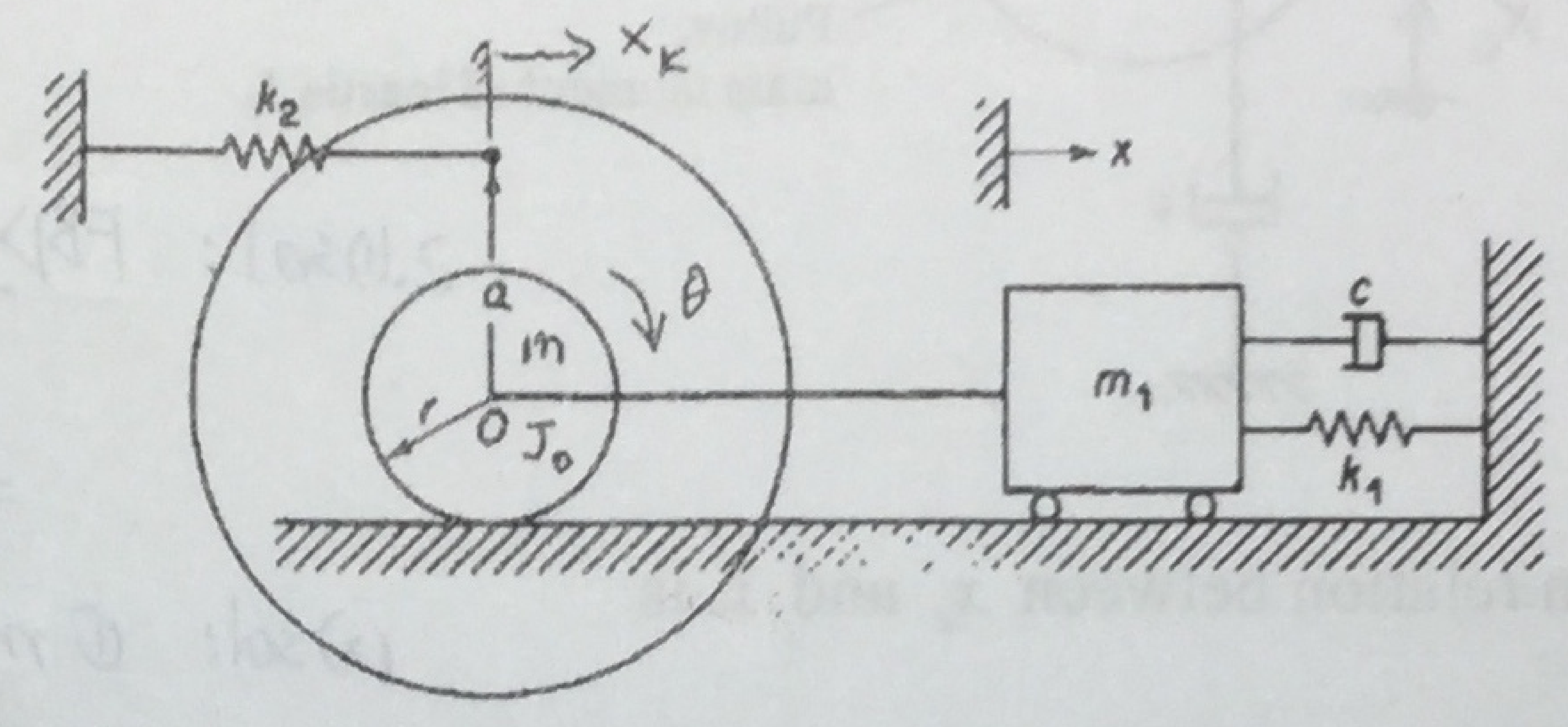
ME 370.1 Fall 2018 Name(s)

