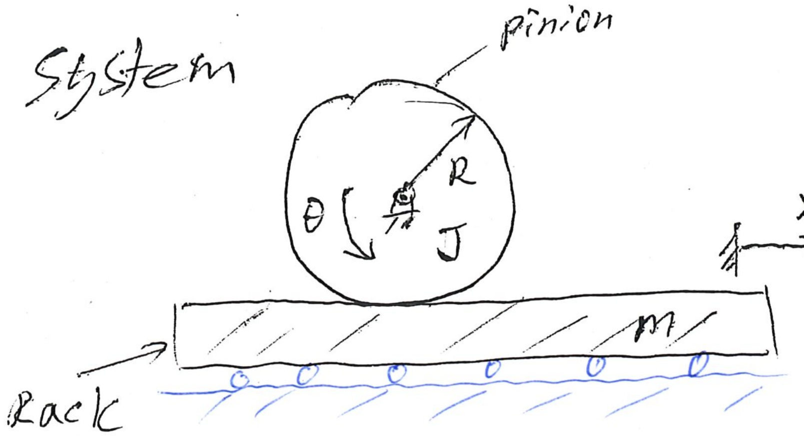
Determine ~~equivalent~~ (model) m and/or J for machine ele's

$$\boxed{E_K \text{ stored in system} = E_K \text{ stored in model}}$$

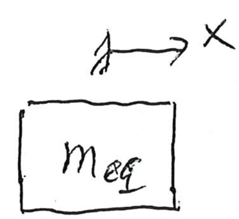
Example: Rack-and-pinion set

System



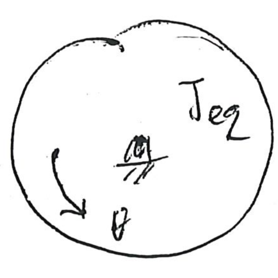
$x = R\theta$

model 1



$m_{eq} = ?$

model 2



$J_{eq} = ?$

model 1: $E_K = \frac{1}{2} m_{eq} \dot{x}^2$ (1)

model 2: $E_K = \frac{1}{2} J_{eq} \dot{\theta}^2$ (2)

$x = R\theta$

$\dot{x} = R\dot{\theta}$

system: $E_K = \frac{1}{2} J \dot{\theta}^2 + \frac{1}{2} m \dot{x}^2$

$= \frac{1}{2} J \left(\frac{\dot{x}}{R}\right)^2 + \frac{1}{2} m \dot{x}^2$

$= \frac{1}{2} \left(\frac{J}{R^2} + m \right) \dot{x}^2$ (3)

contribution by pinion

compare Eq (3) to Eq. (1) $\Rightarrow$ $\boxed{m_{eq} = m + \frac{J}{R^2}}$

Similarly

$E_K = \frac{1}{2} J \dot{\theta} + \frac{1}{2} m \dot{x}^2$

$= \frac{1}{2} J \dot{\theta} + \frac{1}{2} m (R\dot{\theta})^2$

$= \frac{1}{2} \left(J + mR^2 \right) \dot{\theta}^2$ (4)

