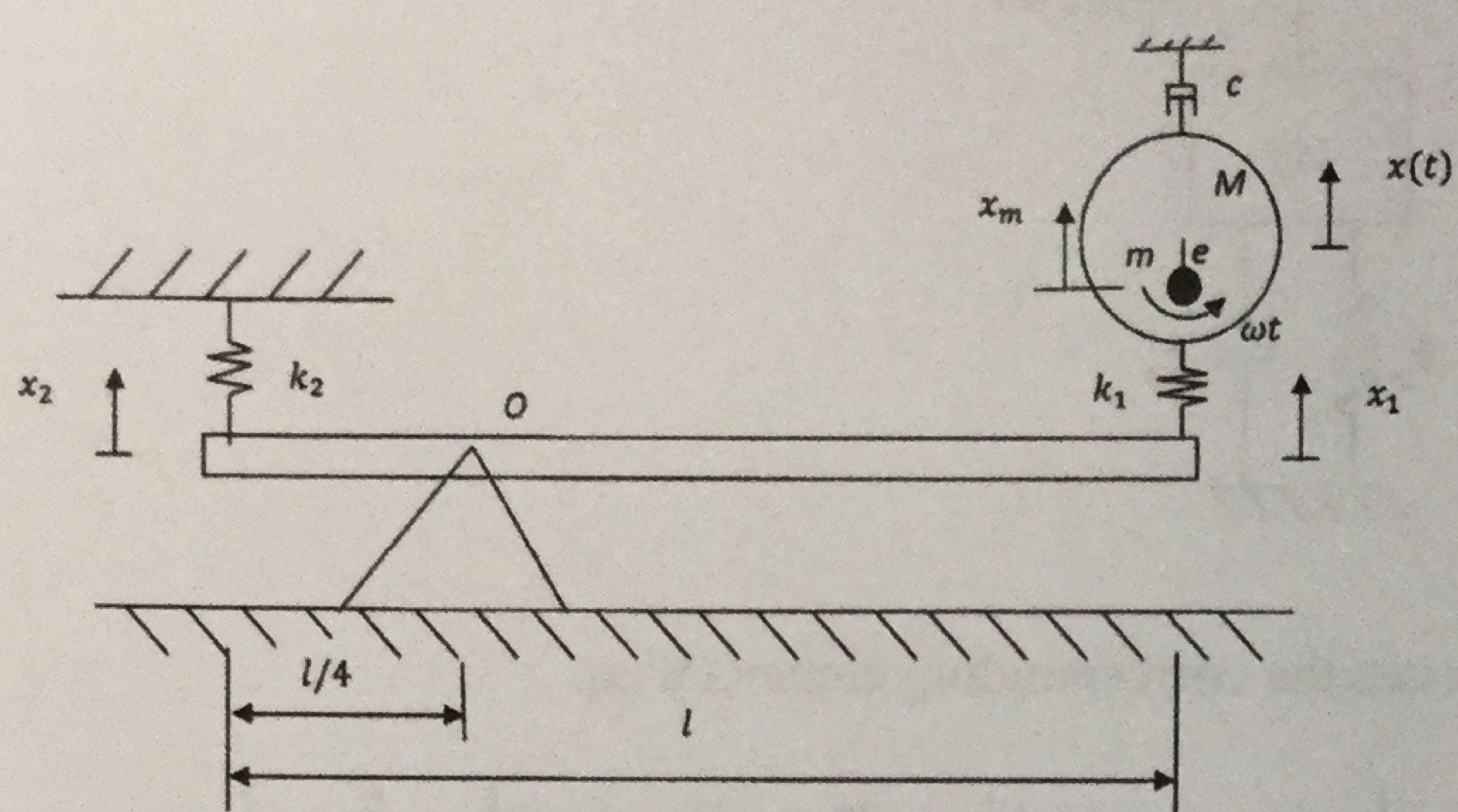
ME 370.1 Fall 2018

Name(s)

