Encoding the RHS of Gov. eqs into $e^{i\omega t}$ form

$$[M]\ddot{\bar{x}} + [C]\dot{\bar{x}} + [K]\bar{x} = \bar{F} = RHS$$

Example 1:

RHS:

$$\begin{bmatrix} F_{10}\sin\omega t \\ 0 \end{bmatrix} \rightarrow \boxed{\begin{bmatrix} F_{10} \\ 0 \end{bmatrix} e^{i\omega t} \Rightarrow \bar{x}(t) = \bar{\bar{X}}^{*} e^{i\omega t} \Rightarrow \begin{bmatrix} X_1 e^{i\phi_1} \\ X_2 e^{i\phi_2} \end{bmatrix}}$$

$$\downarrow$$

$$\bar{x}(t) = \begin{bmatrix} X_1 \sin(\omega t + \phi_1) \\ X_2 \sin(\omega t + \phi_2) \end{bmatrix}$$

Example 2:

$$\begin{bmatrix} 5\cos(\omega t + 2) \\ 0 \end{bmatrix} \longrightarrow \begin{bmatrix} 5 \\ 0 \end{bmatrix} e^{i(\omega t + 2)} = \begin{bmatrix} 5e^{i2} \\ 0 \end{bmatrix} e^{i\omega t}$$

$$\Rightarrow \bar{x}(t) = \bar{\bar{X}}^{*} e^{i\omega t} \Rightarrow \begin{bmatrix} X_1 e^{i\phi_1} \\ X_2 e^{i\phi_2} \end{bmatrix}$$

$$\downarrow$$

$$\bar{x}(t) = \begin{bmatrix} X_1 \cos(\omega t + \phi_1) \\ X_2 \cos(\omega t + \phi_2) \end{bmatrix}$$

1st

$\downarrow$

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Alternative.

Example 3:

$$e^{i(\omega t+2)}$$

$$\begin{bmatrix} 5\cos(\omega t+2) \\ 4\sin\omega t \end{bmatrix} = \begin{bmatrix} 5\cos(\omega t+2) \\ 4\cos(\omega t-\frac{\pi}{2}) \end{bmatrix} \Longrightarrow \begin{bmatrix} 5e^{i2} \\ 4e^{-i\frac{\pi}{2}} \end{bmatrix} e^{i\omega t}$$

$$e^{i(\omega t-\frac{\pi}{2})}$$

$$\Rightarrow \begin{bmatrix} \bar{X}_1 e^{i\varphi_1} \\ \bar{X}_2 e^{i\varphi_2} \end{bmatrix} \quad \cdots\cdots \longrightarrow \quad \bar{x}(t) = \begin{bmatrix} \bar{X}_1 \cos(\omega t+\varphi_1) \\ \bar{X}_2 \cos(\omega t+\varphi_2) \end{bmatrix}$$

Alternatively for example 2:

$$\begin{bmatrix} 5\cos(\omega t+2) \\ 0 \end{bmatrix} \longrightarrow \begin{bmatrix} 5\cos(\omega t) \\ 0 \end{bmatrix} \longrightarrow \begin{bmatrix} 5 \\ 0 \end{bmatrix} e^{i\omega t} \Longrightarrow$$

$$\bar{x}(t) = \bar{X}^{*} e^{i\omega t} \Rightarrow \begin{bmatrix} \bar{X}_1 e^{i\varphi_1} \\ \bar{X}_2 e^{i\varphi_2} \end{bmatrix}$$

$$\Downarrow$$

$$\bar{x}(t) = \begin{bmatrix} \bar{X}_1 \cos(\omega t+2+\varphi_1) \\ \bar{X}_2 \cos(\omega t+2+\varphi_2) \end{bmatrix}$$

Example 4

$$\begin{bmatrix} F_1 \cos(\omega_1 t + \alpha_1) \\ \\ F_2 \sin \omega_2 t \end{bmatrix} = \begin{bmatrix} F_1 \cos(\omega_1 t + \alpha_1) \\ \\ 0 \end{bmatrix} + \begin{bmatrix} 0 \\ \\ F_2 \sin \omega_2 t \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} F_1 e^{i\alpha_1} \\ \\ 0 \end{bmatrix} e^{i\omega_1 t} + \begin{bmatrix} 0 \\ \\ F_2 \end{bmatrix} e^{i\omega_2 t}$$

$\Downarrow$ solving separately $\Downarrow$

$$\bar{X}^*_{\omega_1} = \begin{bmatrix} X_1 e^{i\psi_1} \\ \vdots \\ X_n e^{i\psi_n} \end{bmatrix}_{\omega_1} \qquad \bar{X}^*_{\omega_2} = \begin{bmatrix} X_1 e^{i\psi_1} \\ \vdots \\ X_n e^{i\psi_n} \end{bmatrix}_{\omega_2}$$

Total follow-up response ~~steady-state vibration~~ is

$$\bar{x}_{ss}(t) = \begin{bmatrix} X_1 \cos(\omega_1 t + \psi_1) \\ \vdots \\ X_n \cos(\omega_1 t + \psi_n) \end{bmatrix}_{\omega_1} + \begin{bmatrix} X_1 \sin(\omega_2 t + \psi_1) \\ \vdots \\ X_n \sin(\omega_2 t + \psi_n) \end{bmatrix}_{\omega_2}$$

Finally total response is:

$\bar{x}(t) = \bar{x}_h + \bar{x}_p$