$\therefore$ $\boxed{J_{eq} = J + mR^2}$

by Rack

Dynamic equivalence of a floating link in plane motion

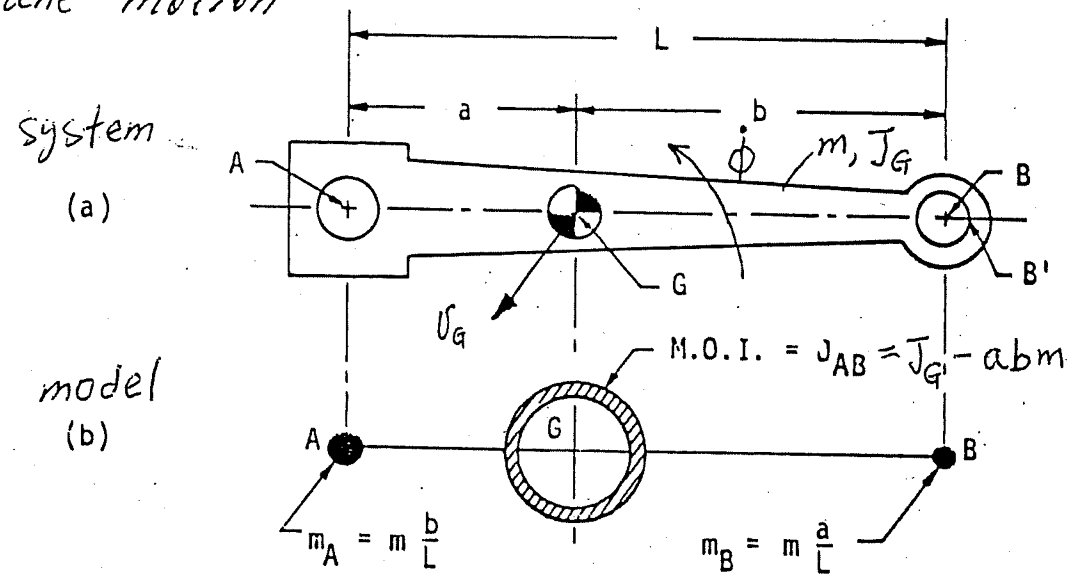


Figure 12.31-1. (a) Given link; (b) Dynamically equivalent replacement.

Model →

$$KE = \frac{1}{2} m_A v_A^2 + \frac{1}{2} m_B v_B^2 + \frac{1}{2} J_{AB} \dot\phi^2 \qquad \text{A}$$

system →

$$KE = \frac{1}{2} m v_G^2 + \frac{1}{2} J_G \dot\phi^2 \qquad \text{B}$$

$$= \frac{1}{2} m \left(\frac{b}{L} v_A^2 + \frac{a}{L} v_B^2 - ab\dot\phi^2 \right) + \frac{1}{2} J_G \dot\phi^2$$

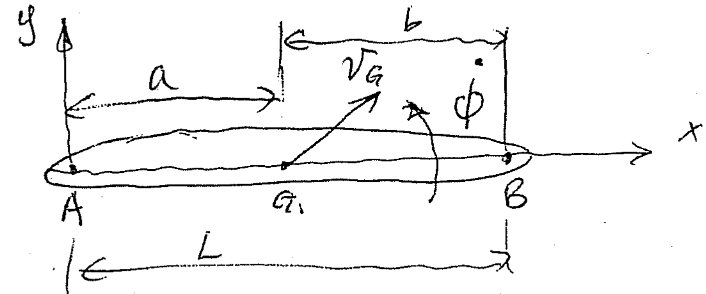
$$= \frac{1}{2}\left(m\frac{b}{L}\right) v_A^2 + \frac{1}{2}\left(m\frac{a}{L}\right) v_B^2 + \frac{1}{2}(J_G - abm)\dot\phi^2$$

Eq. A = Eq. B

$$\Rightarrow \boxed{m_A = m\frac{b}{L}, \quad m_B = m\frac{a}{L}, \quad J_{AB} = J_G - abm}$$

Derivation of V_G^2 in Eq. B

Kinematics gives:



$$v_{Ax} = v_{Gx} = v_{Bx}$$

velocities along the bar

$$v_{By} = v_{Gy} + b\dot{\phi}$$

velocities ⊥ the bar

$$v_{Ay} = v_{Gy} - a\dot{\phi}$$

$\therefore$

$$v_A^2 = v_{Ax}^2 + v_{Ay}^2 = \overbrace{v_{Gx}^2 + v_{Gy}^2}^{v_G^2} - 2a v_{Gy}\dot{\phi} + a^2\dot{\phi}^2 \qquad (3)$$

similarly

$$v_B^2 = v_G^2 + 2b v_{Gy}\dot{\phi} + b^2\dot{\phi}^2 \qquad (4)$$

$b \cdot$ Eq. (3) $+ a \cdot$ Eq. (4) $\Rightarrow$

$$b v_A^2 + a v_B^2 = (b+a) v_G^2 + (ba^2 + ab^2)\dot{\phi}^2$$

$$= L v_G^2 + abL\dot{\phi}^2$$

$$\therefore \boxed{v_G^2 = \frac{b}{L} v_A^2 + \frac{a}{L} v_B^2 - ab\dot{\phi}^2} \qquad (5)$$

Eq. B $\Rightarrow$ Eq. A