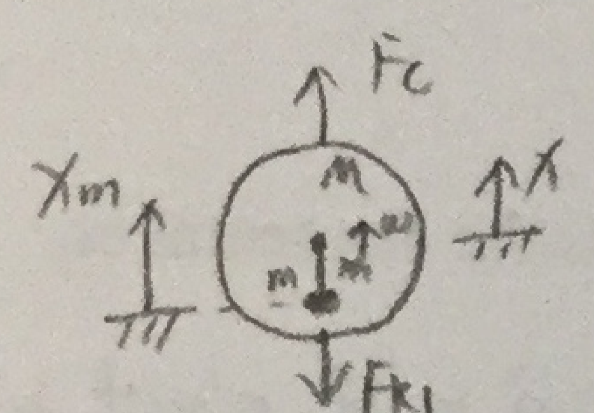
Homework 8

Due: *In class*, Friday, Oct. 19

1. (20 pts) A motor system with its supporting structure is shown below, where the bar of pivot O is considered rigid and massless. The motor is of a total mass M and a rotating mass m with a rotating arm e. The mass rotates at an angular velocity ω and is right below the center of rotation at $t = 0$.

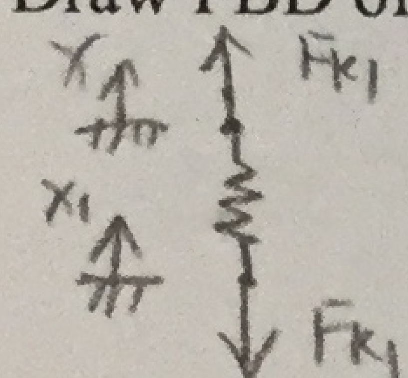


(1) (2 pts) Draw FBD of the motor in x-direction of motion and write the corresponding elemental eq.



ele. eq: $(M-m)\ddot{x} + m\ddot{x}_m = F_c - F_{k_1}$

(2) (2 pts) Draw FBD of spring k_1 consistent with that of the motor in (1) and write its elemental eq.



ele. eq: $F_{k_1} = k_1(x - x_1)$

(3) (2 pts) The elemental eq. relating x_m to x is:

a) $x_m = x + e \cdot \cos\omega t$

b) $x_m = x - e \cdot \sin\omega t$

c) $x_m = x - e \cdot \cos\omega t$

d) $x_m = x + e \cdot \sin\omega t$

(4) (14 pts) The math model of the system in x is:

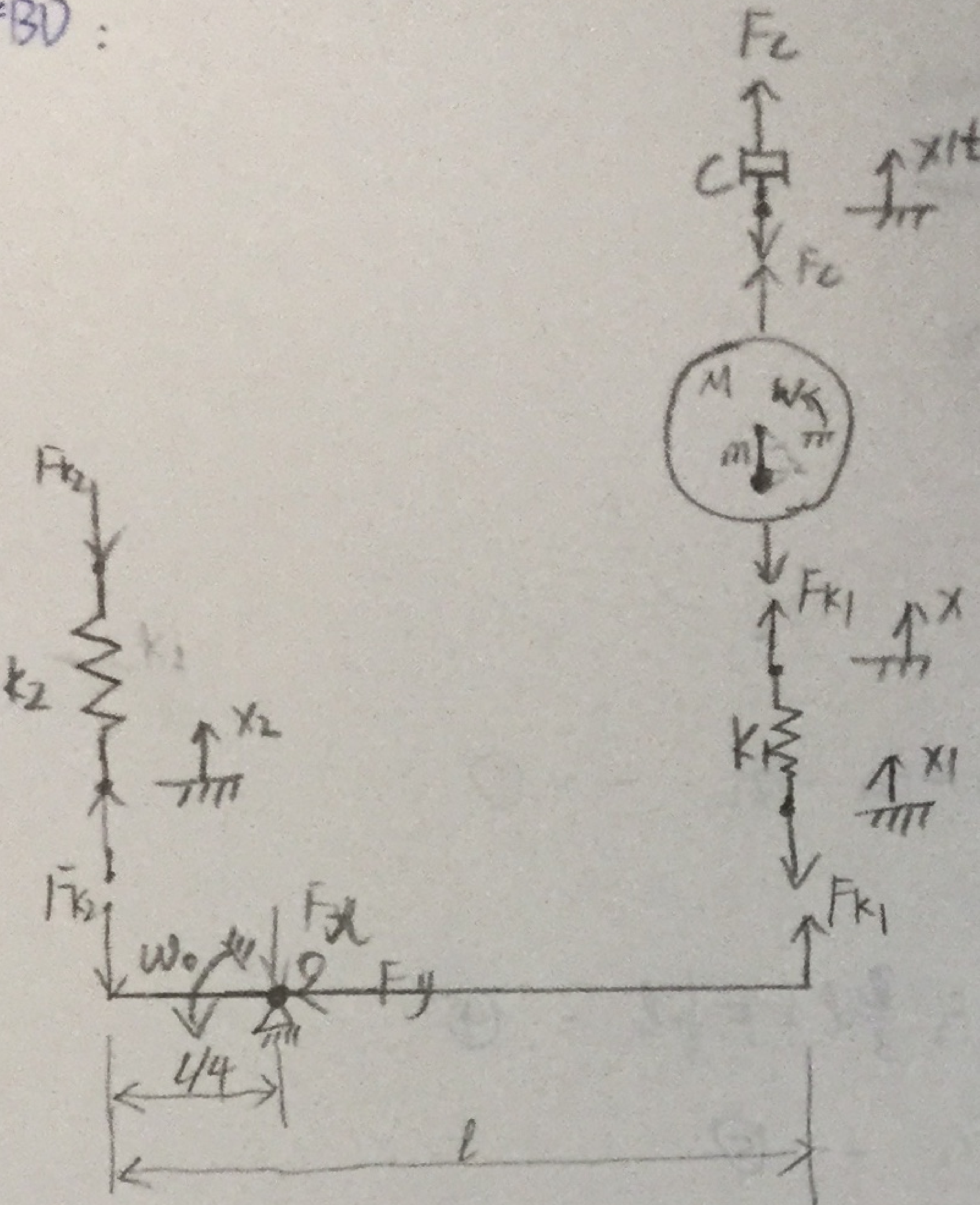
a) $M\ddot{x} + c\dot{x} + \frac{k_1 k_2}{9k_1 + k_2} x = -me\omega^2 \cos\omega t$

