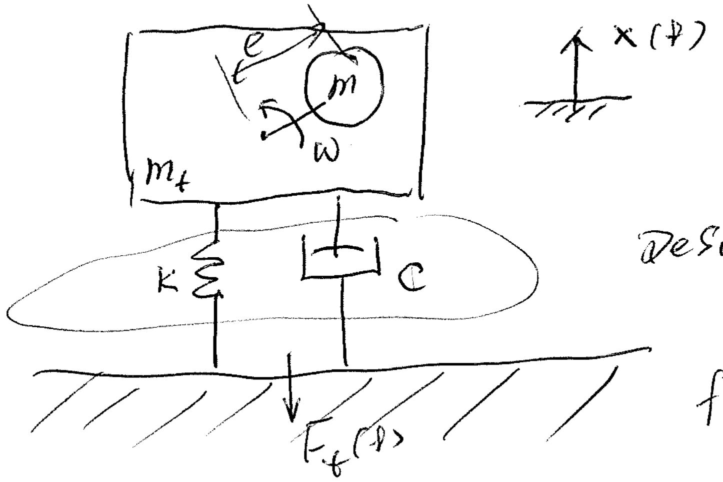
Design applications

Example: Rotating machinery mount design



Design machine mount (isolator) for small X and/or F_{t0} magnitudes

Math model:

$$m_t \ddot{x} + c\dot{x} + kx = me\omega^2 \sin \omega t$$

or

$$\ddot{x} + 2\xi\omega_n \dot{x} + \omega_n^2 x = E \sin \omega t$$

$$\omega_n = \sqrt{\frac{k}{m_t}}, \quad \xi = \frac{1}{2\omega_n}\frac{c}{m_t}, \quad E = \frac{me\omega^2}{m_t}$$

$\Rightarrow \quad x_p(t) = X \sin(\omega t + \psi)$

a) Magnitude of steady-state vibration (by Eq. (m)):

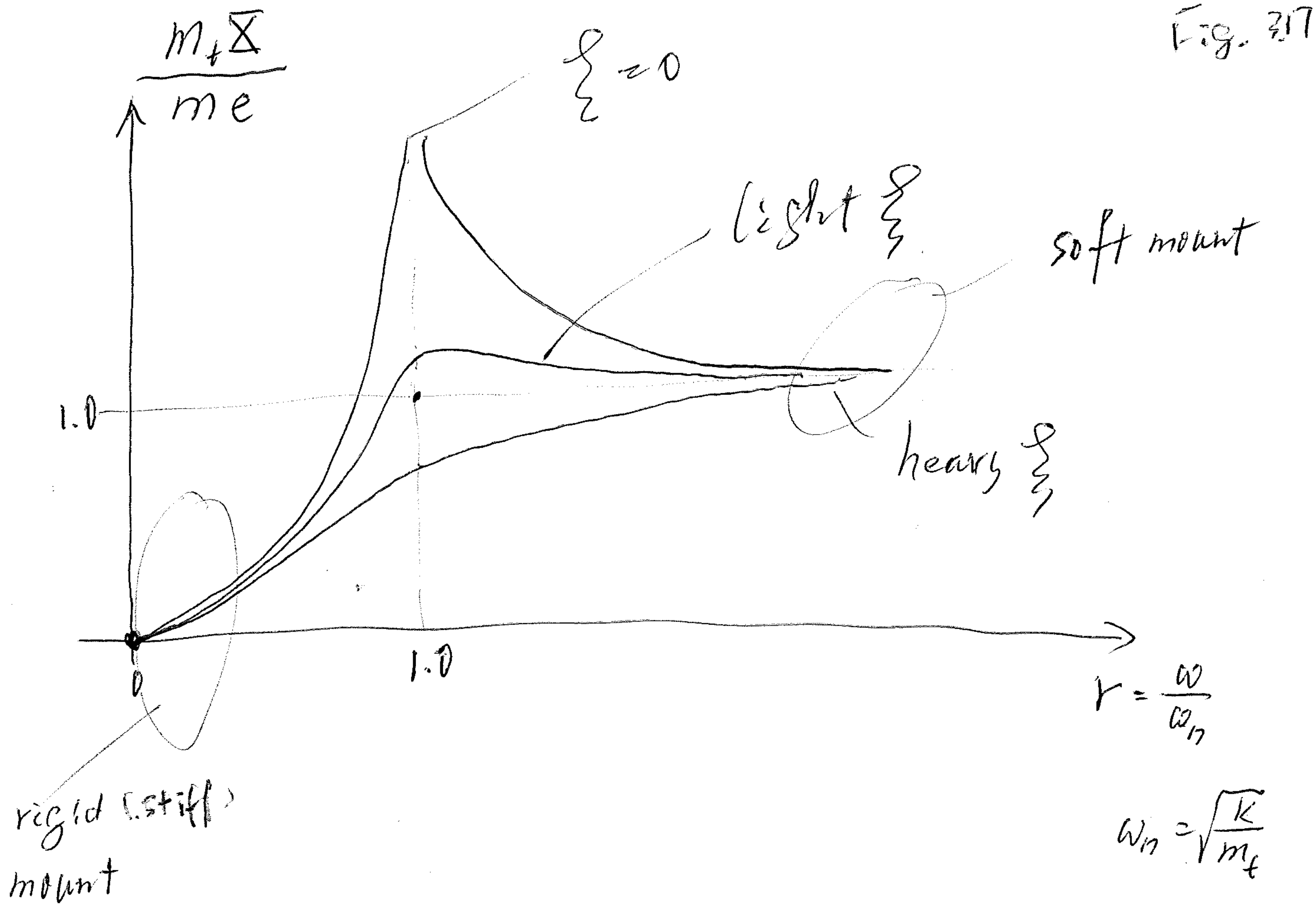
$$\bar{X} = \frac{\left(\frac{me\omega^2}{m_t}\right)/\omega_n^2}{\left[(1-r^2)^2 + (2\xi r)^2\right]^{1/2}}$$