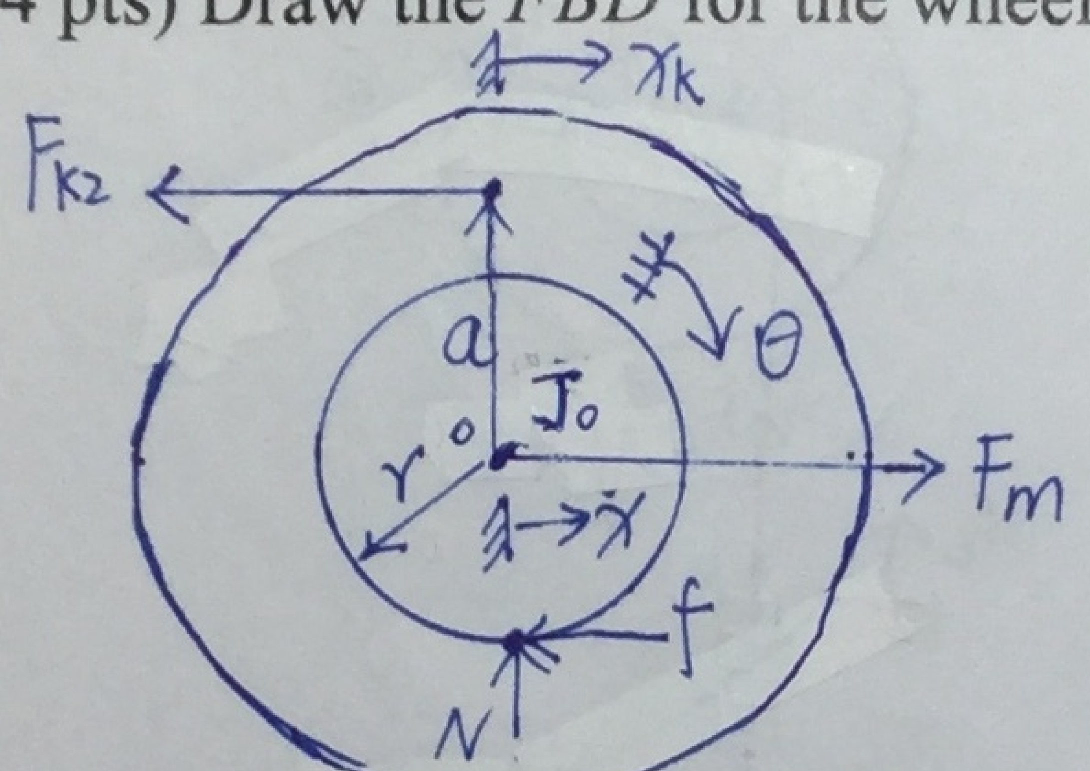
Homework 6

Due: *In class*, Friday, Oct. 5

1. (15 pts) Examine the geometric model shown below. The wheel rolls without slip making the model SDOF. The spring k_2 is connected to the wheel at a location "a" above the center of the wheel.



(1) (4 pts) Draw the *FBD* for the wheel and write the two *ele eqs*, one for m and the other for J_o.



m: $m\ddot{x} = F_m - f - F_{k_2}$

J_o: $J\ddot{\theta} = fr - F_{k_2}a$

(2) (2 pts) The motion relation between x_k and x is

a) $x_k = x$

b) $x_k = (a/r)x$

c) $x_k = (r/a)x$

d) $x_k = ((a+r)/r)x$

e) $x_k = ((a+r)/a)x$

(2) sol: Since: $x = \theta r$

$x_k = \theta r + \theta a$

$\Rightarrow \frac{x_k}{x} = \frac{\theta r + \theta a}{\theta r} = \frac{r+a}{r}$