b) $M\ddot{x} + 2c\dot{x} + \frac{k_1 k_2}{2k_1 + k_2} x = -me\omega^2 \cos\omega t$

c) $M\ddot{x} + c\dot{x} + \frac{2k_1 k_2}{9k_1 + k_2} x = -me\omega^2 \sin\omega t$

d) $M\ddot{x} + 2c\dot{x} + \frac{k_1 k_2}{9k_1 + k_2} x = -me\omega^2 \sin\omega t$

1.(4) sol: FBD:



ele. eq.

motor: $(M-m)\ddot{x} + m\ddot{x}_m = F_c - F_{k_1}$ -- ①

k_1: $F_{k_1} = k_1(x - x_1)$ - - ②

C: $F_c = -c\dot{x}$ - - - ③

bar: $F_{k_1}\frac{3}{4}l + F_{k_2}\frac{l}{4} = 0$ - - ④

k_2: $F_{k_2} = k_2 x_2$ - - - ⑤

motion: $x_1 = -3x_2$ - - - ⑥

$x_m = x - e\cos\omega t$ - - - ⑦

substitute ②⑤⑥ into ④, get $k_1(x-x_1)\frac{3}{4}l + k_2\left(-\frac{1}{3}x_1\right)\frac{l}{4} = 0$

