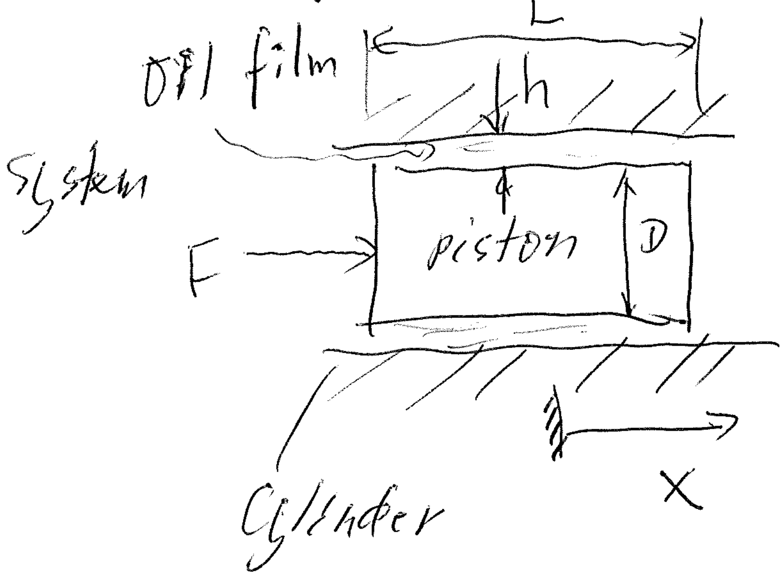
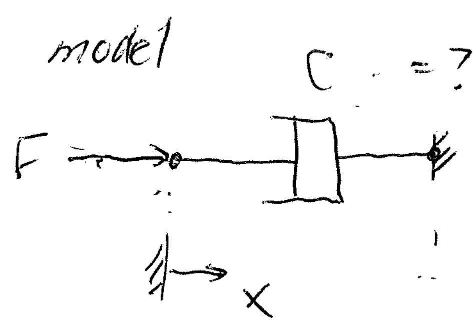
Determine C:

Example: Engine Piston-Cylinder contact





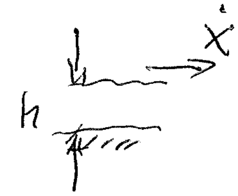
model $\longrightarrow$ $F = C\dot{x}$ (A)

system $\longrightarrow$ $F = (\quad)\dot{x}$ (B)

work out Eq. (B)

$$F = \Sigma \tau \Delta A = \int_A \tau\, dA = \tau A = (\eta \dot{\gamma})(\pi D L)$$

$$= \eta \frac{\dot{x}}{h} \pi D L = \left(\frac{\eta \pi D L}{h}\right)\dot{x} \quad (B)$$

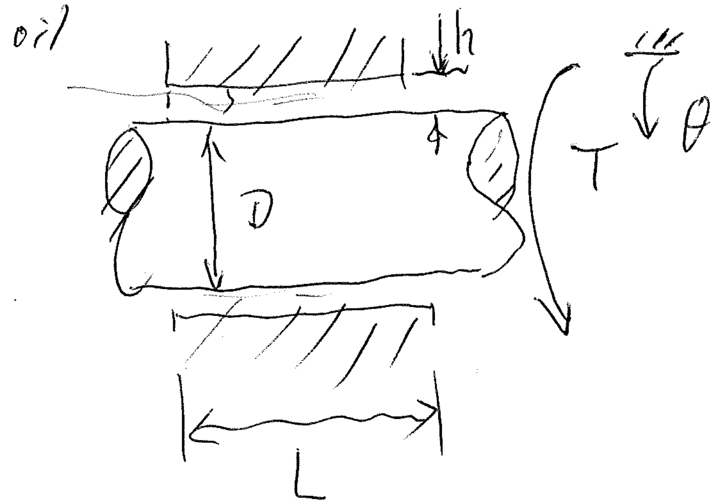


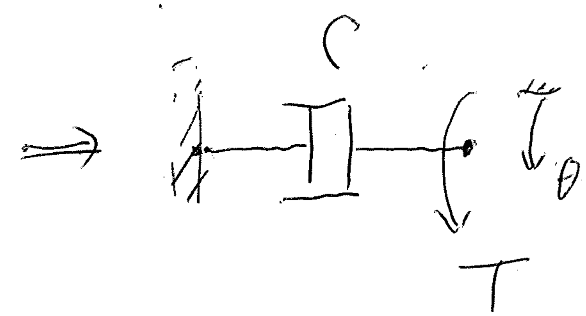
$$\therefore \boxed{C = \frac{\pi D L \eta}{h}}$$