$\Rightarrow x_k = ((a+r)/r)x$

(3) (9 pts) The math model of the system in x is

a) $(m + m_1 + J_o/r^2)\ddot{x} + c\dot{x} + (k_1 + k_2(a+r)^2/r^2)x = 0$

b) $(m + m_1 + J_o/r^2)\ddot{x} + c\dot{x} + (k_1 + k_2(a+r)/r)x = 0$

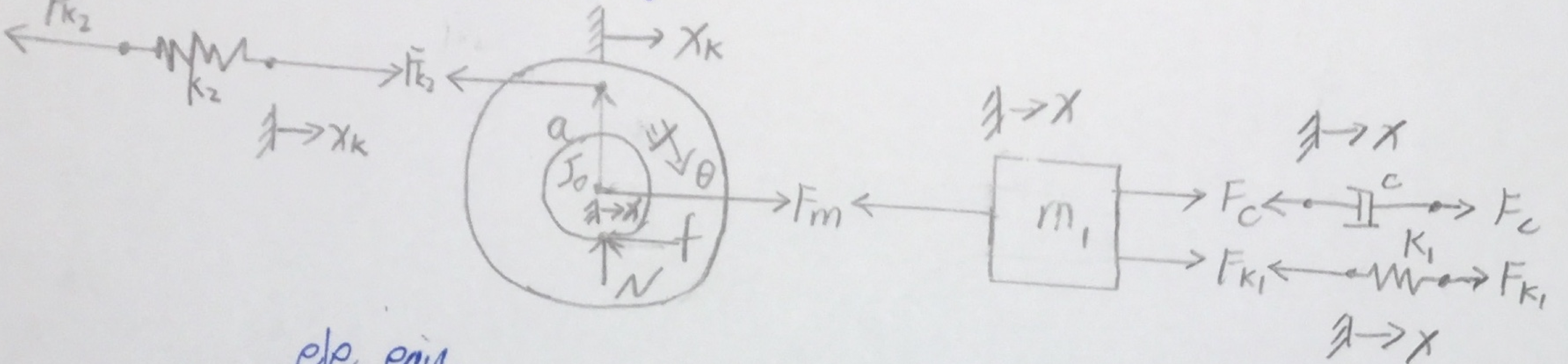
c) $((a/r)m + m_1 + J_o/r^2)\ddot{x} + c\dot{x} + (k_1 + (a+r)^2/r^2)x = 0$

d) $((a/r)m + m_1 + J_o/r^2)\ddot{x} + c\dot{x} + (k_1 + k_2(a+r)/r)x = 0$

e) $(m + (a/r)m_1 + J_o/r^2)\ddot{x} + c\dot{x} + (k_1 + (a+r)^2/r^2)x = 0$

f) $(m + (a/r)m_1 + J_o/r^2)\ddot{x} + c\dot{x} + (k_1 + k_2(a+r)/r)x = 0$

1. (3) sol: ① FBD method



ele. equ.

$k_2: \; F_{k_2} = k_2 x_k$ - - - ①

$m: \; m\ddot{x} = F_m - f - F_{k_2}$ - - - ②

$J_0: \; J_0\ddot{\theta} = fr - F_{k_2}a$ - - - ③

$m_1: \; m_1\ddot{x} = F_c + F_{k_1} - F_m$ - - - ④

$c: \; F_c = -c\dot{x}$ - - - ⑤

$k_1: \; F_{k_1} = -k_1 x$ - - - ⑥

motion relation:

$\theta = \frac{x}{r}$ - - - ⑦

$x_k = \frac{a+r}{r} x$ - - - ⑧

From ③, get $f = \frac{J_0\ddot{\theta} + F_{k_2}a}{r}$ - - - ⑨

From ④⑤⑥, get $F_m = -m_1\ddot{x} - c\dot{x} - k_1 x$ - - ⑩

substitute ①⑨⑩ into ②, get $m\ddot{x} = -m_1\ddot{x} - c\dot{x} - k_1 x - \frac{J_0\ddot{\theta} + k_2 x_k a}{r} - k_2 x_k$ - -

$\Rightarrow m\ddot{x} + m_1\ddot{x} + \frac{J_0}{r}\ddot{\theta} + c\dot{x} + k_1 x + \frac{k_2 a}{r} x_k + k_2 x_k = 0$ - - ⑪

substitute ⑦⑧ into ⑪, get $(m + m_1 + \frac{J_0}{r^2})\ddot{x} + c\dot{x} + k_1 x + \frac{k_2 a}{r} \cdot \frac{a+r}{r} x + k_2 \frac{a+r}{r} x = 0$

$\Rightarrow \boxed{(m + m_1 + \frac{J_0}{r^2})\ddot{x} + c\dot{x} + (k_1 + k_2 \frac{(a+r)^2}{r^2}) x = 0}$ governing equation

② energy method.