$$\Rightarrow x_1 = \frac{9k_1}{9k_1+k_2}x \quad \text{- - -} ⑧$$

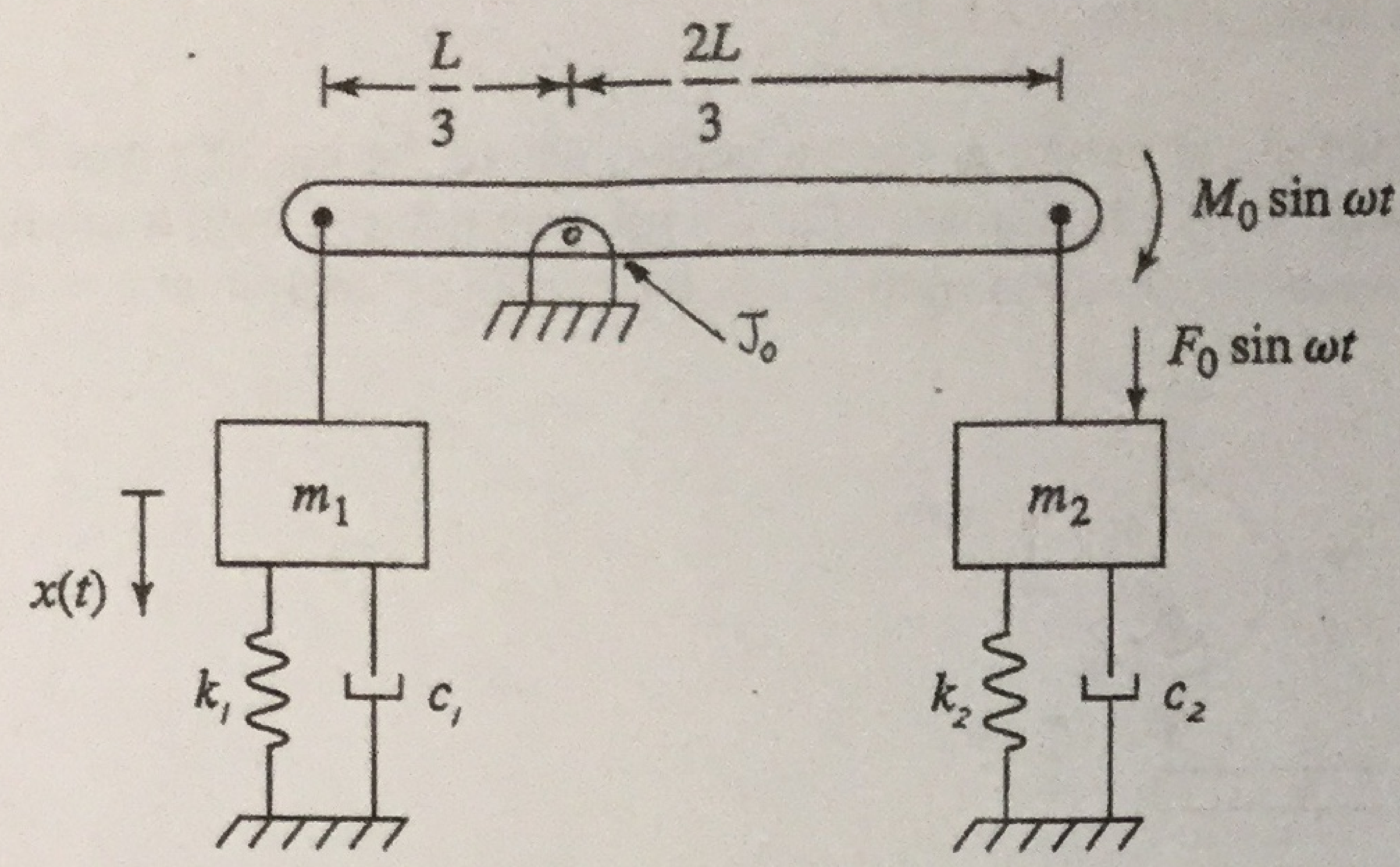
according to ⑦, $\Rightarrow \ddot{x}_m = \ddot{x} + e\omega^2\cos\omega t$ .. ⑨

substitute ②③⑧⑨ into ①, get

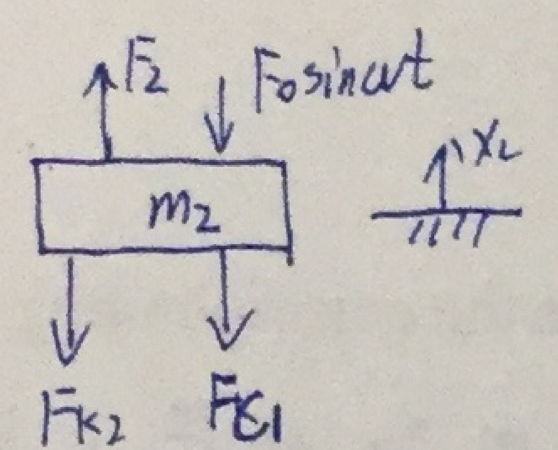
$$(M-m)\ddot{x} + m(\ddot{x} + e\omega^2\cos\omega t) = -c\dot{x} - k_1\left(x - \frac{9k_1}{9k_1+k_2}x\right)$$

$$\Rightarrow M\ddot{x} + c\dot{x} + \frac{k_1k_2}{9k_1+k_2}x = -me\omega^2\cos\omega t.$$

2. (14 pts) A mechanical system is described by the following geometric model. There are two harmonic actions on the system. One is the force on mass m_2 and the other is the moment on the bar J_o.