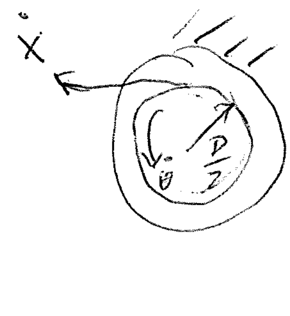
$\dot{\gamma} = \frac{\Delta \dot{x}}{\Delta y} = \frac{\dot{x}}{h}$

h is very small gap

Example: Engine Crank bearing







model → $T = C \cdot \dot{\theta}$ (A)

system → $T = \int_A (\tau dA)\frac{D}{2} = \tau \frac{D}{2} \int dA$

$$= \eta \dot{\gamma} \frac{D}{2} A = \eta \frac{D\dot{\theta}/2}{h} \frac{D}{2} \pi D L$$

$\dot{x} = \frac{D}{2}\dot{\theta}$

$$= \frac{\pi D^3 L \eta}{4h} \dot{\theta}$$ (B)

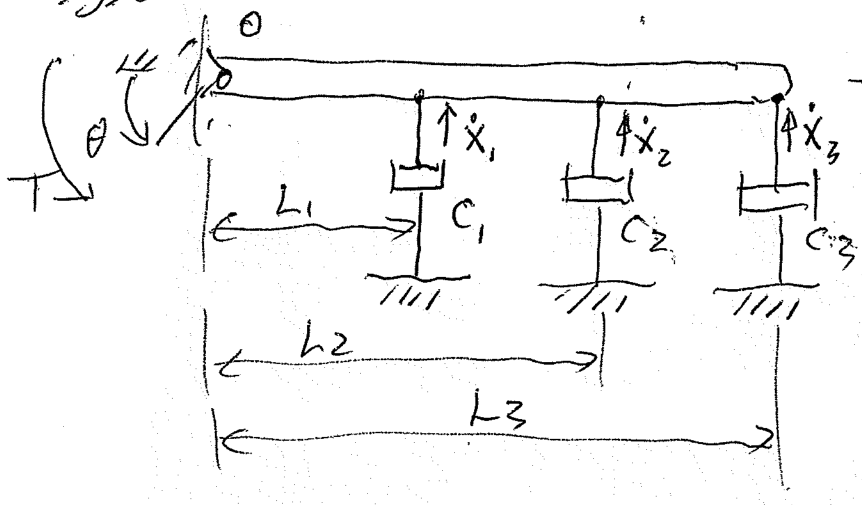
$\therefore$ $\boxed{C = \frac{\pi D^3 L \eta}{4h}}$

check units

c) Other combinations

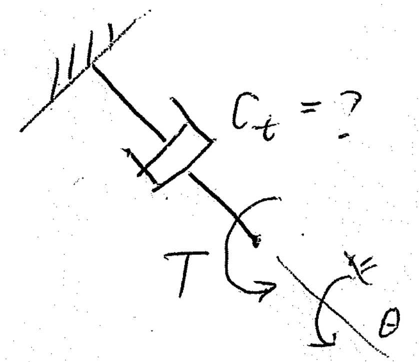
Example:

system



① Determine an equiv. torsional damper at "O"

model



model: $T = C_t \dot{\theta}$ (A)

system: $T = (\quad\quad) \dot{\theta}$ (B)

