ME 370.1 Fall 2018

Name(s)

Homework 7

Due: *In class*, Friday, Oct. 12

1. (10 pts) The bouncing mode of the vertical vibration of a car suspension system is found to be optimal when it is underdamped. The desirable damping constant of the shock absorber is determined by the following design specification:

The car body is lifted up by a small amount above its static equilibrium position (the tires are still in touch with the ground) and then released. During the bouncing motion of the car, its lowest position below static equilibrium should be approximately 20% of the amount of the original lift.

Assume the mass of the car is 2,400 kg and is equally supported by the four tires. Also, assume the stiffness of each suspension spring is $k = 25{,}000$ N/m.

(1) (7 pts) The damping ratio, ξ, of the system corresponding to the above design specification should be:

a) $\xi = 0.1332$

b) $\xi = 0.2297$

c) $\xi = 0.4559$

d) $\xi = 0.5732$

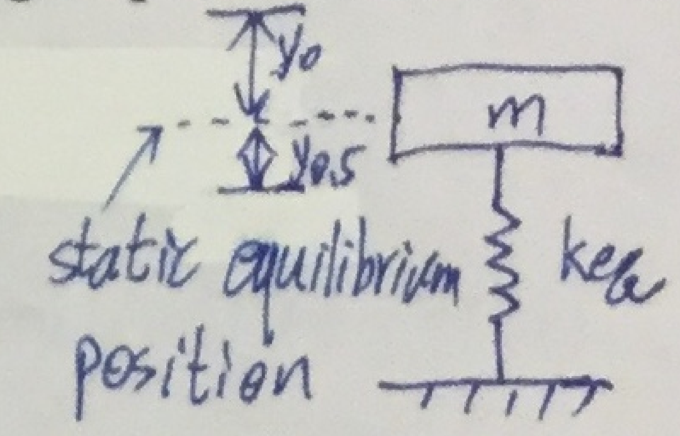
e) $\xi =$ 07249

sol: since $y_{ns} = 20\% \cdot y_0$

$$\frac{y_0}{y_{ns}} = \sqrt{\frac{y_0}{y_1}} = 5$$

Then $\delta = \ln\frac{y_0}{y_1} = \ln 25$

$$\text{Thus } \xi = \frac{\delta}{\sqrt{4\pi^2+\delta^2}} = \frac{\ln 25}{\sqrt{4\pi^2+(\ln 25)^2}} \approx 0.4559$$



(2) (3 pts) The shock absorber of the suspension system should give a damping constant, *c* in unit N-s/m, of:

a) c = 11114

b) c = 13492

c) c = 14126

d) c = 15943

e) c = 18258

sol: $\xi = \frac{1}{2\omega_n} \cdot \frac{c_{eq}}{m_{eq}}$, where $\omega_n = \sqrt{\frac{k_{eq}}{m_{eq}}}$