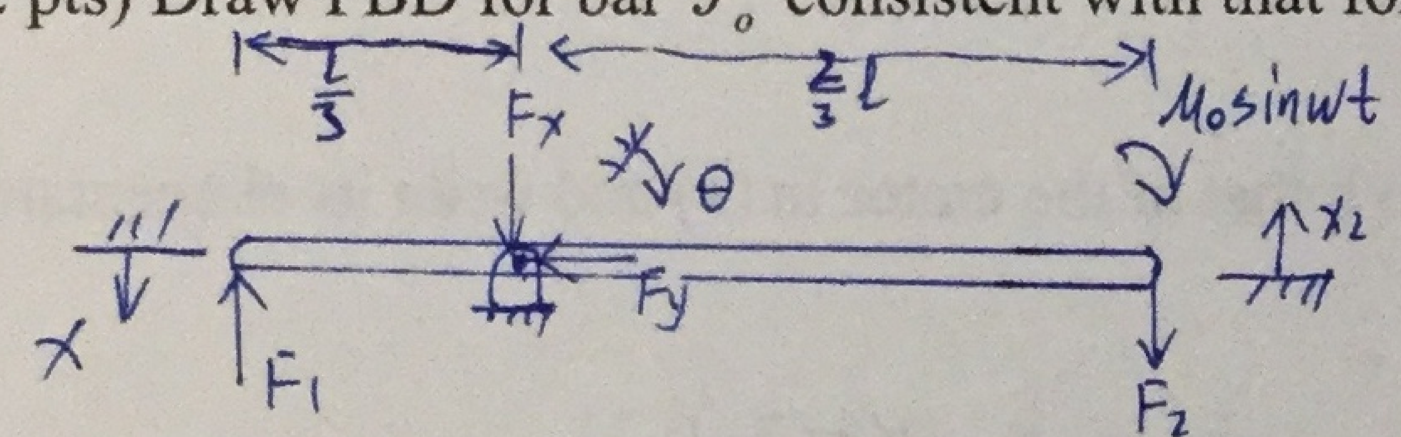


(1) (2 pts) Draw FBD of mass m_2 and writes the corresponding elemental eq.



ele. eq: $m_2 \ddot{x}_2 = F_2 - F_0 \sin\omega t - F_{k2} - F_{c2}$

(2) (2 pts) Draw FBD for bar J_o consistent with that for m_2 in (1) and writes the elemental eq.



ele. eq: $J_o \ddot{\theta} = M_o \sin\omega t + F_2 \frac{2}{3}L + F_1 \frac{1}{3}L$

(3) (10 pts) The math model of the system in x is:

a) $\left(m_1 + 2m_2 + \frac{9}{L^2}J_o\right)\ddot{x} + (c_1 + 2c_2)\dot{x} + (k_1 + 2k_2)x = -\left(\frac{3M_o}{L} + 2F_o\right) sin\omega t$

b) $\left(m_1 + 4m_2 + \frac{9}{L^2}J_o\right)\ddot{x} + (c_1 + 4c_2)\dot{x} + (k_1 + 4k_2)x = -\left(\frac{3M_o}{L} + 2F_o\right) sin\omega t$