$\dot{x} = L \dot{\theta}$

Work out Eq. (B)

$$\Rightarrow \quad T = L_1 F_{c_1} + L_2 F_{c2} + L_3 F_{c3}$$

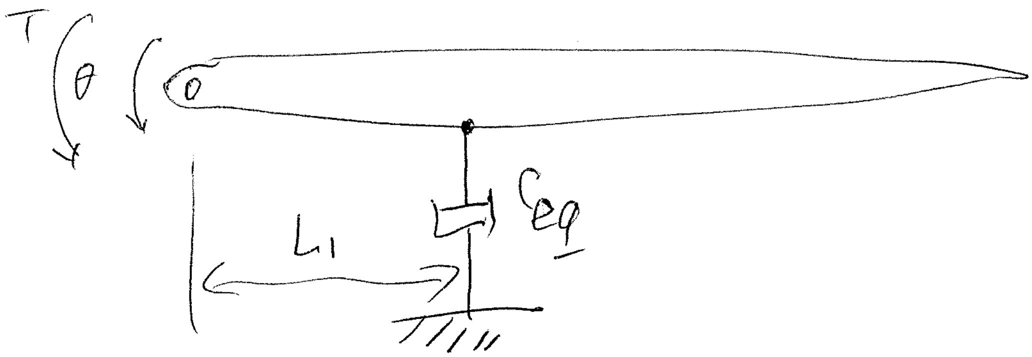
$$= L_1 c_1 \dot{x}_1 + L_2 C_2 \dot{x}_2 + L_3 C_2 \dot{x}_3$$

$$= L_1 C_1 (L_1 \dot{\theta}) + L_2 C_2 (L_2 \dot{\theta}) + L_3 C_3 (L_3 \dot{\theta})$$

$$= \underbrace{\left(c_1 L_1^2 + c_2 L_2^2 + C_3 L_3^2 \right)}_{C_t} \dot{\theta}$$

(2) Determine an equivalent translation damper at location c_1

Model



Model:

$$F_c = C_{eq}\,\dot{x}_1 \Rightarrow F_c L_1 = (C_{eq} L_1 \dot{\theta}) L_1$$

$$\Rightarrow T = (C_{eq} L_1^2)\,\dot{\theta} \qquad (A)$$

System:

$$T = (\qquad)\,\dot{\theta} \qquad (B)$$

$$T = L_1 F_{c_1} + L_2 F_{c_2} + L_3 F_{c_3}$$

$$\vdots$$

$$= (c_1 L_1^2 + c_2 L_2^2 + c_3 L_3^2)\,\dot{\theta} \qquad (B)$$

Eq. (A) = Eq. (B) $\Rightarrow$ $\boxed{C_{eq} = c_1 + c_2\left(\frac{L_2}{L_1}\right)^2 + c_3\left(\frac{L_3}{L_1}\right)^2}$

Energy method

$\boxed{\dot{E}_d \text{ in system} = \dot{E}_d \text{ in model}}$

(1) model: $\dot{E}_d = C_t \dot{\theta}^2$

system: $\dot{E}_d = c_1 \dot{x}_1^2 + c_2 \dot{x}_2^2 + c_3 \dot{x}_3^2$

$$= c_1 (L_1 \dot{\theta})^2 + c_2 (L_2 \dot{\theta})^2 + c_3 (L_3 \dot{\theta})^2$$

$$= \underbrace{\left(c_1 L_1^2 + c_2 L_2^2 + c_3 L_3^2 \right)}_{C_t} \dot{\theta}^2$$

(2) Model: $\dot{E}_d = c_{eq} \dot{x}_1^2$

system: $\dot{E}_d = c_1 \dot{x}_1^2 + c_2 \dot{x}_2^2 + c_3 \dot{x}_3^2$

$$= \left(c_1 L_1^2 + c_2 L_2^2 + c_3 L_3^2 \right) \dot{\theta}^2 \quad \left(\frac{\dot{x}_1}{L_1}\right)^2$$

$$= \underbrace{\frac{c_1 L_1^2 + c_2 L_2^2 + c_3 L_3^2}{L_1^2}}_{c_{eq}} \dot{x}_1^2$$