since four suspension springs are parallel to each other,

$k_{eq} = 4k = 100{,}000$ N/m

$$\Rightarrow c_{eq} = 2\omega_n m_{eq} \xi$$

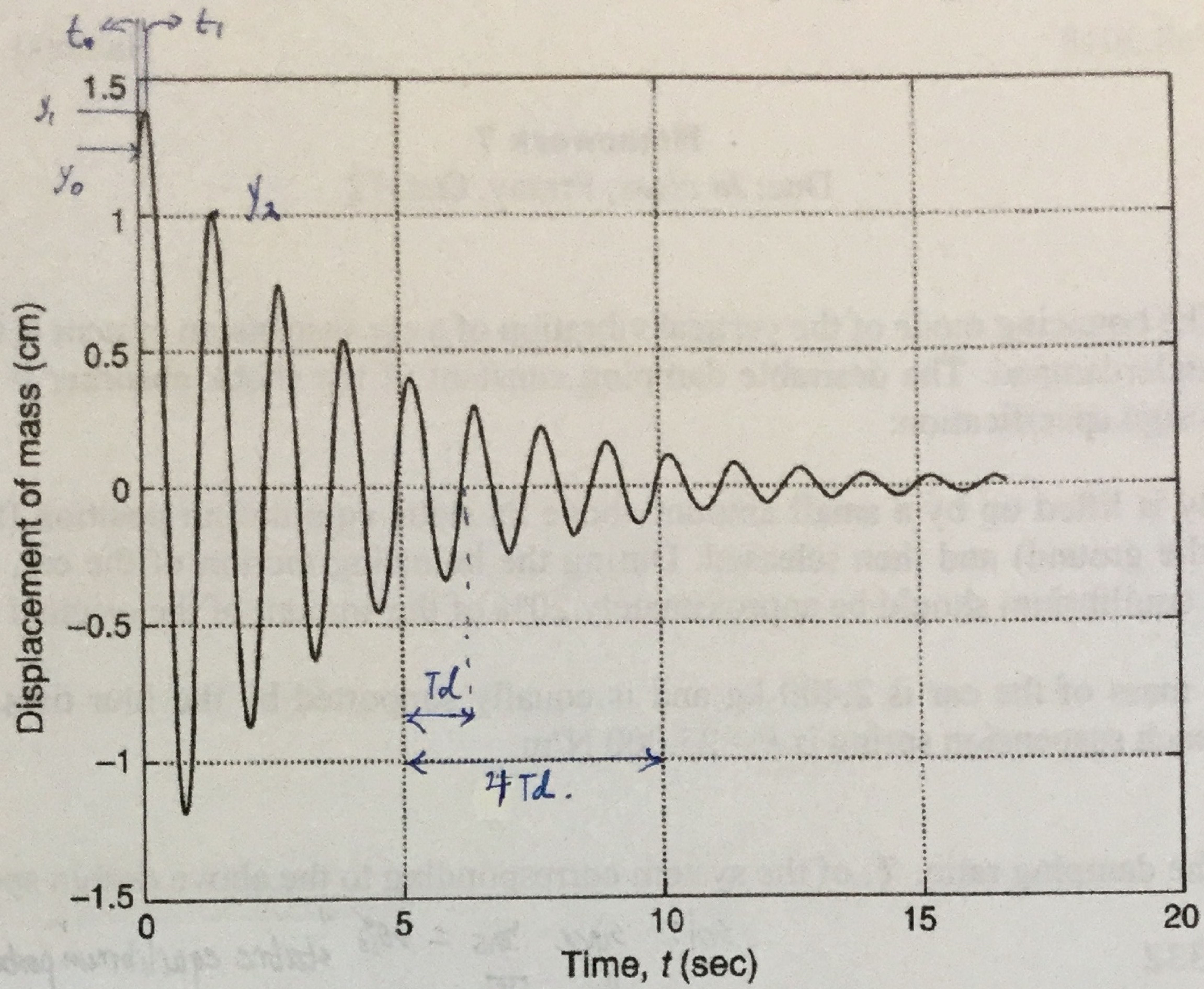
$$= 2\sqrt{\frac{k_{eq}}{m_{eq}}} \cdot m_{eq} \cdot \xi$$

$$= 2 \cdot \sqrt{\frac{100000}{2400}} \cdot 2400 \cdot 0.4559$$

$$\doteq 14125.54$$

$$\approx 14126$$

2. (20 pts) Below is the record of a free vibration of a mass-spring-damper system:



(1) (5 pts) The damping ratio of the system is about equal to

a) $\xi = 0.02$ b) $\xi = 0.035$ c) $\xi = 0.05$ d) $\xi = 0.065$

(2) (5 pts) The natural frequency of the system in rad/s is about equal to

a) $\omega_n = 2.57$ b) $\omega_n = 3.93$ c) $\omega_n = 5.03$ d) $\omega_n = 6.11$

(3) (5 pts) The initial velocity of the mass in cm/s is estimated ***from the graph*** to be

a) $\dot{y}_o = -10$ b) $\dot{y}_o = 1.0$ c) $\dot{y}_o = 10$ d) $\dot{y}_o = 20$

(4) (5 pts) The expression, $y(t) = Ae^{-\omega_n \xi t}\sin(\omega_d t + \phi)$, that describes the displacement of the mass is approximately given by

a) $y(t) = 1.28e^{-\xi\omega_n t}\sin(\omega_d t + 1.12)$

b) $y(t) = 1.28e^{-\xi\omega_n t}\sin(\omega_d t + 1.37)$

c) $y(t) = 1.35e^{-\xi\omega_n t}\sin(\omega_d t + 1.37)$

d) $y(t) = 1.35e^{-\xi\omega_n t}\sin(\omega_d t + 1.76)$

e) $y(t) = 1.14e^{-\xi\omega_n t}\sin(\omega_d t + 1.76)$

f) $y(t) = 1.14e^{-\xi\omega_n t}\sin(\omega_d t + 1.12)$

Problem 2.

