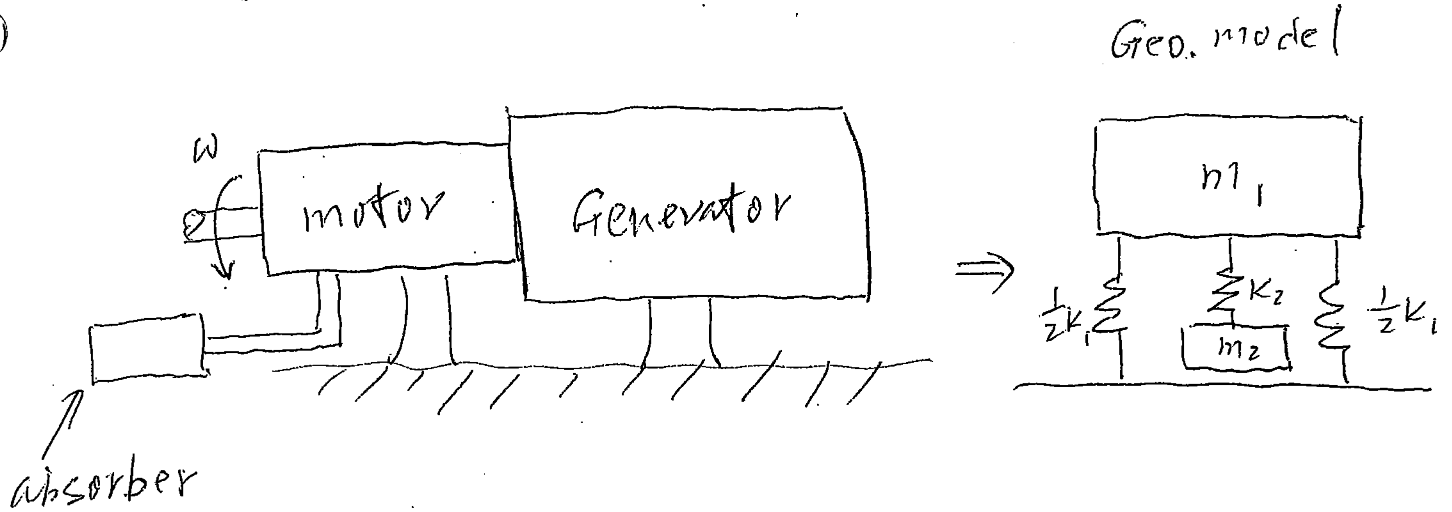
Example: Absorber for a motor-generator set



Given:

① speed range of machine operation 2000 ~ 4000 rpm

② System vibrates heavily at 3000 rpm w/o the absorber.

An absorber is to be designed to suppress vibration at this speed, $\omega = 3000\ rpm = 314.16\ rad/s$

③ when a trial absorber of $m_t = 2\ kg$ and $\sqrt{\frac{k_t}{m_t}} = 314.16\ rad/s$ is used, the critical speeds (nat. freqs) of the resulting system are $\omega_{n1} = 2500\ rpm$ and $\omega_{n2} = 3500\ rpm$.

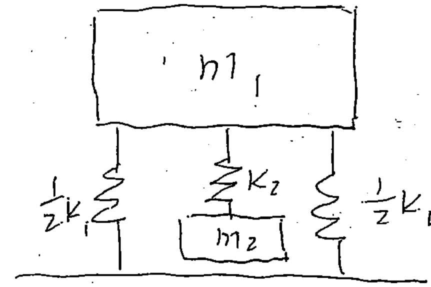
Objective:

Design an absorber (m_2, k_2) to

① suppress vibration at $\omega = 3000\ rpm$

② critical speeds lie outside of $\omega = 2000 \sim 4000\ rpm$

Solution: Geo. model



① Determine k_1 and m_1 of the system model

Recall the char. eq:

$$\mathrm{Det}\left[[k]-\omega_n^2[m]\right] = 0$$

$$\Rightarrow (k_1+k_2-\omega_n^2 m_1)(k_2-\omega_n^2 m_2)-k_2^2 = 0 \quad (1)$$

Or $$k_1-\omega_n^2 m_1 = \frac{k_2^2}{k_2-\omega_n^2 m_2} - k_2 \quad (2)$$

Using the data of the trial absorber:

$m_2 = m_t = 2\ kg$

$k_2 = k_t = 314.16^2 m_t = 197393\ N/m$

$\omega_{n1} = 2500\ rpm = 261.8\ rad/s$

$\omega_{n2} = 3500\ rpm = 366.5\ rad/s$

Eq. (2) $\Rightarrow$

$$K_1 - 261.8^2 m_1 = 448621 \quad (3) \quad w/\ \omega_n = \omega_{n1}$$

$$K_1 - 366.5^2 m_1 = -744244 \quad (4) \quad w/\ \omega_n = \omega_{n2}$$

$$\Rightarrow \boxed{m_1 = 18.13\ kg \quad \text{and} \quad K_1 = 1691018\ N/m}$$

(2) Determine K_2 and m_2 to satisfy design objectives.

