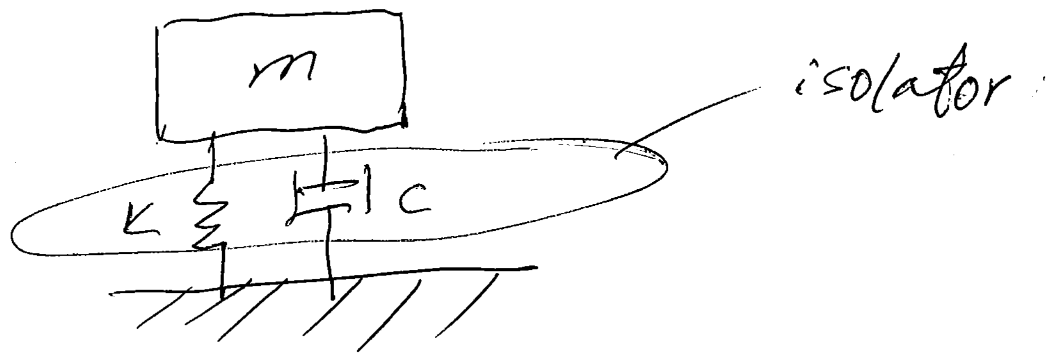
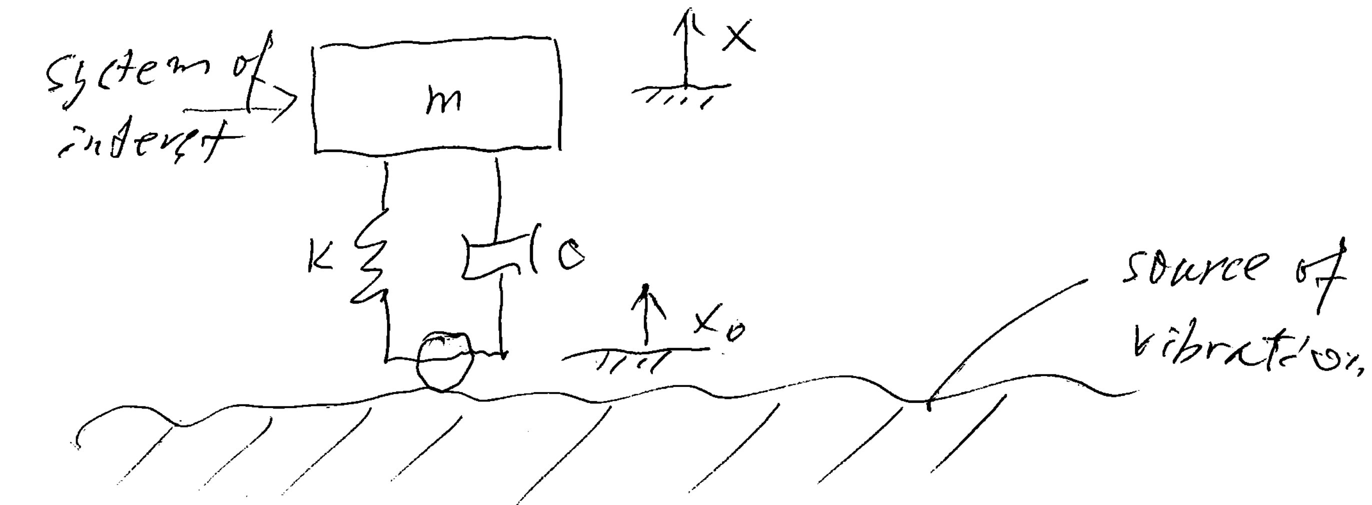
4.1 Vibration isolation

Theory of SDOF may be used for 1st-order design approximation (calculation)



