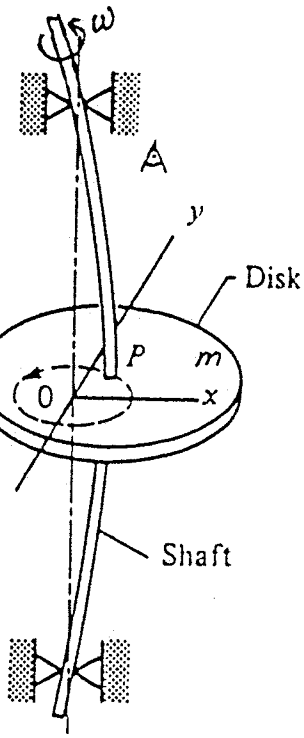
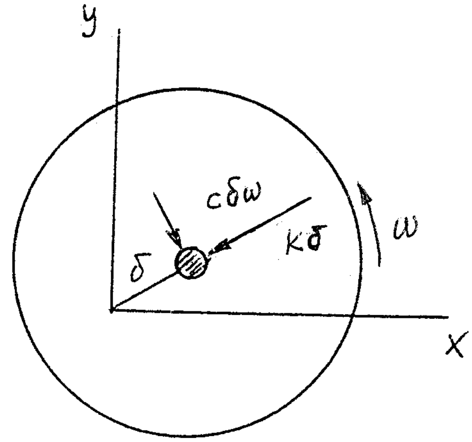
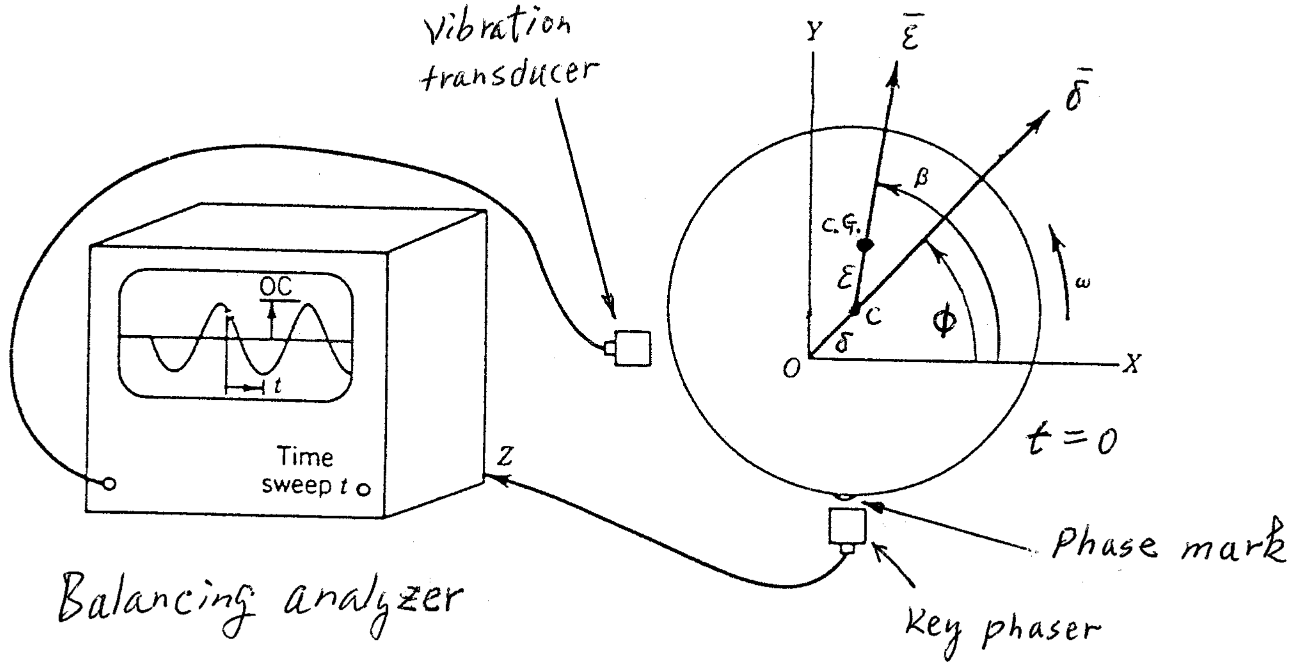