Select $\omega_{n1} = 1800\ rpm = 188.5\ rad/s$

Eq. (1) $\Rightarrow$

$$\boxed{K_1 - \omega_n^2 m_1 - \omega_n^2 K_1 \frac{m_2}{K_2} - \omega_n^2 m_2 + \omega_n^4 m_1 \frac{m_2}{K_2} = 0} \quad (5)$$

Suppressing vibration at $\omega = 3000\ rpm$ (314.16 rad/s) requires =

$$\frac{K_2}{m_2} = \omega^2$$

Or

$$\boxed{\frac{m_2}{K_2} = \frac{1}{314.16^2}} \quad (6)$$

(So, the only unknown in Eq. (5) is m_2)

Eq. 5 $\Rightarrow$

$$\boxed{m_2 = \frac{1}{\omega_{n1}}\left[k_1 - \omega_{n1}^2 m_1 - \omega_{n1}^2 k_1 \frac{m_2}{k_2} + \omega_{n1}^4 m_1 \frac{m_2}{k_2}\right] = 18.85 \text{ kg}}$$

Then Eq. 6 $\Rightarrow$

$$\boxed{k_2 = \omega^2 m_2 = 1860887 \text{ N/m}}$$

3 Check where ω_{n2} is:

$\mathrm{Det}\left[[K] - \omega_n^2 [M]\right] = 0$ or Eq. 5 $\Rightarrow$

$$\left(m_1 \frac{m_2}{k_2}\right)\omega_n^4 - \left(m_1 + k_1 \frac{m_2}{k_2} + m_2\right)\omega_n^2 + k_1 = 0 \qquad (7)$$

Or

$$0.0001844\, \omega_n^4 - 54.11\, \omega_n^2 + 1691018 = 0$$

$\Rightarrow$

$$\boxed{\begin{array}{cc} \omega_{n1} = 188.5 \text{ rad/s} & \omega_{n2} = 508.5 \text{ rad/s} \\ \Downarrow & \Downarrow \\ 1800 \text{ rpm} & 4855 \text{ rpm} \end{array}}$$

(4) Check how big $\bar{X}_2$ is

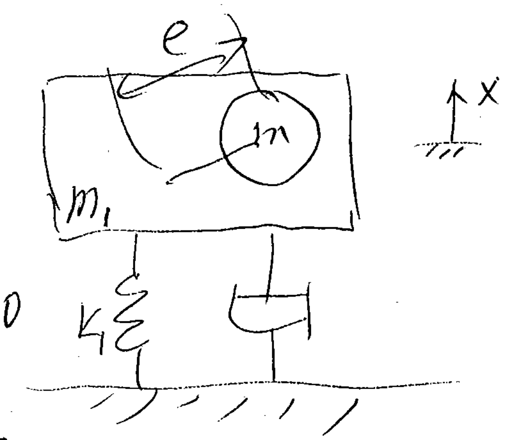
Recall

$$\bar{X}_2 = \left| \frac{k_2 F_0}{(k_1+k_2-\omega^2 m_1)(k_2-\omega^2 m_2)-k_2^2} \right| = \frac{\overset{me\omega^2}{F_0}}{k_2} \le \bar{X}_{2max}$$

$$\therefore \quad \bar{X}_2 = \frac{me\omega^2}{k_2} \qquad (8)$$

Need to determine machine unbalance : me

Recall (w/o the absorber)



$$\frac{m_1 \bar{X}}{me} = \frac{r^2}{[(1-r^2)^2+(2\zeta r)^2]^{1/2}} \simeq 1.0$$

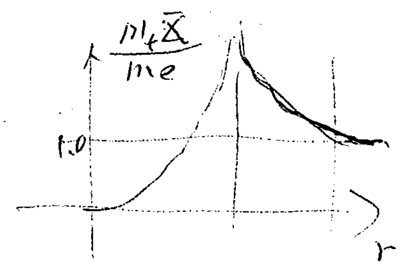
w/ $r \gg 1.0$

$\bar{X}$ may be measured ~~by a vibrometer~~

then

$$me = m_1 \bar{X}$$

measured at a high speed of $\omega \gg \omega_n$ using an accelerometer



Eq. (8) $\Rightarrow$

$$\boxed{\bar{X}_2 = \frac{m_1 \bar{X} \omega^2}{k_2}}$$