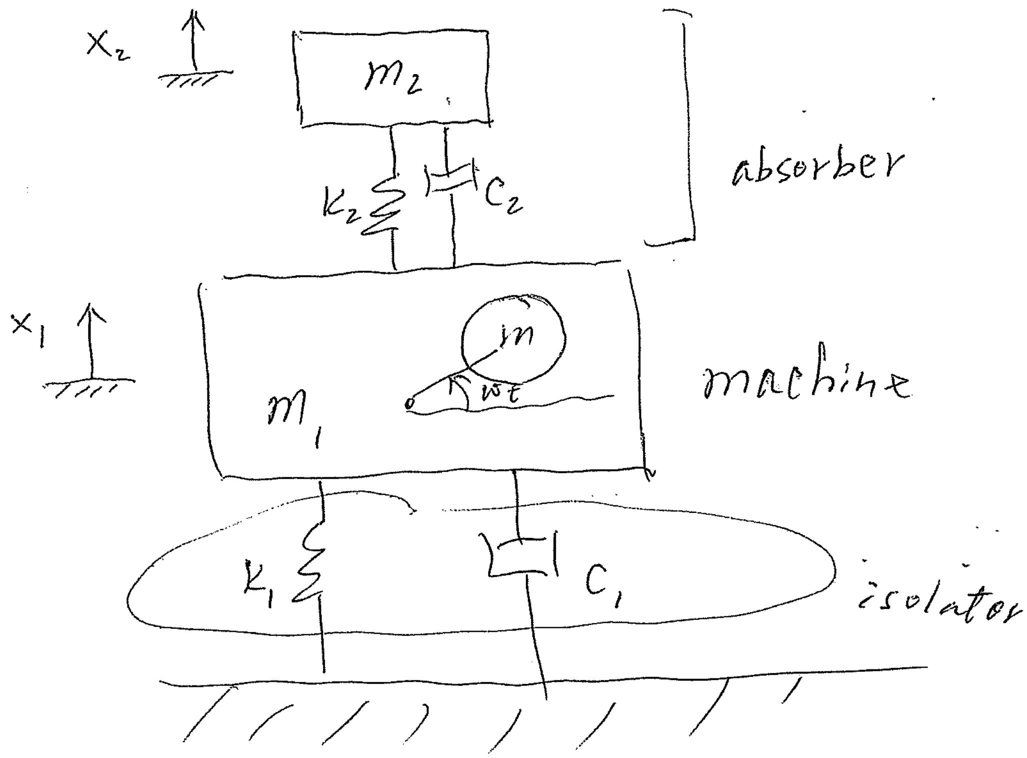
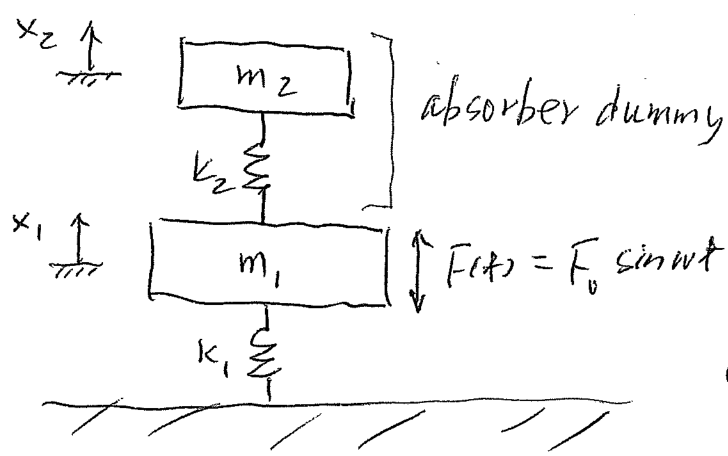
4.2 Vibration absorption

A typical model is:



Design an absorber to reduce $\bar{X}_1$.