(1) sol: we denote: y_0 : initial position
y_1 : position at first peak
y_2 : position at second peak

then logarithm decrement: $\delta = \ln\frac{y_1}{y_2} = \ln\frac{1.4}{1} = \ln 1.4 = 0.33647$

From the graph, $4T_d = 5\,s \Rightarrow T_d = 1.25\,s$

Thus $\xi = \frac{\delta}{\sqrt{4\pi^2+\delta^2}} = \frac{\ln 1.4}{\sqrt{4\pi^2+(\ln 1.4)^2}} \approx 0.0535 \approx 0.05$

(2) sol: $\omega_n = \frac{2\pi}{T_d\sqrt{1-\xi^2}} = \frac{2\pi}{1.25\sqrt{1-0.05^2}} \approx 5.0328 \approx 5.03$

(3) sol: denote t_0 : initial time at y_0, $t_0=0$
t_1 : time at y_1, t_1 is approximately $\frac{T_d}{8}$

then $\dot{y}_0$ is approximately the slope of segment between t_0 and t_1.

i.e. $\dot{y}_0 \approx \frac{y_1-y_0}{t_1-t_0} = \frac{1.4-1.25}{\frac{1.25}{8}} = 0.96 \approx 1.0$

(4) sol: $\begin{cases} B_1 = y(0) = 1.25 \\ B_2 = \frac{\dot{y}(0)+\xi\omega_n y(0)}{\omega_d} = \frac{\dot{y}(0)+\xi\omega_n y(0)}{\frac{2\pi}{T_d}} = \frac{1+0.05\cdot 5.03\cdot 1.25}{\frac{2\pi}{1.25}} = 0.2615 \end{cases}$

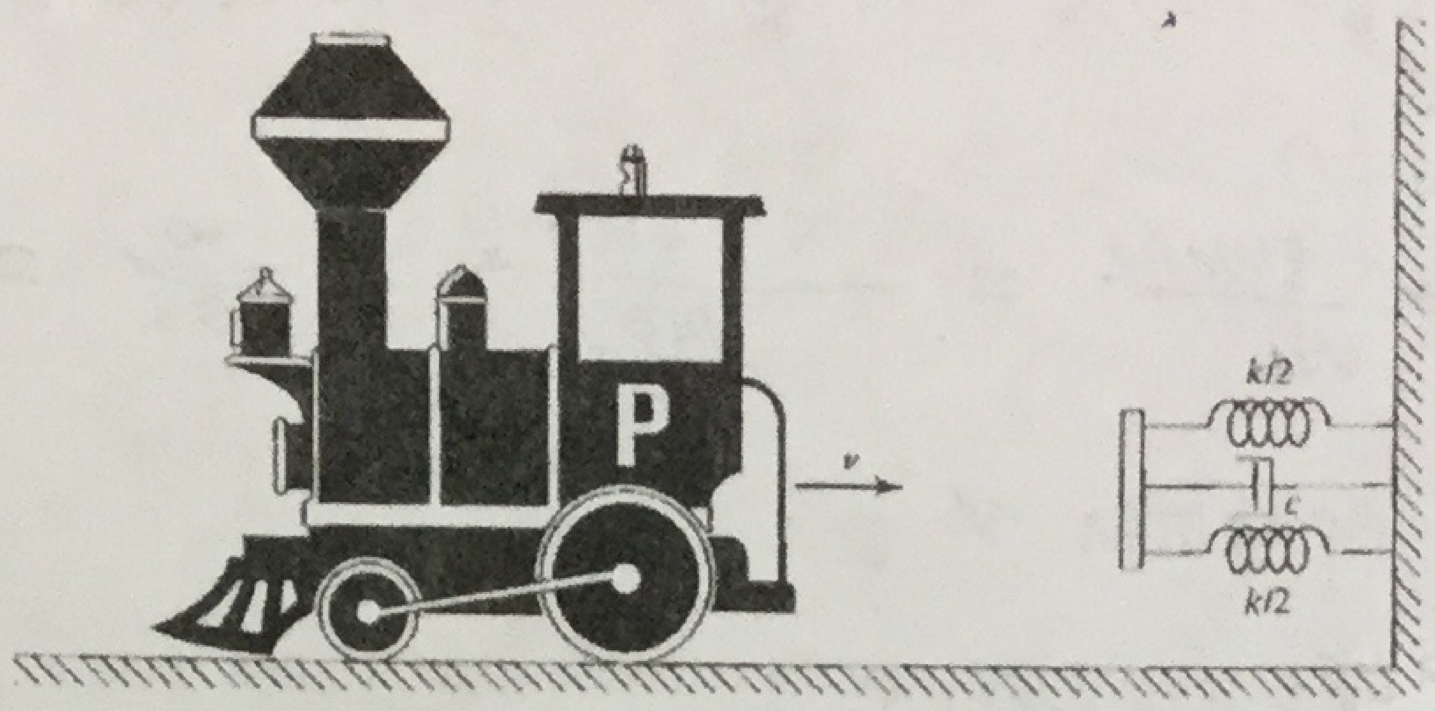
$A = \sqrt{B_1^2+B_2^2} = \sqrt{1.25^2+0.2615^2} = 1.277 \approx 1.28$