Isolation from external source

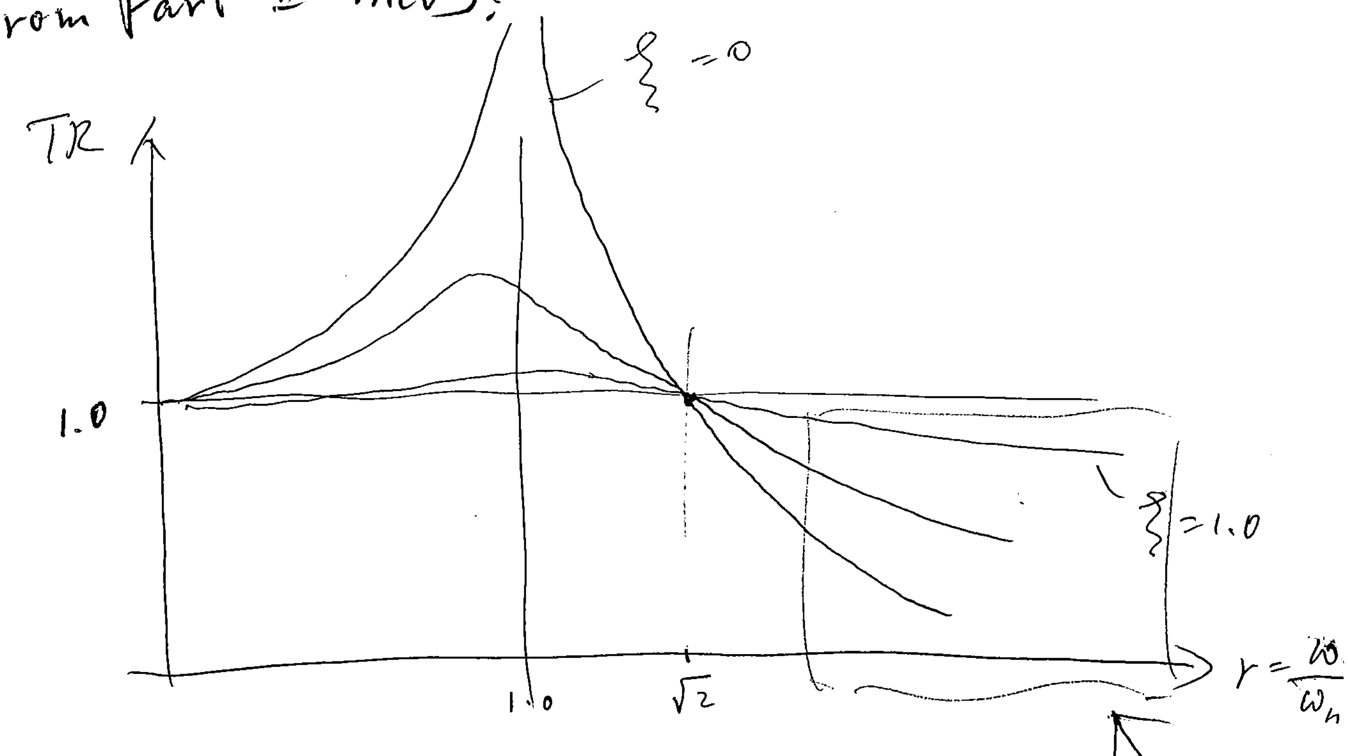
Example – vehicle suspension



Isolation is measured by motion transmissibility

$$TR = \frac{X}{X_0}$$

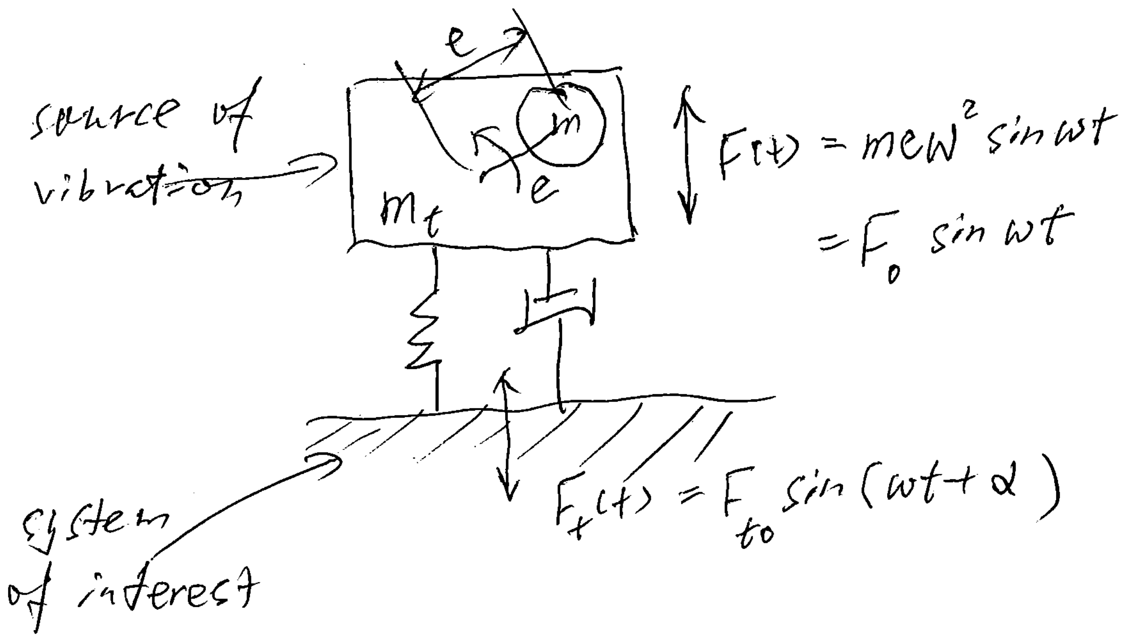
From Part II theory:



Design the system w/ light ξ ($\xi = 0.5$) and small ω_n (soft spring, small c) to achieve $TR < 1.0$

$$TR = \frac{\left[1 + (2\xi r)^2\right]^{1/2}}{\left[(1-r^2)^2 + (2\xi r)^2\right]^{1/2}}$$

Example — rotating machinery



Isolation is measured by force transmissibility.

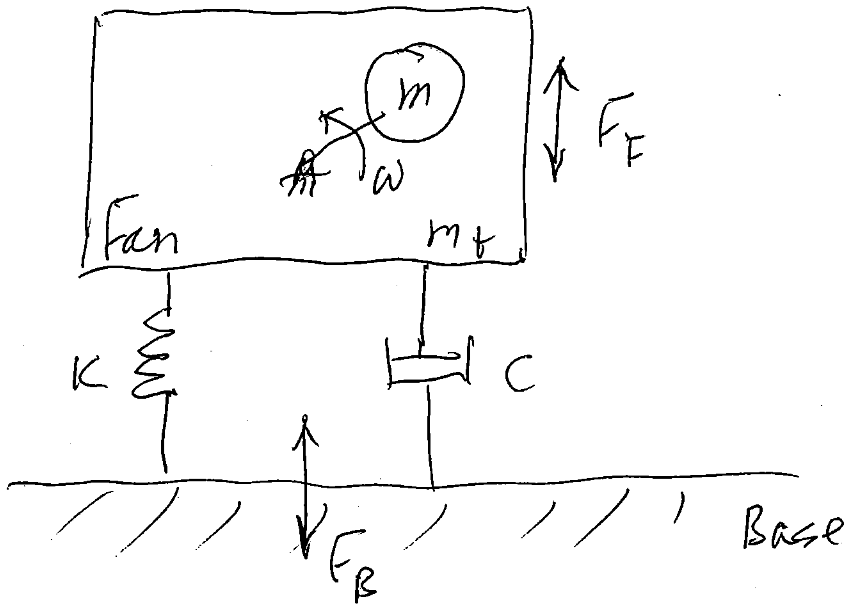
$$TR = \frac{F_{t0}}{F_0}$$

From Part II

$$TR = \frac{\left[1 + (2\zeta r)^2\right]^{1/2}}{\left[(1-r^2)^2 + (2\zeta r)^2\right]^{1/2}}, \quad r = \frac{\omega}{\omega_n}$$

$r > \sqrt{2}$ to achieve $TR < 1.0$

Example: Fan mounting design calculation



Given: $m_f = 40\,kg$, $\omega = 1000\,rpm = 104.7\,rad/s$

Design fan mounting (K, C) so that no more than 10% of unbalance force F_F is transmitted to base.

Design equation:

$$TR = \frac{F_B}{F_F} = \frac{[1+(2\xi r)^2]^{1/2}}{[(1-r^2)^2+(2\xi r)^2]^{1/2}} \le 10\% = 0.1 \quad (A)$$

No unique answer — choose (r, ξ) pair to achieve obj.