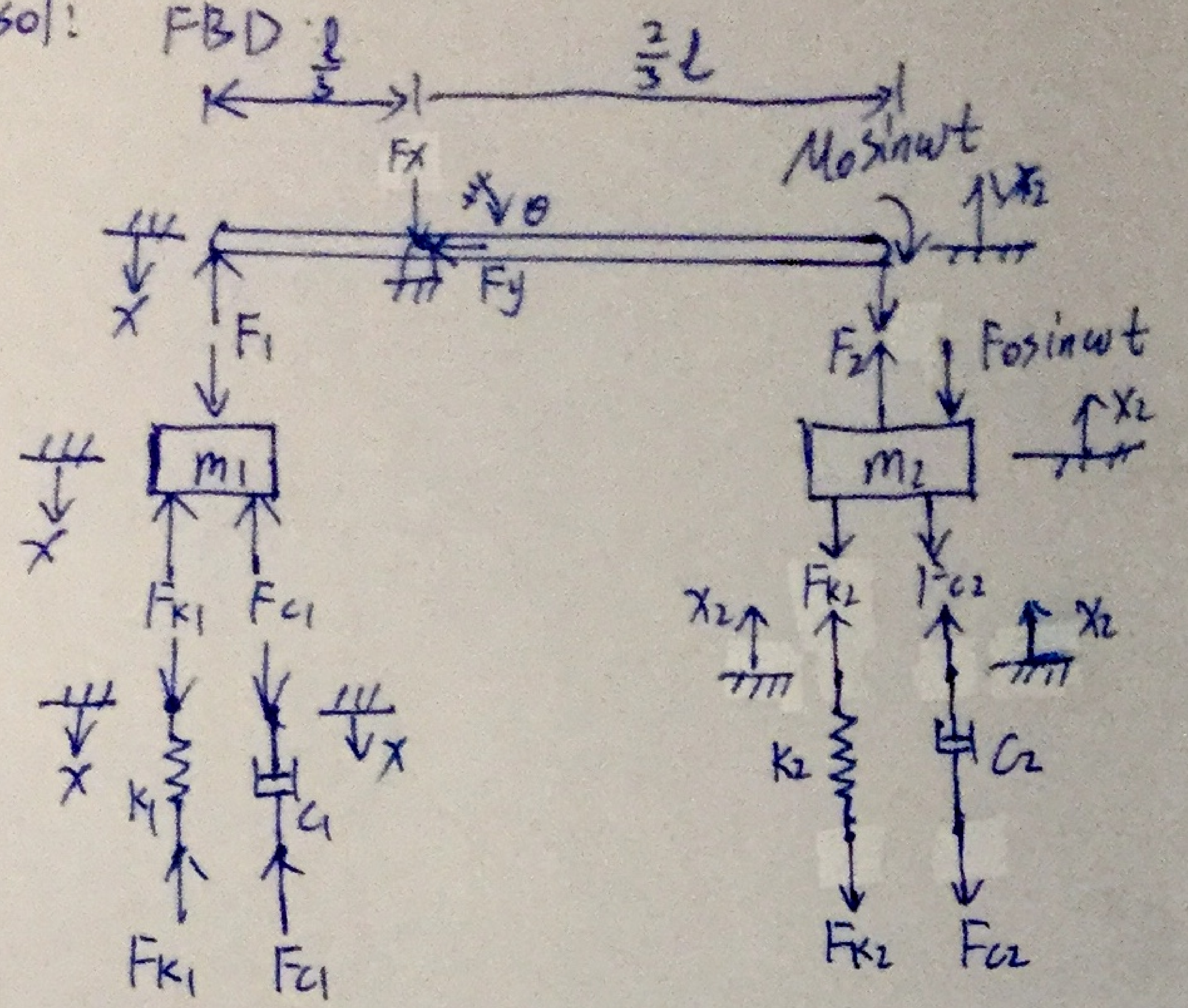
c) $\left(m_1 + 4m_2 + \frac{3}{2L^2}J_o\right)\ddot{x} + (c_1 + 2c_2)\dot{x} + (k_1 + 2k_2)x = -\left(\frac{3M_o}{L} + F_o\right) sin\omega t$

d) $\left(m_1 + 4m_2 + \frac{9}{4L^2}J_o\right)\ddot{x} + (c_1 + 4c_2)\dot{x} + (k_1 + 4k_2)x = -\left(\frac{3M_o}{L} + F_o\right) sin\omega t$

e) $\left(m_1 + 2m_2 + \frac{9}{L^2}J_o\right)\ddot{x} + (c_1 + 2c_2)\dot{x} + (k_1 + 2k_2)x = -\left(\frac{3M_o}{L} + F_o\right) sin\omega t$

f) $\left(m_1 + 2 + \frac{3}{2L^2}J_o\right)\ddot{x} + (c_1 + 4c_2)\dot{x} + (k_1 + 4k_2)x = -\left(\frac{3M_o}{L} + 2F_o\right) sin\omega t$

2. (3) sol: FBD



ele eqs.

