ME 370.1 Fall 2018 Name(s)

Homework 12

Due: *In class*, Friday Nov. 30

1. [25 pts] Consider the function given below (where $s = i\omega$):

$$T(s) = \frac{5}{6s^3 + 11s^2 + 6s + 1}$$

(1) (5 pts) Obtain expressions of the amplitude and phase angle of T as functions of ω.

$$T = \frac{5}{\sqrt{(1-11w^2)^2 + (6w - 6w^3)^2}}, \quad \phi = \tan_2^{-1} \frac{6w^3 - 6w}{1 - 11w^2}$$

(2) (2 pts) The amplitude of $T(i0.3)$ is

a) $A = 0.721$ b) $A = 2.192$

c) $A = 3.053$ d) $A = 5.334$

(3) (2 pts) The phase angle (in rad) of $T(i0.3)$ is

a) $\psi =$ -0.525 b) $\psi =$ -1.565

c) $\psi =$ -2.078 d) $\psi =$ -2.912

(4) (2 pts) The amplitude of $T(i1.0)$ is

a) $A = 0.2$ b) $A = 1.0$

c) $A = 0.5$ d) $A = 2.0$

(5) (2 pts) The phase angle (in rad) of $T(i1.0)$ is

a) $\psi =$ -π/3 b) $\psi =$ -π

c) $\psi =$ -π/2 d) $\psi =$ -3π/2

(6) (12 pts) Plot in matlab Im(T) *vs.* Re(T) in the complex plane as ω increases from 0 to 5 rad/s. In your plot, use matlab data curser tool to help obtain:

a) Amplitude of T when the curve crosses the negative real axis, $A|_{\psi=\pi} =$ 0.5

b) Phase angle of T when the amplitude of T is reduced to unity, $\psi|_{A=1.0} =$ -2.7046

(Determine it using the data curser with $Re^2(T) + Im^2(T) = 1$ and $\tan_2^{-1}\frac{Im}{Re}$).

Submit your matlab program along with the plot with data curser marks.

1. sol:

(1) $T(s) = T(jw) = \dfrac{5}{6(jw)^3 + 11(jw)^2 + 6(jw) + 1} = \dfrac{5}{(1-11w^2) + i(6w - 6w^3)}$

$Amp[T(iw)] = \dfrac{Amp(5)}{Amp[(1-11w^2) + i(6w-6w^3)]} = \dfrac{5}{\sqrt{(1-11w^2)^2 + (6w-6w^3)^2}}$

$Angle[T(iw)] = angle(5) - angle((1-11w^2) + i(6w-6w^3))$

$= 0 - \tan_2^{-1} \dfrac{6w - 6w^3}{1-11w^2}$

$= \tan_2^{-1} \dfrac{6w^3 - 6w}{1-11w^2}$

(2)(3) $Amp[T(i0.3)] = \dfrac{5}{\sqrt{(1-11 \cdot 0.3^2)^2 + (6 \cdot 0.3 - 6 \cdot 0.3^3)^2}} = 3.053$

$Angle[T(i0.3)] = \tan_2^{-1} \dfrac{6 \cdot 0.3^3 - 6 \cdot 0.3}{1 - 11 \cdot 0.3^2} = \tan_2^{-1} \dfrac{-1.638}{0.01} = -1.565$

(4)(5) $Amp[T(i1.0)] = \dfrac{5}{\sqrt{(1-11 \cdot 1^2)^2 + (6 \cdot 1 - 6 \cdot 1^3)^2}} = \dfrac{5}{10} = 0.5$

$Angle[T(i1.0)] = \tan_2^{-1} \dfrac{6 \cdot 1^3 - 6 \cdot 1}{1 - 11 \cdot 1^2} = \tan_2^{-1} \dfrac{0}{-10} = -\pi$

(6) $T(s) = T(jw) = \dfrac{5}{(1-11w^2) + i(6w - 6w^3)} = \dfrac{5[(1-11w^2) - i(6w-6w^3)]}{(1-11w^2)^2 + (6w-6w^3)^2}$