Consider $\xi = 0.15?$

Eq. (A) $\Rightarrow$

$$\frac{1}{|1-r^2|} \le 0.1 \qquad (B)$$

~~From isolation theory (the chart earlier), we know $r > \sqrt{2}$~~

$\therefore r > \sqrt{2}$

Eq. (B) $\Rightarrow$

$$\frac{1}{r^2-1} \le 0.1 \Rightarrow r^2 \ge \frac{1}{0.1} + 1 = 11$$

$$\therefore \boxed{r \ge \sqrt{11}}, \quad r = \frac{\omega}{\omega_n} = \frac{104.7}{\sqrt{k/m_t}} \ge \sqrt{11}$$

$$\Rightarrow \boxed{K \le \frac{104.7^2}{11} m_t = \frac{104.7^2}{11} \times 60 \ N/m}$$

$$\therefore \boxed{K_{max} = 39862 \ N/m}$$

With $\xi \neq 0$

From the TR chart ~~earlier~~, we see a larger r or softer k is needed than with $\xi = 0$.

Eq. (A) $\Rightarrow$

$$\frac{[1+(2\xi r)^2]}{[(1-r^2)^2+(2\xi r)^2]} = 0.01$$

or

$$(1-r^2)^2 + (2\xi r)^2 = 100[1+(2\xi r)^2]$$

$\Rightarrow$

$$1 - 2r^2 + r^4 + 4\xi^2 r^2 = 100 + 400\xi^2 r^2$$

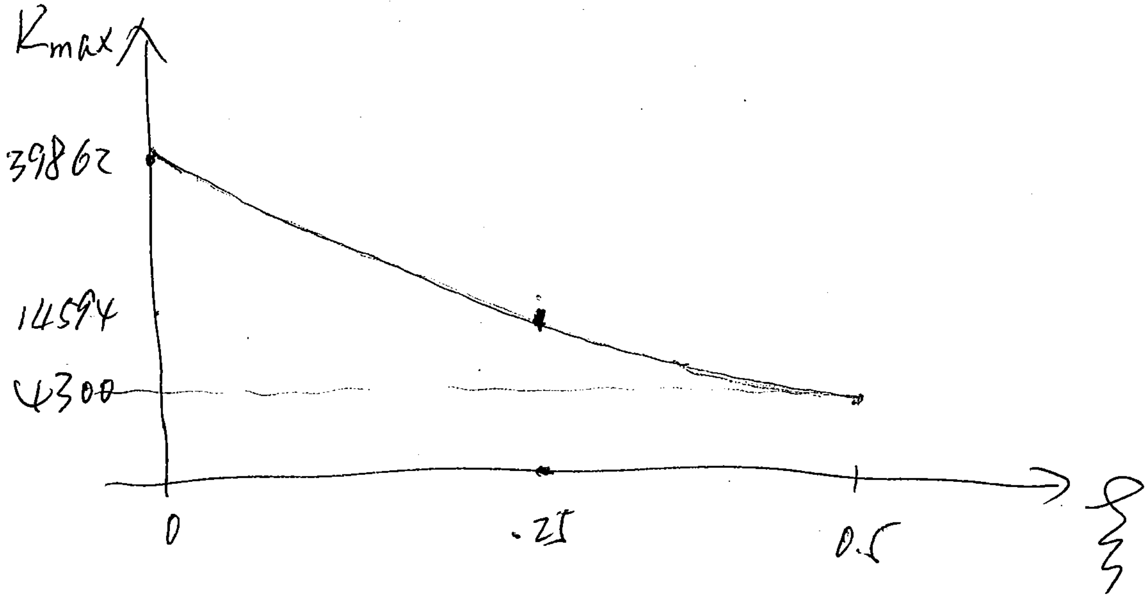
$\Rightarrow$

$$\boxed{r^4 - (396\xi^2+2)r^2 - 99 = 0}$$

$\therefore$

$$\underset{r_{min}}{r^2} = \frac{(396\xi^2+2) \pm \sqrt{(396\xi^2+2)^2 + 4\times 99}}{2} = \left(\frac{\omega}{\omega_n}\right)^2 = \frac{104.7^2}{\underset{k_{max}}{k/m_t}}$$

$$\therefore \boxed{K_{max} = \frac{2 \times 104.7^2 \overbrace{m_t}^{=40\,kg}}{(396\xi^2+2)+\sqrt{(396\xi^2+2)^2+4\times 99}}}$$