$\phi = \tan_2^{-1}\frac{B_1}{B_2} = \tan^{-1}\frac{B_1}{B_2} = \tan^{-1}\frac{1.25}{0.26} = 1.3658 \approx 1.37$

so the displacement equation is:

$y(t) = 1.28\,e^{-\xi\omega_n t}\sin(\omega_d t + 1.37)$

3. (25 pts) A railroad car of mass 2000 kg traveling at a velocity of 10 m/s is to be stopped at the end of the track by a spring-damper assembly, as shown below. The stiffness of the assembly is $k = 40{,}000$ N/m and the damping constant is $c = 10{,}000$ N-s/m.



(1) (3 pts) *Write down* the governing equation of the system *with initial conditions* corresponding to the moment when the railroad car engages with the spring-damper assembly.

(1)sol: It is a simple mass-spring-damper system

Thus governing equation: $m\ddot{x} + c\dot{x} + kx = 0$

Initial conditions: $x_0 = 0$, $\dot{x}_0 = 10$

(2) (2 pts) The motion of the car is given by

(2)sol: $\omega_n = \sqrt{\frac{k}{m}} = \sqrt{\frac{40000}{2000}} = 2\sqrt{5}$

$\xi = \frac{1}{2\omega_n}\frac{c}{m} = \frac{1}{4\sqrt{5}} \cdot \frac{10000}{2000} = \frac{\sqrt{5}}{4} < 1$

Thus, it's a underdamped system

Motion equation form is (C)

a) $x(t) = A_1 e^{s_1 t} + A_2 e^{s_2 t}$

b) $x(t) = (A_1 + A_2 t)e^{-\omega_n t}$

c) $x(t) = e^{-\xi\omega_n t}(A_1 \cos\omega_d t + A_2 \sin\omega_d t)$

(3) (10 pts) Perform necessary calculations and then generate a smooth matlab plot of $x(t)$ with labels, making sure it satisfies ICs (submit the plot). From the plot, identify the maximum displacement of the car and the time taken to reach it from the choices below (x_{max} in meter t_{max} in second):

a) $x_{max} = 0.97$ and $t_{max} = 0.19$

b) $x_{max} = 1.16$ and $t_{max} = 0.26$

c) $x_{max} = 1.72$ and $t_{max} = 0.35$

d) $x_{max} = 2.23$ and $t_{max} = 0.31$

e) $x_{max} = 2.69$ and $t_{max} = 0.42$

f) $x_{max} = 3.71$ and $t_{max} = 0.63$

(4) (10 pts) Also, generate and submit a smooth matlab plot of the contact force between the car and the stopping assembly (ie. the sum of $F_k(t)$ and $F_c(t)$). From the plot, identify the maximum contact force during the stopping process from the choices below (F_{max} in kN):

a) $F_{max} = 11.2$

b) $F_{max} = 40.4$

c) $F_{max} = 100$

d) $F_{max} = 205.5$

e) $F_{max} = 400$

f) $F_{max} = 500$

Each plot is 7 pts.