$\Rightarrow Im(T^*) = \dfrac{-5(6w-6w^3)}{(1-11w^2)^2 + (6w-6w^3)^2}$

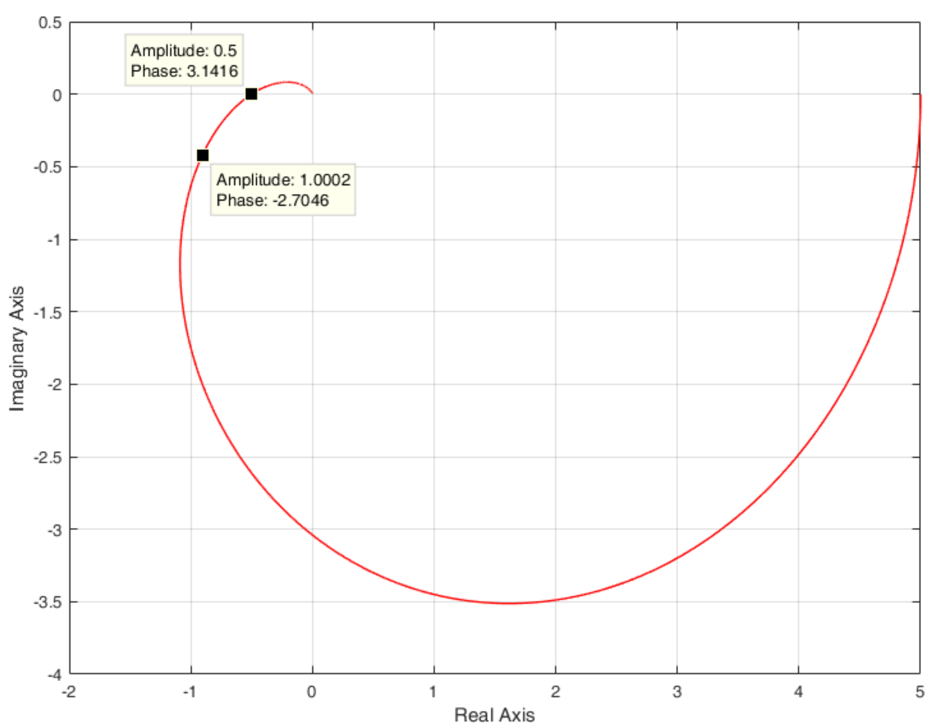
$Re(T^*) = \dfrac{5(1-11w^2)}{(1-11w^2)^2 + (6w-6w^3)^2}$

a) $A|_{\varphi=\pi} = 0.5$

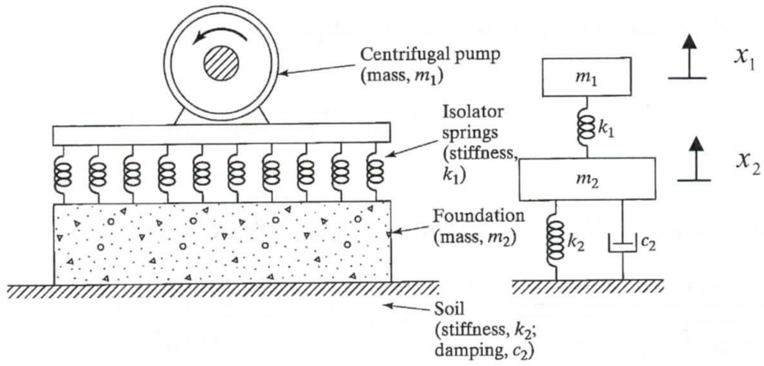
b) $\varphi|_{A=1.0} = -2.7046$

matlab program and plot are shown in next page.

```
%{
 * Course:ME370
 * Name:Liming Gao
 * Date: Nov. 14, 2018
 *
 * Program Description: Problem 1.06, hw12.
 %}
clear all
close all

w = 0:0.001:5; %frequency
Im = -5 * (6*w - 6*w.^3 ) ./ ((1 - 11*w.^2).^2 + (6*w - 6*w.^3).^2);
%Imaginary part
Re = 5 * (1 - 11*w.^2) ./ ((1 - 11*w.^2).^2 + (6*w - 6*w.^3).^2); % Real part
T = Re+i*Im;

fig = figure(1);
plot(T,'r','LineWidth',1)
xlabel('Real Axis');
ylabel('Imaginary Axis');
grid on

% interactive data cursor
datacursormode on
dcm_obj = datacursormode(fig);
set(dcm_obj,'UpdateFcn',@myupdatefcn)
function txt = myupdatefcn(empt,event_obj)

% Customizes text of data tips
pos = get(event_obj,'Position');
txt = { ['Amplitude: ',num2str(sqrt(pos(1)^2+ pos(2)^2))],...
          ['Phase: ',num2str(atan2(pos(2),pos(1)))]};
end
```



2. [28 pts] A centrifugal pump and its mounting foundation is modeled by a geometrical model shown below. The mass of the rotating part of the pump is m with its center of gravity, e, offset from the axis of rotation. This imbalance, me, generates a rotating centrifugal force, inducing a vertical vibration of the system.