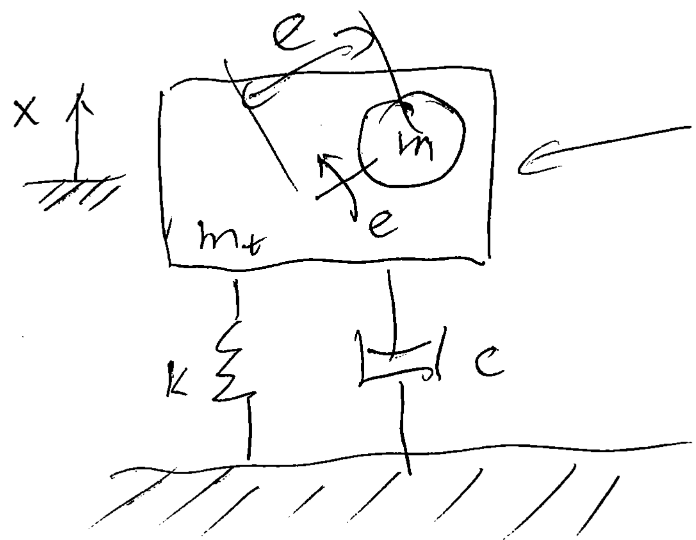
Then

$$\frac{c}{m_t} = 2\xi\omega_n$$

$$\Rightarrow \boxed{c = 2\xi\omega_n m_t = 2\xi\sqrt{K m_t}}$$

Isolation from internal source

A typical problem is rotating machinery

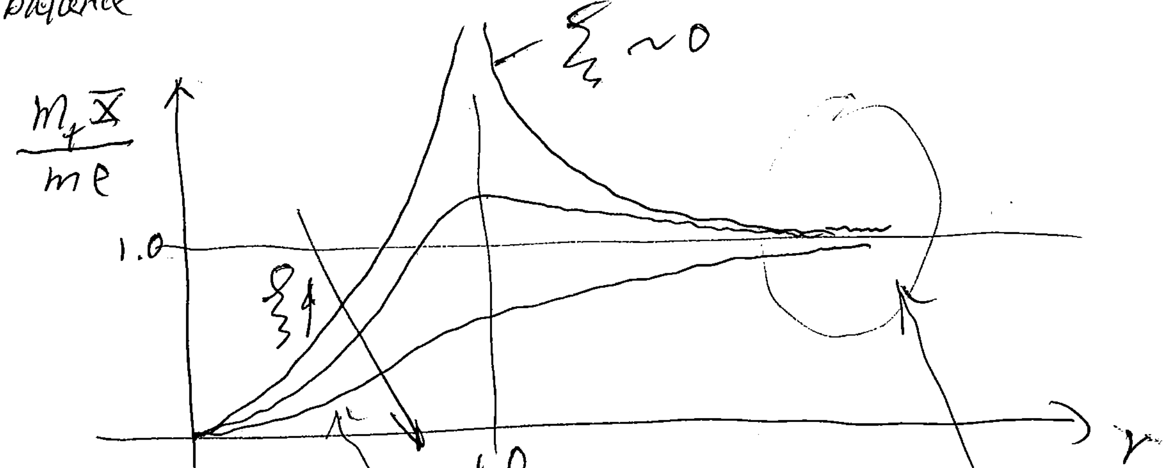


Both source of vibration and system of interest

From Part II, vibration is measured by

$$\frac{m_t X}{me} = \frac{r^2}{\left[(1-r^2)^2+(2\xi r)^2\right]^{1/2}}$$

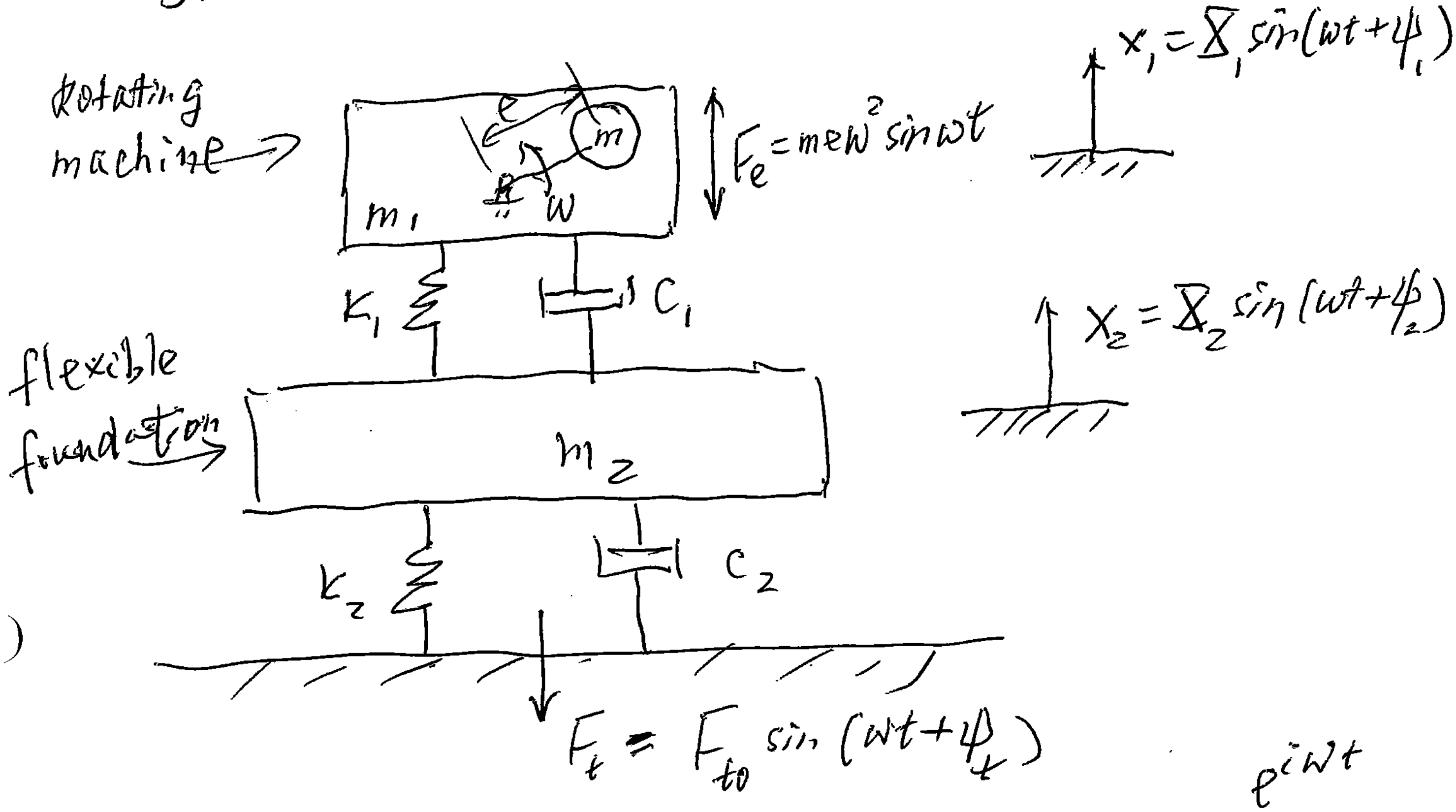
unbalance



To reduce X, use large ω_n and ξ or small ω_n and light ξ.

Isolator design calculation with MDOF model

A typical problem is



e^{iwt}

Model:

$$[M]\ddot{\bar{x}} + [C]\dot{\bar{x}} + [K]\bar{x} = \begin{bmatrix} mew^2 \\ 0 \end{bmatrix} \sin wt$$

Use a matlab program to determine k_1 and c_1 so that the vibration of the machine X_1 and the force transmitted to surrounding F_{t0} are acceptable for the range of designed machine speed.