3. (3) sol: From (2), it's motion equation is

$$x(t) = e^{-\xi\omega_n t}(A_1\cos\omega_d t + A_2\sin\omega_d t)$$

and $\omega_n = 2\sqrt{5}$, $\xi = \frac{\sqrt{5}}{4} \approx 0.5590$

$$\Rightarrow \omega_d = \omega_n\sqrt{1-\xi^2} = 2\sqrt{5}\cdot\sqrt{1-\left(\frac{\sqrt{5}}{4}\right)^2} = \frac{\sqrt{55}}{2} \approx 3.7081$$

since
$$\begin{cases} A_1 = x_0 = 0 \\ A_2 = \dfrac{\dot{x}_0 + \xi\omega_n x_0}{\omega_d} = \dfrac{10 + \xi\omega_n\cdot 0}{\frac{\sqrt{55}}{2}} = \dfrac{20}{\sqrt{55}} \approx 2.6968 \end{cases}$$

Thus, motion equation is

$$x(t) = \frac{20}{\sqrt{55}} e^{-\frac{5}{2}t}\sin\left(\frac{\sqrt{55}}{2}t\right)$$

$$\approx 2.6968\, e^{-2.5t}\sin(3.7081t)$$

plot the curve $x(t)$ vs. t using Matlab, shown in Figure 1.

From the Figure 1, $\boxed{X_{max} = 1.16,\ t_{max} = 0.26}$

(4) sol: ① method one

$$F_{ck} = -m\ddot{x}(t)$$

Plot graph of $-m\ddot{x}(t)$ vs. t

② method two

$$F_{ck} = F_c + F_k = c\dot{x}(t) + kx(t)$$

plot the graph of $F_{ck} = c\dot{x}(t) + kx(t)$ vs. t

The contact force graph is shown in Figure 2.