m_2: $m_2\ddot{x}_2 = F_2 - F_0\sin\omega t - F_{k2} - F_{c2}$ --- ①

k_2: $F_{k2} = k_2 x_2$ --- ②

c_2: $F_{c2} = c_2\dot{x}_2$ --- ③

bar: $J_0\ddot{\theta} = M_0\sin\omega t + F_2\frac{2}{3}l + F_1\frac{1}{3}l$ -- ④

m_1: $m_1\ddot{x} = F_1 - F_{k1} - F_{c1}$ -- ⑤

k_1: $F_{k1} = k_1 x$ -- ⑥

c_1: $F_{c1} = c_1 x$ -- ⑦

motion: $x_2 = 2x$ -- ⑧

$\theta = -\frac{3}{l}x$ -- ⑨

substitute ②③⑧ into ①, get $2m_2\ddot{x} + F_0\sin\omega t + 2k_2 x + 2c_2\dot{x} = F_2$ -- ⑩

substitute ⑥⑦ into ⑤ get $m_1\ddot{x} + k_1 x + c_1\dot{x} = F_1$ -- ⑪

substitute ⑩⑪⑨ into ④, get

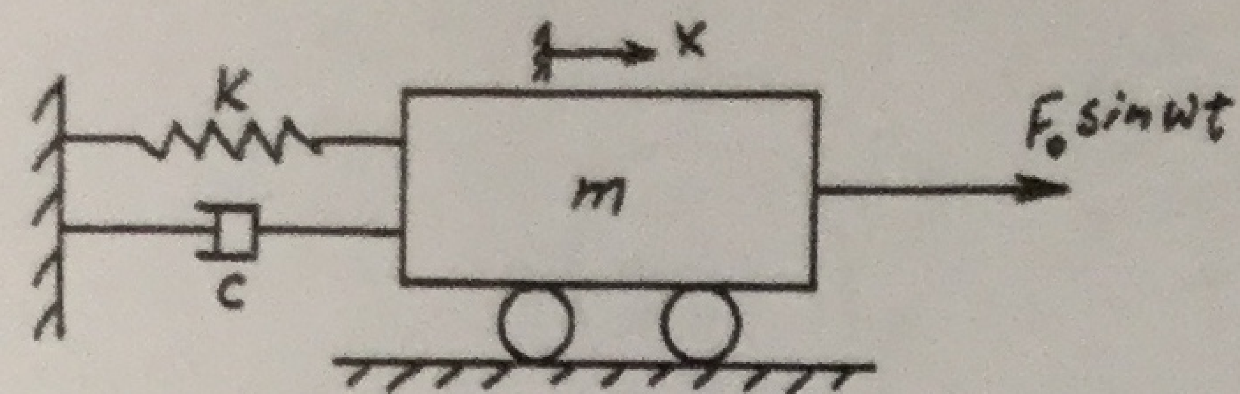
$$J_0\left(-\frac{3}{l}\right)\ddot{x} = M_0\sin\omega t + \frac{2l}{3}(2m_2\ddot{x} + F_0\sin\omega t + 2k_2 x + 2c_2\dot{x}) + \frac{l}{3}(m_1\ddot{x} + c\dot{x} + k_1 x)$$

multiple $\frac{3}{l}$ on both side, get

$$-J_0\frac{9}{l^2}\ddot{x} = \frac{3M_0}{l}\sin\omega t + 2(2m_2\ddot{x} + F_0\sin\omega t + 2k_2 x + 2c_2\dot{x}) + (m_1\ddot{x} + c\dot{x} + k_1 x)$$

$$\Rightarrow \left(m_1 + 4m_2 + \frac{9}{l^2}J_0\right)\ddot{x} + (c_1 + 4c_2)\dot{x} + (k_1 + 4k_2)x = -\left(\frac{3M_0}{l} + 2F_0\right)\sin\omega t.$$

3. (16 pts) A mass-spring-damper system is shown below, whose governing equation was derived in class. Let m = 10 kg, k = 4000 N/m, c = 40 N-s/m, F_o = 100 N, ω = 25 rad/s, $x(0) = 0.1$ and $\dot{x}(0) = -2.0$ m/s.