The math model of the system is given by

$$\begin{bmatrix} m_1 & 0 \\ 0 & m_2 \end{bmatrix}\begin{bmatrix} \ddot{x}_1 \\ \ddot{x}_2 \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ 0 & c_2 \end{bmatrix}\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} + \begin{bmatrix} k_1 & -k_1 \\ -k_1 & k_1+k_2 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} me\omega^2\cos(\omega t+\alpha) \\ 0 \end{bmatrix}$$

Let $m_1 = 300\,\text{kg}$, $m_2 = 4000\,\text{kg}$, $me = 0.1\,\text{kg-m}$, $k = 1{,}000\,\text{kN/m}$, $k_2 = 10{,}000\,\text{kN/m}$, $c_2 = 25{,}000\,\text{N-s/m}$, $\omega = 1200\,\text{rpm}$ and $\alpha = \pi/3$.

(1) (3 pts) The expression describing the stead-state vibration of the pump mass is (where ψ is phase lag behind the physical driving action):

a) $x_1(t) = X_1 \sin(\omega t + \psi_1)$

b) $x_1(t) = X_1 \cos(\omega t + \psi_1)$

c) $x_1(t) = X_1 \sin(\omega t + \alpha + \psi_1)$

d) $x_1(t) = X_1 \cos(\omega t + \alpha + \psi_1)$

e) $x_1(t) = X_1 e^{i(\omega t+\alpha+\psi_1)}$

f) $x_1(t) = X_1 e^{i(\omega t+\psi_1)}$

(2) (3pts) Write down the numerical values of the impedance matrix:

$$[Z] = \left[[k] - \omega^2[M] + i\omega[C]\right] = 10^6 \times \begin{bmatrix} -3.737 & -1 \\ -1 & -52.165 + i3.142 \end{bmatrix}$$

(3) (3 pts) The natural frequencies of the system in rad/s are:

$\omega_{n1} = 45.149$ $\qquad \omega_{n2} = 63.099$

(4) (2 pts) The amplitude of vibration of the pump in millimeters is

a) $X_1 = 0.224$ b) $X_1 = 0.324$

c) $X_1 = 0.424$ d) $X_1 = 0.524$

(5) (2 pts) The amplitude of vibration of the foundation in millimeters is

a) $X_2 = 0.008$ b) $X_2 = 0.018$

c) $X_2 = 0.028$ d) $X_2 = 0.038$

(6) (2 pts) The phase lag in rad of $x_1(t)$ from the excitation is

a) $\psi_1 = 0.57$ b) $\psi_1 = -1.57$

c) $\psi_1 = 2.57$ d) $\psi_1 = -3.14$

(7) (2 pts) The phase lag in rad of $x_2(t)$ from the excitation is

b) $\psi_2 = 0.06$ b) $\psi_2 = -0.43$

c) $\psi_2 = 1.57$ d) $\psi_2 = -2.34$

(8) (5 pts) The magnitude of the force in newton transmitted to the soil ground is

a) $F_{to} = 63.2$ b) $F_{to} = 85.2$

c) $F_{to} = 97.2$ d) $F_{to} = 110.2$

(9) (3 pts) If the speed of the pump is reduced to one half of the current speed, the magnitude of the force transmitted to the soil ground would be:

$F_{to} = 12800$

(10) (3 pts) Explain to the point why the force in Step (9) is so much larger than that in Step (8).

Ans: The half of current speed is $20\pi = 62.8$ rad/s, which is very close to the 2nd natural frequency, thus resonance occurs.

2. sol:

(1) since RHS = $\begin{bmatrix} m e \omega^2 \cos(\omega t+\alpha) \\ 0 \end{bmatrix}$, then $x_1(t) = \underline{X_1} \cos(\omega t + \alpha + \varphi_1)$

(2) $[Z] = [-\omega^2[M] + i\omega[C] + [k]]$

where $[M] = \begin{bmatrix} m_1 & 0 \\ 0 & m_2 \end{bmatrix} = \begin{bmatrix} 300 & 0 \\ 0 & 4000 \end{bmatrix}$, $[C] = \begin{bmatrix} 0 & 0 \\ 0 & c_2 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 25000 \end{bmatrix}$

$[k] = \begin{bmatrix} k_1 & k_1 \\ -k_1 & k_1+k_2 \end{bmatrix} = 10^6 \cdot \begin{bmatrix} 1 & -1 \\ -1 & 11 \end{bmatrix}$, $\omega = 1200\,rpm = 40\pi$ rad/s

$\Rightarrow [Z] = 10^6 \cdot \begin{bmatrix} -3.737 & -1 \\ -1 & -52.165 + i\,3.142 \end{bmatrix}$

(3) ignore [C]