From it, $\boxed{F_{max} = 100\,kN}$ at $t=0$

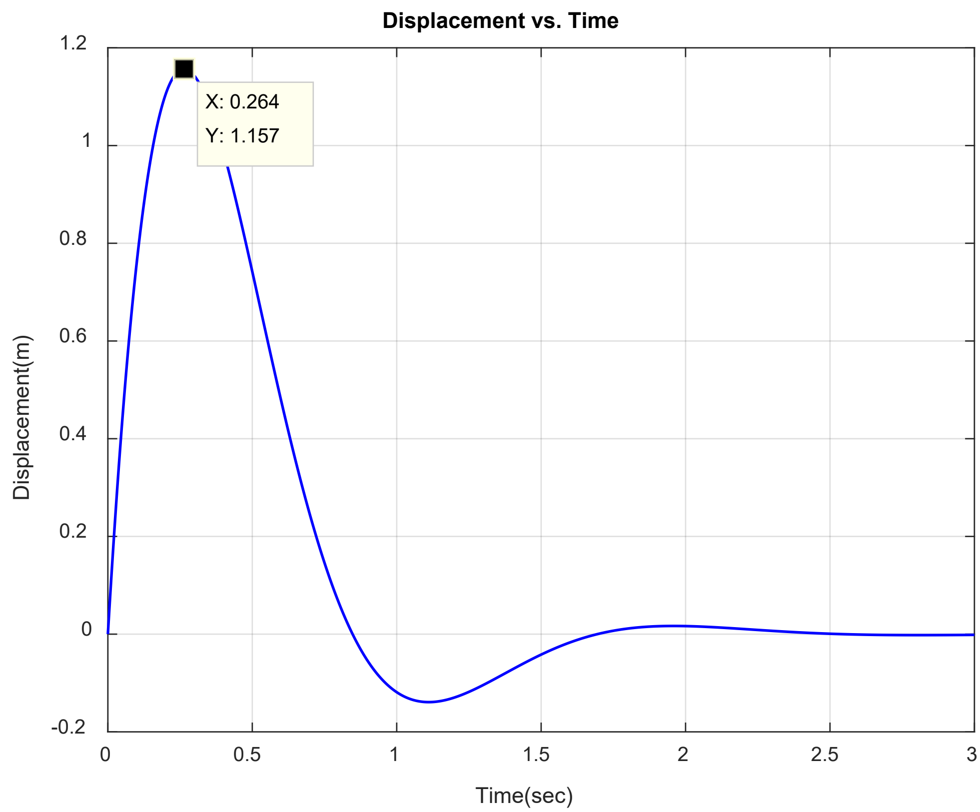


Figure 1

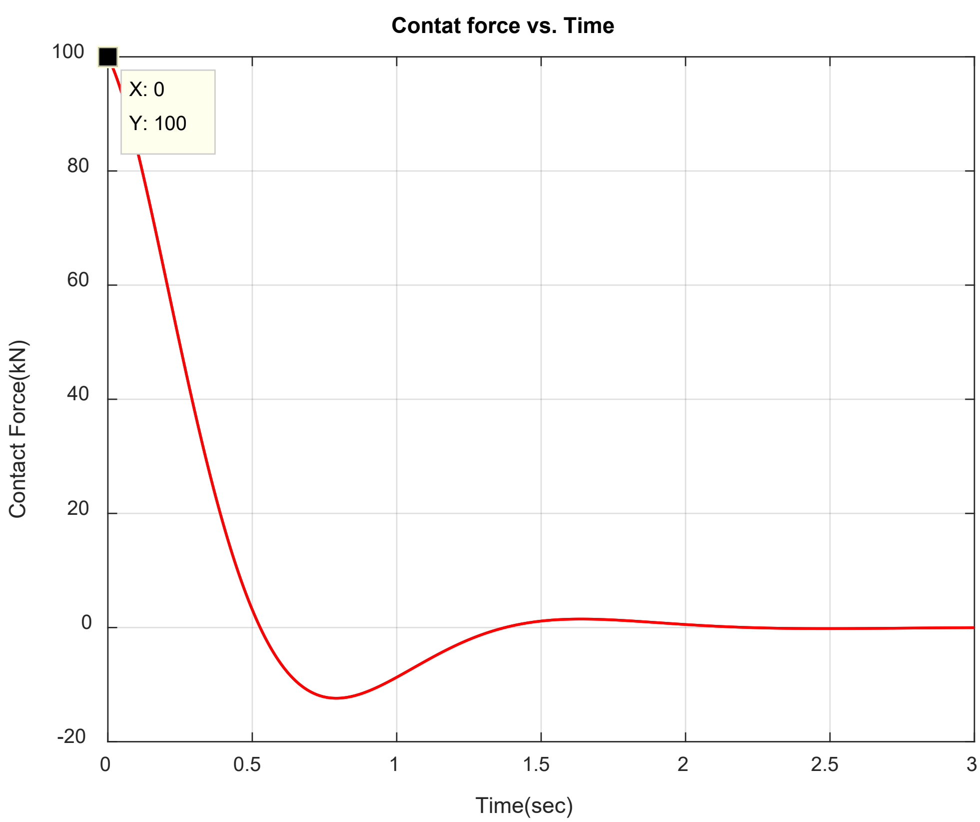


Figure 2

Code demo:

1. Plot motion and force curve

```
%{
 * Course:ME370
 * Name:Liming Gao
 * Date: October, 2018
 *
 * Program Description: Solution for Hw07 Problem 3.
 %}

clear all
close all

t= 0:0.001:3; %Set time sequences
xt = (20/(55^0.5))*exp(-2.5*t).*sin(55^0.5*t/2+0); %Motion equation
vt = (2*exp(-(5*t)/2).*(55*cos((55^(1/2)*t)/2) -
5*55^(1/2)*sin((55^(1/2)*t)/2)))/11; %The derivative of Motion equation
at = -(10*exp(-(5*t)/2).*(55*cos((55^(1/2)*t)/2) +
3*55^(1/2)*sin((55^(1/2)*t)/2)))/11; %The Second derivative of Motion equation
F = (40000*xt+10000*vt)/1000;
% or Contact force is
% F = (-2000*at)/1000;

%Plot the figure
figure(1)
plot(t,xt,'b','LineWidth',1); %plot the curve

xlabel('Time(sec)')
ylabel('Displacement(m)')
title('Displacement vs. Time')
grid on

figure(2)
plot(t,F,'r','LineWidth',1); %plot the curve
xlabel('Time(sec)')
ylabel('Contact Force(kN)')
title('Contat force vs. Time')
grid on
```

2. Derivative calculation

```
%{
 * Course:ME370
 * Name: Liming Gao
 * Date: October, 2018
 *
 * Program Description: Calculate the derivative
 %}

syms t

xt=(20/(55^0.5))*exp(-2.5*t).*sin(55^0.5*t/2+0);

vt=simplify(diff(xt,t))
at= simplify(diff(xt,t,2))
```