$\Sigma KE = \frac{1}{2} m\dot{x}^2 + \frac{1}{2} J_0 \dot{\theta}^2 + \frac{1}{2} m_1 \dot{x}^2 = \frac{1}{2}(m + \frac{J_0}{r^2} + m_1)\dot{x}^2$

$\Sigma PE = \frac{1}{2} k_2 x_k^2 + \frac{1}{2} k_1 x^2 = \frac{1}{2} k_2 (\frac{a+r}{r} x)^2 + \frac{1}{2} k_1 x^2 = \frac{1}{2}[k_1 + (\frac{a+r}{r})^2 k_2] x^2$

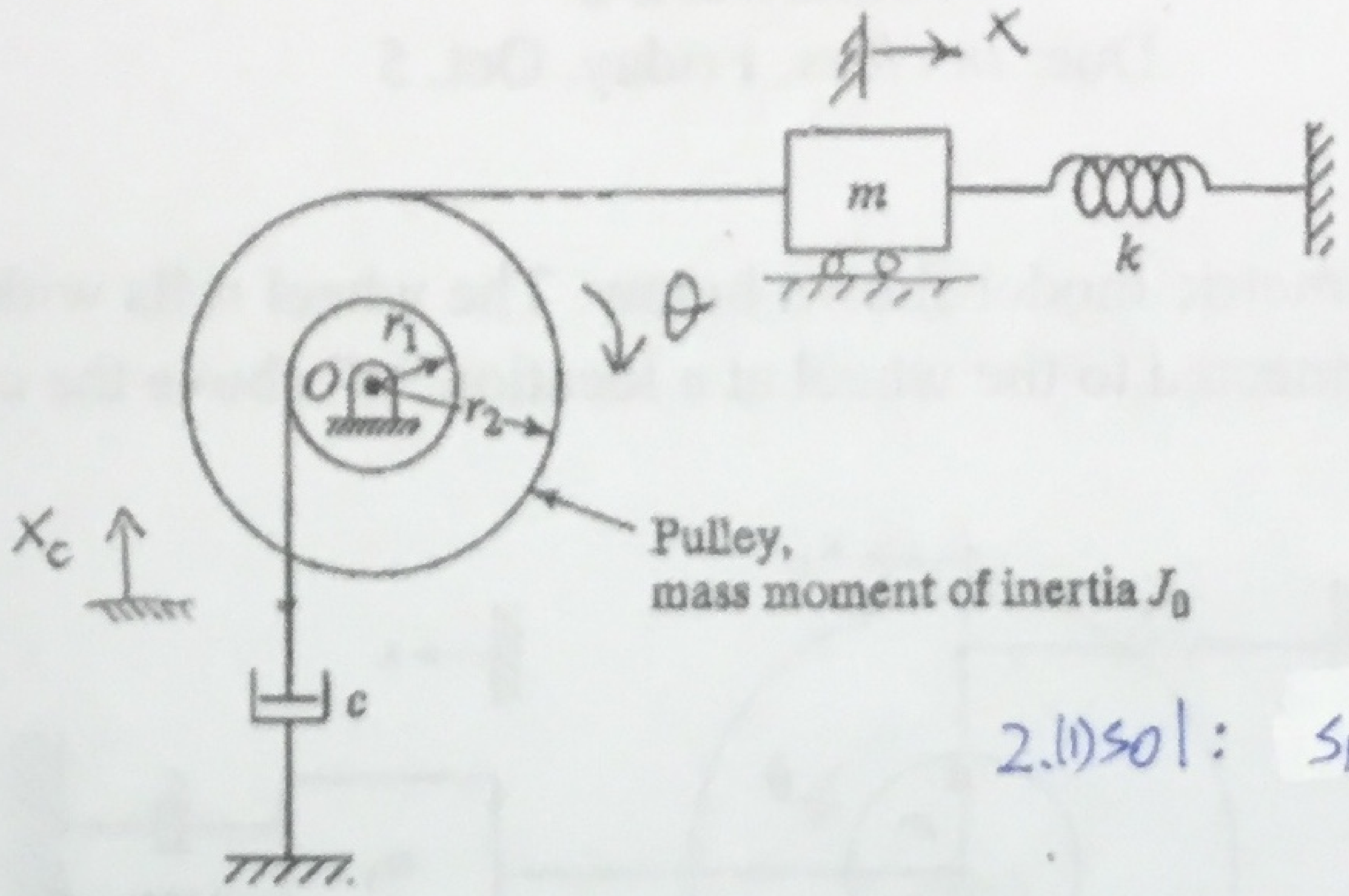
$\Sigma \dot{E}_d = c\dot{x}^2$

since $\frac{d}{dt}(\Sigma KE + \Sigma PE) = -\Sigma \dot{E}_d$

$\Rightarrow \frac{1}{2}(m + m_1 + \frac{J_0}{r^2}) 2\dot{x}\ddot{x} + \frac{1}{2}[k_1 + (\frac{a+r}{r})^2 k_2] 2x\dot{x} = -c\dot{x}^2$

