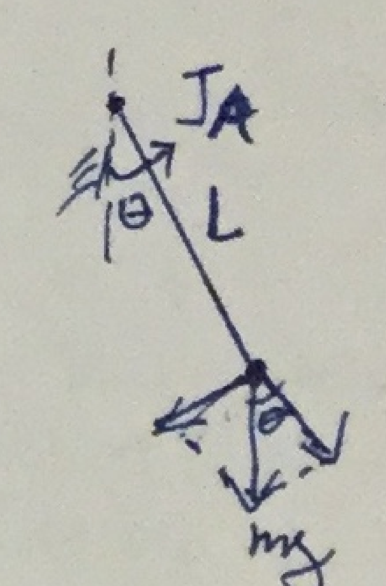
b) $L\ddot{\theta} + g\theta = 0$

c) $2J_A\ddot{\theta} + mgL\theta = 0$

d) $2L\ddot{\theta} + g\theta = 0$

e) $J_A\ddot{\theta} + 2mgL\theta = 0$

Sol: FBD.



ele. eq:

$J_A\ddot{\theta} = -mgL\sin\theta$ -- ②

small angle: $\sin\theta = \theta$

$\Rightarrow$ Governing equation:

$J_A\ddot{\theta} + mgl\theta = 0$ -- ③

(3) (5 pts) Let $m = 2.0$ kg and $L = 0.2$ m. The moment of inertia of the object ***about its center of mass*** in kg-m^2 is about equal to

a) $J_{cg} = 0.009$

b) $J_{cg} = 0.019$

c) $J_{cg} = 0.038$

d) $J_{cg} = 0.076$

e) $J_{cg} = 0.098$

sol: From (1)(2).

$W_n = \sqrt{\frac{mgL}{J_A}} = 2\pi$

$\Rightarrow J_A = \frac{mgL}{4\pi^2}$, take values into it, $J_A \approx 0.099$ kg·m^2

According to parallel theorm,

$J_c = J_A - mL^2$

$J_c = 0.099\ kg\,m^2 - 2kg\cdot(0.2m)^2 = 0.019\ kg\,m^2.$

As worked out in class, the free vibration (ie. pendulum swing) is given by $\theta(t) = A\sin(\omega_n t + \phi)$. Suppose the pendulum is displaced by an angle of $\theta_o = 0.1$ rad and is released with a velocity of $\dot{\theta}_o = -0.5$ rad/s.

(4) (2pts) The amplitude of the pendulum swing in rad is about equal to

a) $A = 0.050$

b) $A = 0.098$

c) $A = 0.128$

d) $A = 0.187$

e) $A = 0.245$

sol: Use the results introduced in our class,

$$A = \sqrt{y_0^2 + \left(\frac{\dot{y}_0}{\omega_n}\right)^2}$$, where y_0 and $\dot{y}_0$ are ICs

From (1), $\omega_n = 2\pi$

$$\Rightarrow A = \sqrt{0.1 + \left(\frac{-0.5}{2\pi}\right)^2} \approx 0.128$$

(5) (2pts) The phase angle in rad of the pendulum swing is about equal to

a) $\phi = \pi/4$

b) $\phi = 0.94$

c) $\phi = 2.24$

d) $\phi = -1.54$

e) $\phi = -2.76$

sol: Also use the results introduced in our class,

$$\phi = \tan_2^{-1}\left(\frac{\omega_n y_0}{\dot{y}_0}\right) = \begin{cases} \tan^{-1}\frac{\omega_n y_0}{\dot{y}_0}, & \dot{y}_0 > 0 \\ \left(\tan^{-1}\frac{\omega_n y_0}{\dot{y}_0}\right) + \pi, & \dot{y}_0 < 0 \end{cases}$$

$$\Rightarrow \phi = \tan_2^{-1}\frac{0.1\cdot 2\pi}{-0.5} = \tan^{-1}\frac{0.2\pi}{-0.5} + \pi \approx 2.24$$

(5) (10pts) Write a simple matlab program to calculate θ vs. t and plot the pendulum swing for two periods of pendulum swing. Make sure your plot shows a smooth curve. Also, make sure your plot matches the given ICs to be correct. Label your plot including units. Submit your matlab program along with results. (the class website has a simple matlab guide)

sol: shown as next page.

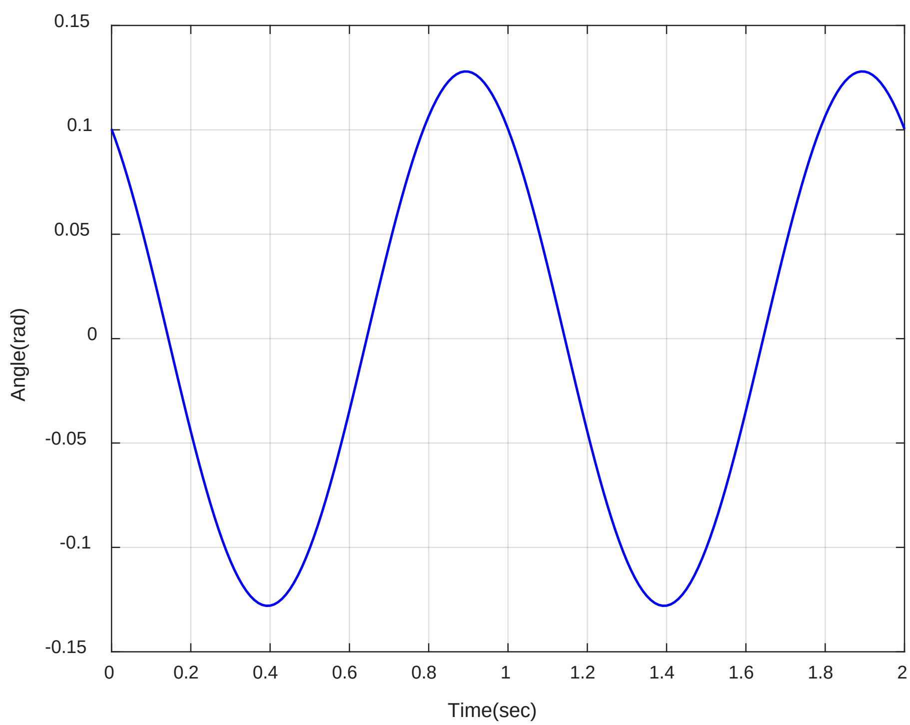
(5) Sol:

```
%{
 * Course:ME370
 * Name:Liming Gao
 * Date: September 04, 2018
 *
 * Program Description: Plot the vibration for pendulum swing.
 %}

clear all
close all

t= 0:0.01:2; %Set time of two periods
y=0.128*sin(2*pi*t+2.24); %Use vibration equation to calculate angle

plot(t,y,'b','LineWidth',1); %plot the curve
xlabel('Time(sec)')
ylabel('Angle(rad)')
grid on
```



Fiugre1 The curve of θ vs. t