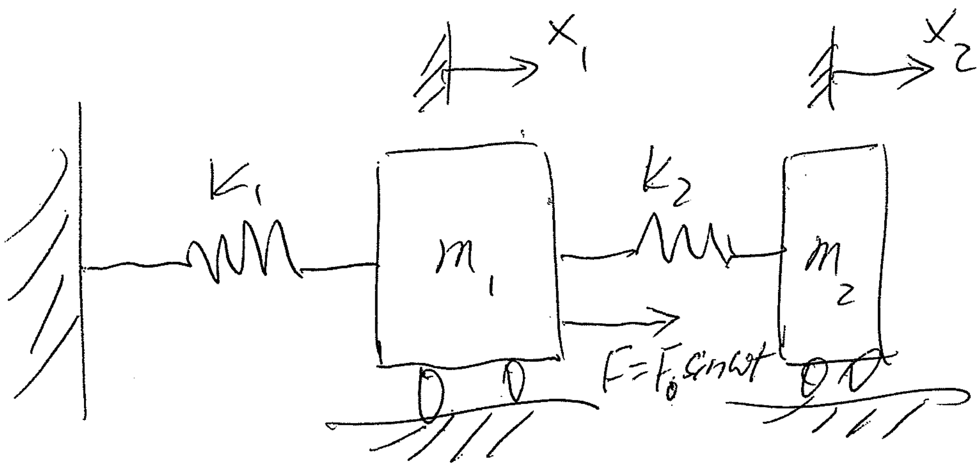
Example: Mass-spring system



Math model:

replaced by or encoded with

$$\begin{bmatrix} m_1 & 0 \\ 0 & m_2 \end{bmatrix}\begin{bmatrix} \ddot{x}_1 \\ \ddot{x}_2 \end{bmatrix} + \begin{bmatrix} (k_1+k_2) & -k_2 \\ -k_2 & k_2 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} F_0 \sin\omega t \\ 0 \end{bmatrix} \Rightarrow \begin{bmatrix} F_0 \\ 0 \end{bmatrix} e^{i\omega t}$$

Follow-up response is

$$\bar{x}(t) = \begin{bmatrix} X_1^* \\ X_2^* \end{bmatrix} e^{i\omega t} \qquad (2)$$

Eq. (2) into Eq. (1) $\Rightarrow$ $\left[[K] - \omega^2 [M]\right]$

$$\begin{bmatrix} -\omega^2 m_1 + (K_1+K_2) & -K_2 \\ -K_2 & -\omega^2 m_2 + K_2 \end{bmatrix} \begin{bmatrix} X_1^* \\ X_2^* \end{bmatrix} = \begin{bmatrix} F_0 \\ 0 \end{bmatrix} \qquad (3)$$

$\therefore$

$$\begin{bmatrix} X_1^* \\ X_2^* \end{bmatrix} = \left[[K] - \omega^2 [M]\right]^{-1} \begin{bmatrix} F_0 \\ 0 \end{bmatrix}$$

$$\Downarrow$$

$$= \frac{1}{Det\left[[K] - \omega^2 [M]\right]} \begin{bmatrix} -\omega^2 m_2 + K_2 & K_2 \\ K_2 & -\omega^2 m_1 + (K_1+K_2) \end{bmatrix} \begin{bmatrix} F_0 \\ 0 \end{bmatrix}$$

$\therefore$

$$X_1^* = \frac{-\omega^2 m_2 + K_2}{Det\left[[K] - \omega^2 [M]\right]} F_0 = \pm X_1 \qquad (A) \quad X_1 > 0$$

$$X_2^* = \frac{K_2}{Det[\quad]} F_0 = \pm X_2 \qquad (B) \quad (\text{or } -X_2)$$

Two observations:

① If $\omega = \omega_{n1}$ or ω_{n2},

$$\text{Det}\left[[K] - \omega^2 [M]\right] = 0$$

$$\Rightarrow \bar{X}_1, \bar{X}_2 \longrightarrow \infty$$

② The results of Eq. Ⓐ and Ⓑ offer some insights into vibration control: absorbing vibration of m_1 by m_2.

Choose m_2 and k_2 to make

$$-\omega^2 m_2 + k_2 = 0 \Longrightarrow \bar{X}_1 = 0$$