$\Rightarrow \boxed{(m + m_1 + \frac{J_0}{r^2})\ddot{x} + c\dot{x} + (k_1 + (\frac{a+r}{r})^2 k_2) = 0}$ math model

2. (12 pts) Refer to the geometric model below.



2.(1) sol: since $\begin{cases} x = r_2\theta & \text{--- ①} \\ x_c = r_1\theta & \text{--- ②} \end{cases}$

$\Rightarrow x_c = \frac{r_1}{r_2}x$

(1) (2 pts) The motion relation between x_c and x is

a) $x_c = x$

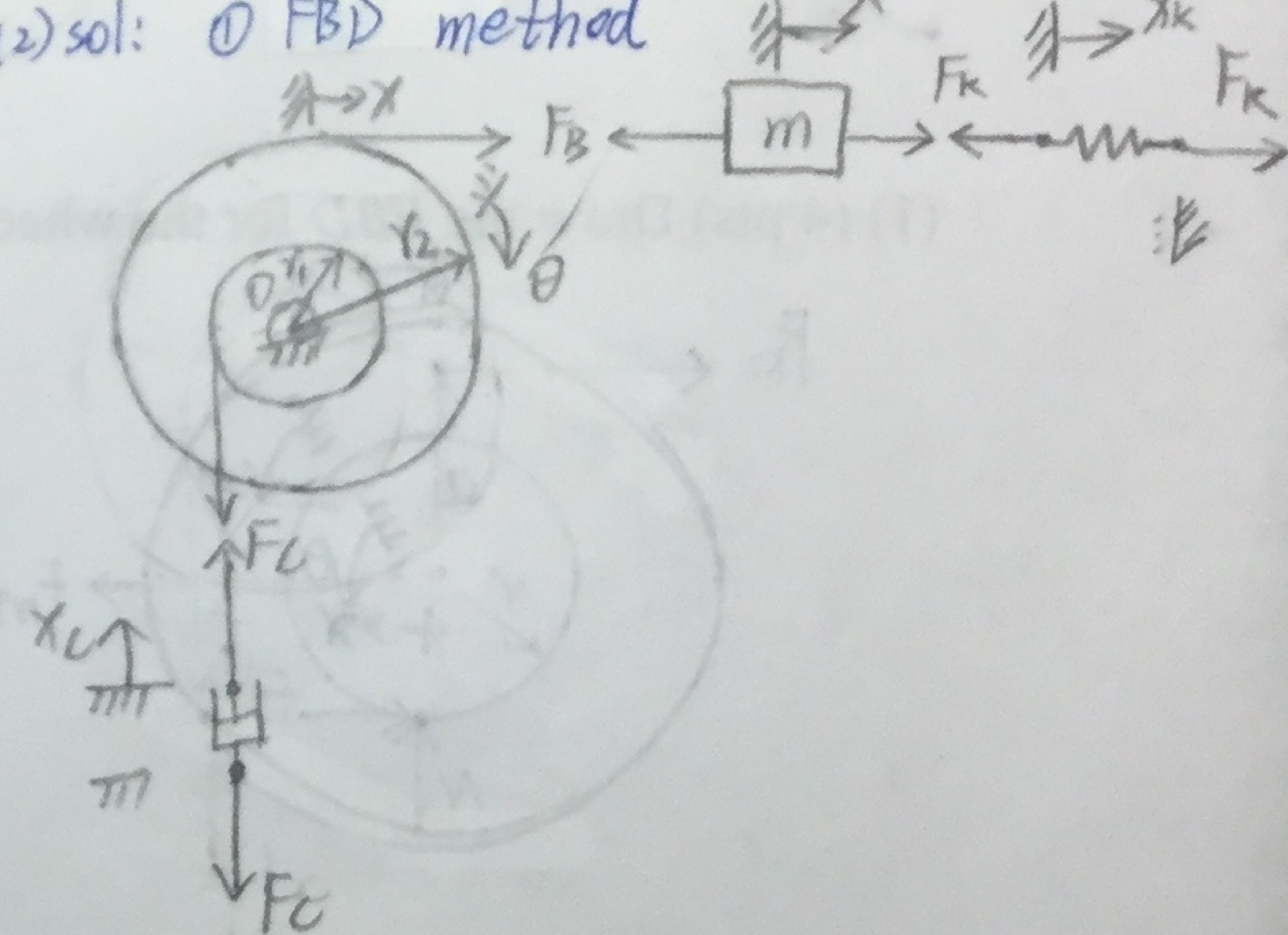
b) $x_c = \frac{r_1}{r_2}x$

c) $x_c = \frac{r_2}{r_1}x$

d) $x_c = \frac{r_1 + r_2}{r_2}x$

e) $x_c = \frac{r_2}{r_1 + r_2}x$

(2) sol: ① FBD method



(2) (10 pts) Let $m = 9\,\text{kg}$, $J_o = 4\,\text{kg-m}^2$, $r_1 = 0.1\,\text{m}$ and $r_2 = 0.25\,\text{m}$. The math model of the system in θ is

a) $656.25\ddot{\theta} + 2c\dot{\theta} + 16.74k\theta = 0$

b) $656.25\ddot{\theta} + c\dot{\theta} + 16.74k\theta = 0$

c) $456.25\ddot{\theta} + c\dot{\theta} + 6.25k\theta = 0$

d) $456.25\ddot{\theta} + 4c\dot{\theta} + 26.83k\theta = 0$

e) $352.74\ddot{\theta} + 2c\dot{\theta} + 6.25k\theta = 0$

f) $352.74\ddot{\theta} + 4c\dot{\theta} + 26.83k\theta = 0$

ele. equ.

k: $F_K = -kx$ --- ③