(1) (3 pts) The magnitude of excitation, E, and the frequency ratio, r, as defined in class notes are

a) $E = 10$ & $r = 0.85$ b) $E = 100$ & $r = 1.25$

c) $E = 100$ & $r = 0.85$ d) $E = 10$ & $r = 1.25$

e) $E = 200$ & $r = 0.92$ f) $E = 200$ & $r = 1.45$

Write down the units of E N/kg and r no units

3.(1) sol. governing Equation:

$m\ddot{x} + c\dot{x} + kx = F_o \sin\omega t$

i.e. $\ddot{x} + \frac{c}{m}\dot{x} + \frac{k}{m}x = \frac{F_o}{m}\sin\omega t$

$\Rightarrow E = \frac{F_o}{m} = \frac{100N}{10kg} = 10\ N/kg$

$r = \frac{\omega}{\omega_n} = \frac{25\ rad/s}{\sqrt{\frac{4000N/m}{10kg}}} = \frac{25}{20} = 1.25$

where, $\omega_n = \sqrt{\frac{k}{m}} = 20$

(2) (6 pts) The expression for the system *follow-up response* in meter is

a) $x_p(t) = 0.041\sin(25t - 2.723)$ b) $x_p(t) = 0.041\sin(25t + 0.418)$

c) $x_p(t) = 0.041\sin(20t - 2.723)$ d) $x_p(t) = 0.041\sin(20t + 0.418)$

e) $x_p(t) = 0.082\sin(25t - 2.723)$ f) $x_p(t) = 0.082\sin(25t + 0.418)$

3.(2) sol: $\xi = \frac{1}{2\omega_n}\frac{c}{m} = \frac{1}{2\times 20}\cdot\frac{40}{10} = 0.1$

Thus:

$$Y = \frac{E/\omega_n^2}{[(1-r^2)^2 + (2\xi r)^2]^{\frac{1}{2}}} = \frac{10/20^2}{[(1-1.25^2)^2 + (2\times 0.1\times 1.25)^2]^{\frac{1}{2}}} \approx 0.041$$

$$\varphi = -\tan_2^{-1}\frac{2\xi r}{1-r^2} = -\tan_2^{-1}\frac{2\times 0.1\times 1.25}{1-1.25^2} = -\left[\tan^{-1}\frac{2\times 0.1\times 1.25}{1-1.25^2} + \pi\right] = -2.723$$

$x_p(t) = Y\sin(\omega t + \varphi)$

(3) (7 pts) The expression for the system *total response* is

a) $x(t) = e^{-2t}(0.067\cos 19.9t + 0.174\sin 19.9t) + x_p(t)$

b) $x(t) = e^{-2t}(0.067\cos 19.9t - 0.054\sin 19.9t) + x_p(t)$

c) $x(t) = e^{-2t}(0.117\cos 19.9t + 0.033\sin 19.9t) + x_p(t)$

d) $x(t) = e^{-2t}(0.117\cos 19.9t - 0.042\sin 19.9t) + x_p(t)$

e) $x(t) = e^{-2t}(0.347\cos 19.9t + 0.154\sin 19.9t) + x_p(t)$

3.(3) sol: since $\xi = 0.1 < 1$, $x_h(t) = e^{-\xi\omega_n t}(B_1\cos\omega_d t + B_2\sin\omega_d t)$

$x(t) = x_h(t) + x_p(t)$, $\omega_d = \omega_n\sqrt{1-\xi^2} = 20\cdot\sqrt{1-0.1^2} \approx 19.9$

$$\Rightarrow \begin{cases} x_h(0) = x(0) - x_p(0) = x(0) - Y\sin\varphi = 0.1 - 0.041\sin(-2.723) \approx 0.117 \\ \dot{x}_h(0) = \dot{x}(0) - \dot{x}_p(0) = \dot{x}(0) - \omega Y\cos\varphi = -2 - 25\cdot 0.041\cdot\cos(-2.723) \approx -1.060 \end{cases}$$

$$\Rightarrow \begin{cases} B_1 = x_h(0) = 0.117 \\ B_2 = \frac{\dot{x}_h(0) + \xi\omega_n x_h(0)}{\omega_d} = \frac{-1.060 + 0.1\cdot 20\cdot 0.117}{19.9} \approx -0.042 \end{cases}$$

thus.

$x(t) = x_h(t) + x_p(t)$

$= e^{-2t}(0.117\cos 19.9t - 0.042\sin 19.9t) + x_p(t)$