ω
A
y
Disk
P
m
0
x
Shaft



y
cδω
ω
δ
kδ
X



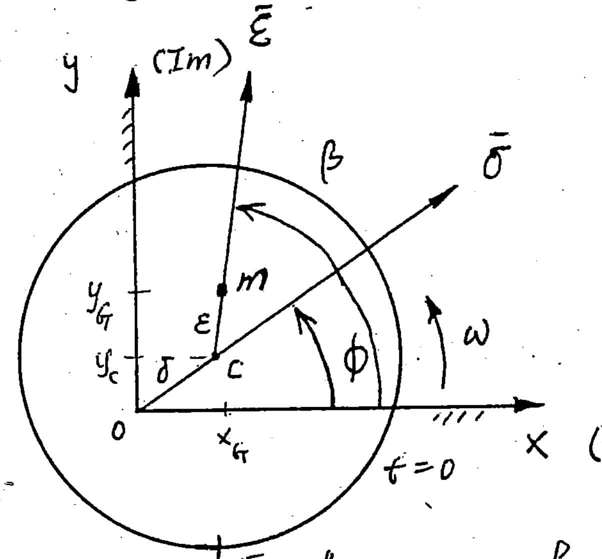
Vibration transducer
Y
$\bar{\varepsilon}$
$\bar{\delta}$
β
c.g.
ε
C
Φ
ω
O
δ
X
OC
t
Time sweep t
Z
t = 0
Phase mark
Key phaser
Balancing analyzer

4.3 Balancing of machinery

— Reducing the intensity of source of vibration "me" or "mε"

We focus on the basic single-rotor balancing

Refer to handout:

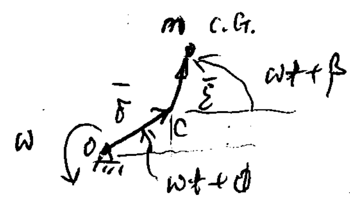


x_c, y_c = location of rotor center

x_G, y_G = rotor center of gravity

$\bar{\delta} = \delta e^{i\phi}$ = rotor center position at ~~the ref. configuration~~ t=0 (measured)

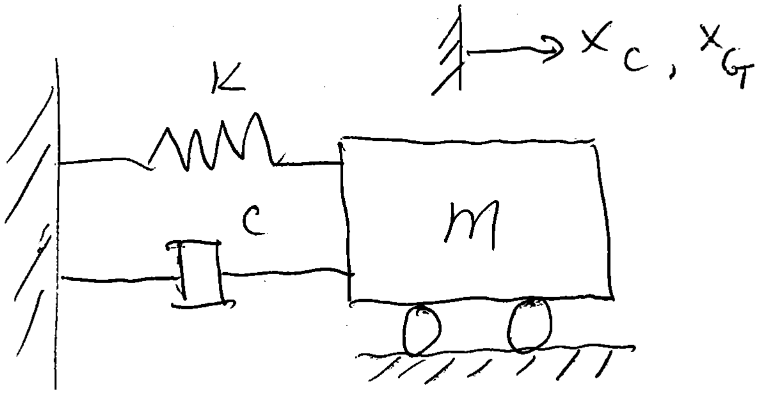
$\bar{\varepsilon} = \varepsilon e^{i\beta}$ = rotor mass-center-offset at t=0 (unknown)



Objective:

Determine $\bar{\varepsilon}$ and nullify it by adding or removing weight(s). marker (t=0)

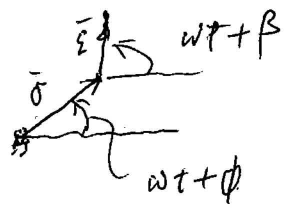
X-direction equation of motion:



Geo. model

$$m\ddot{x}_G = F_{kx} + F_{cx} = -kx_c - c\dot{x}_c \qquad ①$$

Refer to figure:



$t = 0$

$$x_c = \delta\cos(\omega t + \phi)$$

$$x_G = x_c + \varepsilon\cos(\omega t + \beta)$$

Eq. ① $\Rightarrow$

$$m\left(-\delta\omega^2\cos(\omega t+\phi) - \varepsilon\omega^2\cos(\omega t+\beta)\right)$$

$$= -k\delta\cos(\omega t+\phi) + c\omega\sin(\omega t+\phi)$$

Or

$$-m\delta\omega^2\cos(\omega t+\phi) - c\delta\omega\sin(\omega t+\phi) + k\delta\cos(\omega t+\phi)$$

$$= m\varepsilon\omega^2\cos(\omega t+\beta) \quad (2)$$

Similarly, equation of motion in y-direction yields:

$$-m\delta\omega^2\sin(\omega t+\phi) + c\delta\omega\cos(\omega t+\phi) + k\delta\sin(\omega t+\phi)$$

$$= m\varepsilon\omega^2\sin(\omega t+\beta) \quad (3)$$

Eq. (2) + i Eq. (3) $\Rightarrow$

$$-m\delta\omega^2 e^{i(\omega t+\phi)} + c\delta\omega\overbrace{\left[-\sin(\omega t+\phi) + i\cos(\omega t+\phi)\right]}^{ie^{i(\omega t+\phi)}}$$

$$+ k\delta e^{i(\omega t+\phi)} = m\varepsilon\omega^2 e^{i(\omega t+\beta)}$$

$$\Rightarrow \quad -(m\underbrace{\delta e^{i\phi}}_{\bar{\delta}}\omega^2)e^{i\omega t} + i(c\delta e^{i\phi}\omega)e^{i\omega t}$$

$$+ (k\delta e^{i\phi})e^{i\omega t} = (m\underbrace{\varepsilon e^{i\beta}}_{\bar{\varepsilon}}\omega^2)e^{i\omega t}$$

So,

$$-m\omega^2\bar{\delta} + ic\omega\bar{\delta} + k\bar{\delta} = m\bar{\varepsilon}\omega^2$$

$$\Rightarrow \boxed{\bar{\delta} = \underbrace{\frac{\omega^2}{(k-m\omega^2)+ic\omega}}_{\text{call it } \bar{C}}(m\bar{\varepsilon})} \quad (*)$$

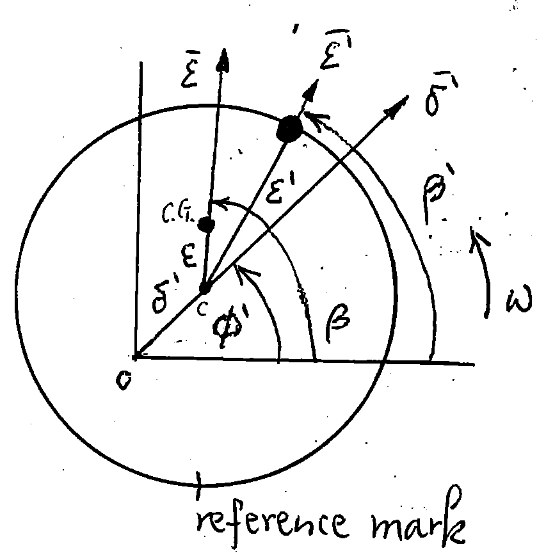
Then

$$\boxed{\bar{\delta} = \bar{C}(m\bar{\varepsilon})} \quad (*)$$

$\bar{\delta}$ is measured. $\bar{C}$ and $(m\bar{\varepsilon})$ are unknown complex numbers. So, need one more complex equation to solve.

Adding a small trial mass, Δm, at a convenient location, $\bar{\varepsilon}'$, on the rotor.

Then run the system and take measurement $\bar{\delta}'$.



x-direction eq. of motion:

$$m\ddot{x}_G' + \Delta m \ddot{x}_{\Delta m} = F_{kx}' + F_{cx}' = -kx_c' - c\dot{x}_c' \quad (5)$$

Ref. fig:

$$x_c' = \delta' \cos(\omega t + \phi')$$

$$x_G' = x_c' + \varepsilon \cos(\omega t + \beta)$$

$$x_{\Delta m} = x_c' + \varepsilon' \cos(\omega t + \beta')$$

Eq. (5) $\Rightarrow$

$$m\left(-\delta'\omega^2 \cos(\omega t + \phi') - \varepsilon\omega^2 \cos(\omega t + \beta)\right)$$

$$+ \Delta m\left(-\delta'\omega^2 \cos(\omega t + \phi') - \varepsilon'\omega^2 \cos(\omega t + \beta')\right)$$

$$= -k\delta' \cos(\omega t + \phi') + c\delta'\omega \sin(\omega t + \phi')$$

Or

$$-(m+\Delta m)\delta'\omega^2\cos(\omega t+\phi') - c\delta'\omega\sin(\omega t+\phi')$$

$$+k\delta'\cos(\omega t+\phi) = m\varepsilon\omega^2\cos(\omega t+\beta) + \Delta m\,\varepsilon'\omega^2\cos(\omega t+\beta') \quad (6)$$