m: $m\ddot{x} = F_K - F_B$ --- ④

J_o: $J_o\ddot{\theta} = F_B r_2 - F_c r_1$ --- ⑤

c: $F_c = c\dot{x}_c$ --- ⑥

①③④ $\Rightarrow F_B = F_K - m\ddot{x}$

$= -kx - m(r_2\ddot{\theta})$

$= -kr_2\theta - mr_2\ddot{\theta}$ --- ⑦

substitute ⑦⑥ into ⑤

$J_o\ddot{\theta} = (-kr_2\theta - mr_2\ddot{\theta})r_2 - c\dot{x}_c r_1$

$= -kr_2^2\theta - mr_2^2\ddot{\theta} - cr_1^2\dot{\theta}$

i.e. math model:

$\boxed{(J_o + mr_2^2)\ddot{\theta} + cr_1^2\dot{\theta} + kr_2^2\theta = 0}$ --- ⑧

substitute the value of m, J_o, r_1, r_2 into ⑧, get

$456.25\ddot{\theta} + c\dot{\theta} + 6.25k\theta = 0$

② energy method.

$\Sigma KE = \frac{1}{2}J_o\dot{\theta}^2 + \frac{1}{2}m\dot{x}^2 = \frac{1}{2}(J_o + mr_2^2)\dot{\theta}^2$

$\Sigma PE = \frac{1}{2}kx^2 = \frac{1}{2}kr_2^2\theta^2$

$\Sigma\dot{E}_d = c\dot{x}_c^2 = cr_1^2\dot{\theta}^2$

since $\frac{d}{dt}(\Sigma KE + \Sigma PE) = -\Sigma\dot{E}_d$

$\Rightarrow \frac{1}{2}(J_o + mr_2^2)2\dot{\theta}\ddot{\theta} + \frac{1}{2}kr_2^2 2\theta\dot{\theta} = -cr_1^2\dot{\theta}^2$

i.e. $(J_o + mr_2^2)\ddot{\theta} + kr_1^2\dot{\theta} + kr_2^2\theta = 0$