① $\mathrm{Det}([k] - \omega_n^2[M]) = 0$

$\Rightarrow \mathrm{Det}\left(\begin{bmatrix} k_1 & k_1 \\ -k_1 & k_1+k_2 \end{bmatrix} - \omega_n^2 \begin{bmatrix} m_1 & 0 \\ 0 & m_2 \end{bmatrix} \right) = \mathrm{Det}\left(10^6 \cdot \begin{bmatrix} 1 & -1 \\ -1 & 11 \end{bmatrix} - \omega_n^2 \begin{bmatrix} 300 & 0 \\ 0 & 4000 \end{bmatrix} \right) = 0$

$\Rightarrow (10^6 - 300\omega_n^2)(11\times10^6 - 4000\omega^2) - 10^{12} = 0$

$\Rightarrow \omega_{n1} = 45.644$, $\omega_{n2} = 63.246$

② or use matlab to calculate, refer the demo code in note 3.2 posted on the course website.

(4)~(7) Replace RHS by $\begin{bmatrix} m e \omega^2 \\ 0 \end{bmatrix} e^{i\omega t} = \begin{bmatrix} F_0 \\ 0 \end{bmatrix} e^{i\omega t} = \bar{F}_0 e^{i\omega t}$

Follow up response is $x(t) = \underline{X}^* e^{i\omega t}$, substitute it into math model,

$\Rightarrow [Z]\,\underline{X}^* = \bar{F}_0$

$\Rightarrow \underline{X}^* = [Z]^{-1} \bar{F}_0 = \dfrac{10^6}{\mathrm{Det}[Z]} \begin{bmatrix} -3.737 & -1 \\ -1 & -52.165 + i\,3.142 \end{bmatrix} \begin{bmatrix} m e \omega^2 \\ 0 \end{bmatrix} = \begin{bmatrix} -4.247\times10^{-4} - i\,1.314\times10^{-7} \\ 8.112\times10^{-6} + i\,4.910\times10^{-7} \end{bmatrix}$

$\underline{X}^* = \begin{bmatrix} X_1 e^{i\varphi_1} \\ X_2 e^{i\varphi_2} \end{bmatrix} \Rightarrow \begin{cases} X_1 = \sqrt{(-4.247\times10^{-4})^2 + (1.314\times10^{-7})^2}\ m = 0.424\,mm, & \varphi_1 = \tan^{-1}\dfrac{-1.314\times10^{-7}}{-4.247\times10^{-4}} = -3.14 \\ X_2 = \sqrt{(8.112\times10^{-6})^2 + (4.910\times10^{-7})^2}\ m = 0.008\,mm, & \varphi_2 = \tan^{-1}\dfrac{4.91\times10^{-7}}{8.112\times10^{-6}} = 0.06 \end{cases}$

(8) $x_2(t) = X_2 \cos(\omega t + \alpha + \varphi_2) = 0.008\times10^{-3} \times \cos(\omega t + \alpha + 0.06)$

$F_{t0} = k_2 x_2 + c_2 \dot{x}_2 = k_2 X_2 \cos(\omega t + \alpha + \varphi_2) - c_2 X_2 \omega \sin(\omega t + \alpha + \varphi_2)$

Magnitude $|F_{t0}| = \sqrt{(k_2 X_2)^2 + (c_2 X_2 \omega)^2} = X_2 \sqrt{k_2^2 + (c_2\omega)^2} = 0.008\times10^{-3} \times \sqrt{(10^7)^2 + (25000 \cdot 40\pi)^2} = 83.85$

(9) If $\omega = \frac{1200 \, rpm}{2} = 20\pi \, rad/s$, following the steps (2)~(8), we get

$F_{t0} = 12800 \, N$

```
%{
 * Course:ME370
 * Name:Liming Gao
 * Date: Nov. 07, 2018
 *
 * Program Description: Problem 2, hw11.
 %}

clear all
close all
format short g
M = [300 0; 0 4000]; % mass matrices
C = [0 0; 0 25000]; % damping matrices
k1 = 1*10^6; % k1
k2 = 1*10^7; %k2
K = [k1 -k1; -k1 k1+k2]; % stiffness matrices
w=1200*(2*pi / 60);    % rotational speed, convert RPM to rad/s

A = inv(M)*K; % system matrix
D = eig(A); %eigen values
nat_frequency = sort(sqrt(D)) %natural frequency

F0=[ w^2*0.1 ;0]; %force
Z = K - w^2*M + i*w*C % Impedance matrix
X = inv(Z)*F0;
X1 = abs(X(1)) %amplitude of vibration of the pump (m)
X2 = abs(X(2)) % amplitude of vibration of the foundation (m)
p1=angle(X(1))    %phase lag of vibration of the pump (rad)
p2=angle(X(2)) %phase lag of vibration of the foundation (rad)

Fto= X2*sqrt((25000*w)^2+(10000000)^2) % magnitude of the force transmitted to the
soil ground (N)
```