Similar work yields an eq. in y-direction:

$$-(m+\Delta m)\delta'\omega^2\sin(\omega t+\phi') + c\delta'\omega\cos(\omega t+\phi')$$

$$+k\delta\sin(\omega t+\phi') = m\varepsilon\omega^2\sin(\omega t+\beta) + \Delta m\,\varepsilon'\omega^2\sin(\omega t+\beta') \quad (7)$$

Then Eq. (6) $+\, i$ Eq. (7) $\Rightarrow$ (similar to earlier part)

$$\left[(k-(m+\Delta m)\omega^2) + ic\omega\right]\bar{\delta}' = \omega^2(m\bar{\varepsilon}) + \omega^2(\Delta m\bar{\varepsilon}')$$

Or $$\bar{\delta}' = \frac{\omega^2}{[k-(m+\Delta m)\omega^2]+ic\omega}\left[(m\bar{\varepsilon})+(\Delta m\bar{\varepsilon}')\right]$$

$$\approx \left(\frac{\omega^2}{(k-m\omega^2)+ic\omega}\right)\left[(m\bar{\varepsilon})+(\Delta m\bar{\varepsilon}')\right], \quad \Delta m \ll m$$

So, we get the second equation:

$$\boxed{\bar{\delta}' = \bar{C}\left[(m\bar{\varepsilon}) + (\Delta m\bar{\varepsilon}')\right]} \quad (8)$$

with the same two unknowns: $\bar{C}$ and $(m\bar{\varepsilon})$

Eqs. (4) and (8) can be used to solve for the two unknowns ($m\bar{\varepsilon}$ and $\bar{C}$), with $\bar{\delta}$ and $\bar{\delta}'$ measured in two test runs w/ or w/o the trial mass, Δm.

Subst. Eq. (4) into Eq. (8) $\Rightarrow$

$$\bar{\delta}' = \frac{\bar{\delta}}{(m\bar{\varepsilon})}\left[(m\bar{\varepsilon}) + (\Delta m\bar{\varepsilon}')\right]$$

Then

$$(m\bar{\varepsilon}) = \frac{\bar{\delta}}{\bar{\delta}' - \bar{\delta}}(\Delta m\bar{\varepsilon}')$$

Or $$\boxed{(m\varepsilon)e^{i\beta} = \frac{\delta e^{i\phi}}{\delta' e^{i\phi'} - \delta e^{i\phi}} (\Delta m\, \varepsilon')e^{i\beta'}} \quad (9)$$

Balancing precedure:

(1) First run — measure δ and ϕ

(2) 2nd run w/ a trial mass Δm added at $\varepsilon' e^{i\beta'}$

— measure δ' and ϕ'

(3) plug in Eq. (9) to solve for $(m\varepsilon)$ and β.

(4) Balancing by

- removing mass Δm_b at location $\varepsilon_b e^{i\beta}$ w/ $\Delta m_b \varepsilon_b = (m\varepsilon)$
- or adding mass Δm_b at location $\varepsilon_b e^{i(\beta+\pi)}$ w/

Example: Runs and calculations for single-rotor balancing

First run: $\delta = 2\ mm$ and $\phi = 45°$

Trial mass: $\Delta m = 0.01\ kg$ @ $\varepsilon' = 200\ mm$ and $\beta' = 60°$

2nd run: $\delta' = 2.5\ mm$ and $\phi' = 30°$

Eq. (9) ⇒

$$(m\varepsilon)e^{i\beta} = \frac{2.0e^{i45°}}{2.5e^{i30°} - 2.0e^{i45°}}(2.0)e^{i60°}$$

$$= \frac{2(\cos 45° + i\sin 45°)}{2.5(\cos 30° + i\sin 30°) - 2(\cos 45° + i\sin 45°)} \times 2(\cos 60° + i\sin 60°)$$

$$= \frac{-1.035 + i3.864}{0.751 - i0.164} = \frac{(1.035^2 + 3.864^2)^{1/2} \operatorname{Exp}\left[i \tan_2^{-1} \frac{3.864}{-1.035}\right]}{(0.751^2 + 0.164^2)^{1/2} \operatorname{Exp}\left(i \tan_2^{-1} \frac{-0.164}{0.751}\right)}$$

$$= \frac{4.0\ e^{i104.5}}{0.769\ e^{-i12.3°}} = 5.2\ e^{i116.8°}$$

$$\therefore \boxed{m\varepsilon = 5.2\ kg\text{-}mm \quad @ \angle 116.8°}$$

Balancing by removing $\Delta m = 0.026\ kg$ at $\bar{\varepsilon}_b = 200\angle 116.8°\ mm$