An undamped 2DOF model may be used for 1st-order analytical design calculations:



$F_0 = me\omega^2$ for rotating machine

Math model is:

$$\begin{bmatrix} m_1 & 0 \\ 0 & m_2 \end{bmatrix}\begin{bmatrix} \ddot{x}_1 \\ \ddot{x}_2 \end{bmatrix} + \begin{bmatrix} (k_1+k_2) & -k_2 \\ -k_2 & k_2 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} F_0 \\ 0 \end{bmatrix} e^{i\omega t}$$

Amplitudes of vibration are:

$$\bar{X}_1 = \left| \frac{k_2 - \omega^2 m_2}{\mathrm{Det}[[K] - \omega^2 [M]]} F_0 \right| \quad (1) \qquad \text{From Part III}$$

$$\mathrm{Det}\left[[K] - \omega^2 [M]\right] = (k_1 + k_2 - \omega^2 m_1)(k_2 - \omega^2 m_2) - k_2^2 \quad (2)$$

and

$$\bar{X}_2 = \left| \frac{k_2}{\mathrm{Det}[[K] - \omega^2 [M]]} F_0 \right| \quad (3)$$

To reduce $\bar{X}_1$,

Eq. (1) $\Rightarrow$ $\quad k_2 - \omega^2 m_2 = 0$

$$\Rightarrow \quad \boxed{\frac{k_2}{m_2} = \omega^2} \quad (4)$$

absorber selection

Demo

Other design considerations:

① Select k_2 and m_2 so that X_2 is not too large — $X_2 \leq X_{2max}$

Eqs ② and ③

$$\Rightarrow \quad X_2 = \left| \frac{1}{(k_1+k_2-\omega^2 m_1)\underbrace{\left(\frac{k_2}{k_2}-\omega^2\frac{m_2}{k_2}\right)}_{=0 \text{ by Eq. ④}} - \frac{k_2^2}{k_2}} F_0 \right| = \frac{F_0}{k_2} \leq X_{2max}$$

So, select k_2 to satisfy:

$$\boxed{k_2 \geq \frac{F_0}{X_{2max}}} \qquad ⑤$$

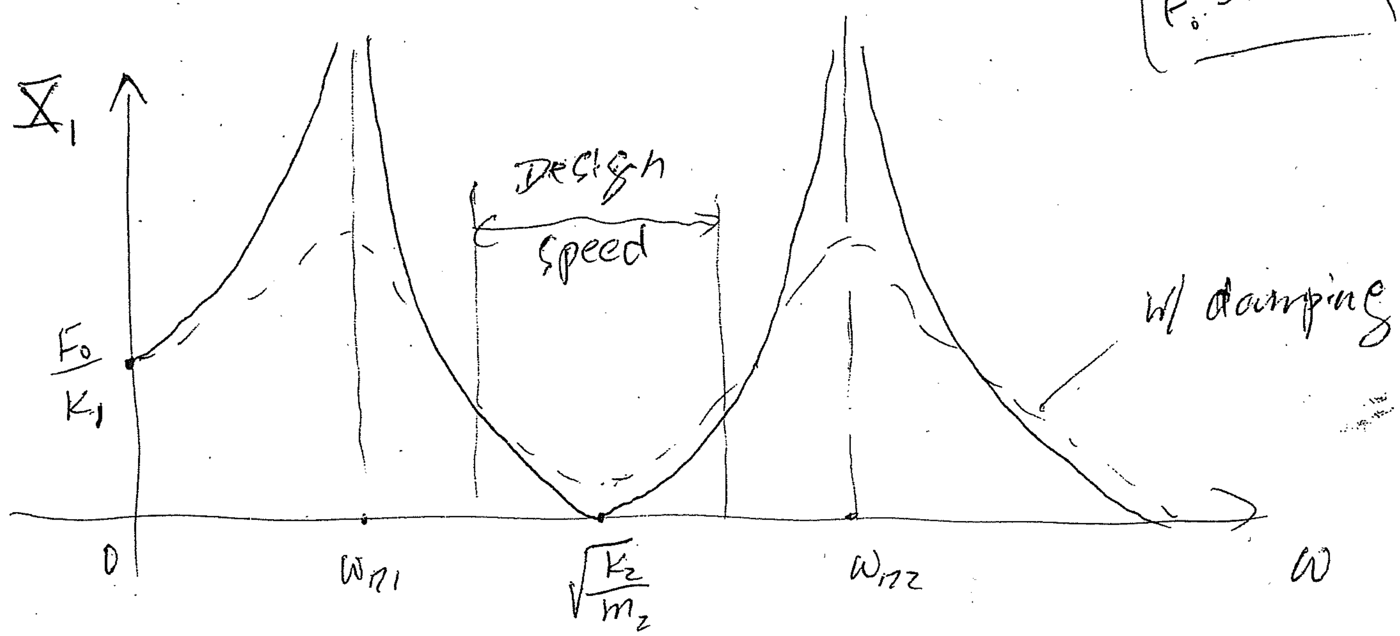
Then Eq. ⑤ ⇒

$$\boxed{m_2 = \frac{k_2}{\omega^2}} \qquad ⑥$$

② Make $\omega_{n1}, \omega_{n2}, \ldots$ sufficiently away from the range of machine speed

For the example: Eq. ① ⇒

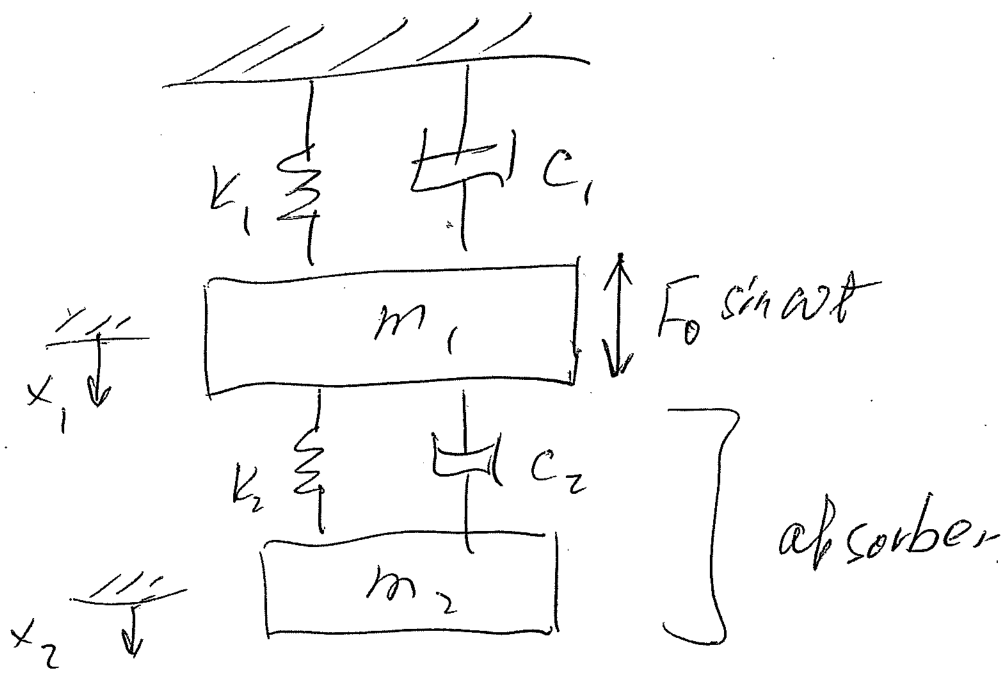
$\boxed{F_0 = me\omega^2}$



machine speed at which vibration is suppresed

ω_n's by $\boxed{\det\left[-\omega_n^2[M] + [K]\right] = 0}$ ⑦

Absorber design considering w/ damping



Model:

$$[M]\ddot{\bar{x}} + [c]\dot{\bar{x}} + [k]\bar{x} = \begin{bmatrix} F_0 \\ 0 \end{bmatrix} e^{i\omega t}$$

Matlab solution —