Or

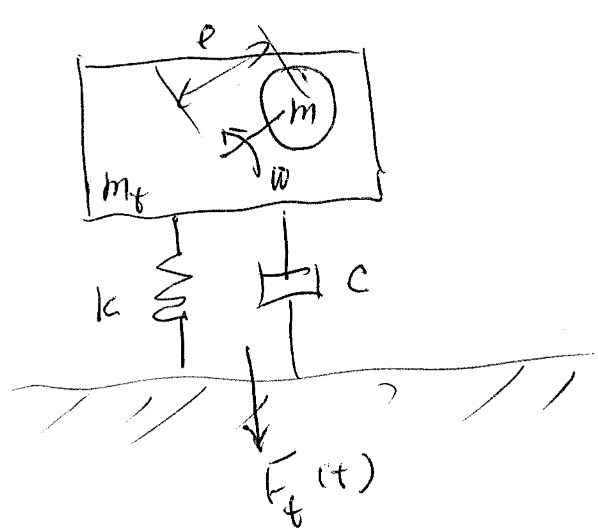
$$\frac{m_t \bar{X}}{me} = \frac{r^2}{\left[(1-r^2)^2+(2\xi r)^2\right]^{1/2}} \qquad (1)$$

Eq. (1) measures how the machine rotating unbalance (me) translates into machine vibration ($m_t \bar{X}$) at various r and ξ. ($r = \frac{\omega}{\omega_n}$, $\omega_n = \sqrt{\frac{k}{m_t}}$)

Eq. (1) may be presented in a chart of a family of curves to reveal some insights for machine design:



b) Magnitude of force transmitted to machine foundation:
(bearings)



$$F_t = F_k + F_c = kx + c\dot{x}$$

At steady running

$$x(t) = X\sin(\omega t + \psi)$$

X by Eq. (m)

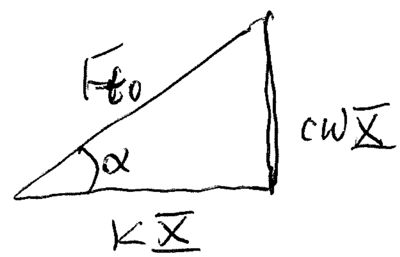
ψ by Eq. (P)

∴

$$F_t = kX\sin(\omega t+\psi) + cX\omega\cos(\omega t+\psi)$$

$$= F_{t0}\left(\frac{kX}{F_{t0}}\sin(\omega t+\psi) + \frac{c\omega X}{F_{t0}}\cos(\omega t+\psi)\right)$$

$$= F_{t0}\sin(\omega t+\psi+\alpha)$$



So, the magnitude of the force (trans.) is

$\xi = \frac{1}{2\omega_n}\frac{a_1}{a_2}$

$$F_{t0} = (k^2 + c^2\omega^2)^{1/2} X$$

$$\overset{\div m_t}{=} \left(\frac{k^2}{m_t^2} + \frac{c^2}{m_t^2}\omega^2\right)^{1/2} m_t X = \left(\omega_n^4 + (2\xi\omega_n)^2\omega^2\right)^{1/2} m_t X$$

↑ Eq. (m)

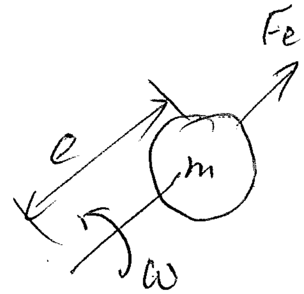
$$= \left(1 + (2\xi r)^2\right)^{1/2} \omega_n^2 m_t \cdot \frac{E/\omega_n^2}{[(1-r^2)^2 + (2\xi r)^2]^{1/2}}$$

↑ ω_n^2

$$E = \frac{me\omega^2}{m_t}$$

$$\therefore \quad F_{to} = \left(1+(2\xi r)^2\right)^{1/2} \frac{\overbrace{me\omega^2}^{F_{eo}}}{\left[(1-r^2)^2+(2\xi r)^2\right]^{1/2}}$$

F_{eo} = magnitude of excitation force



Define a force transmissibility:

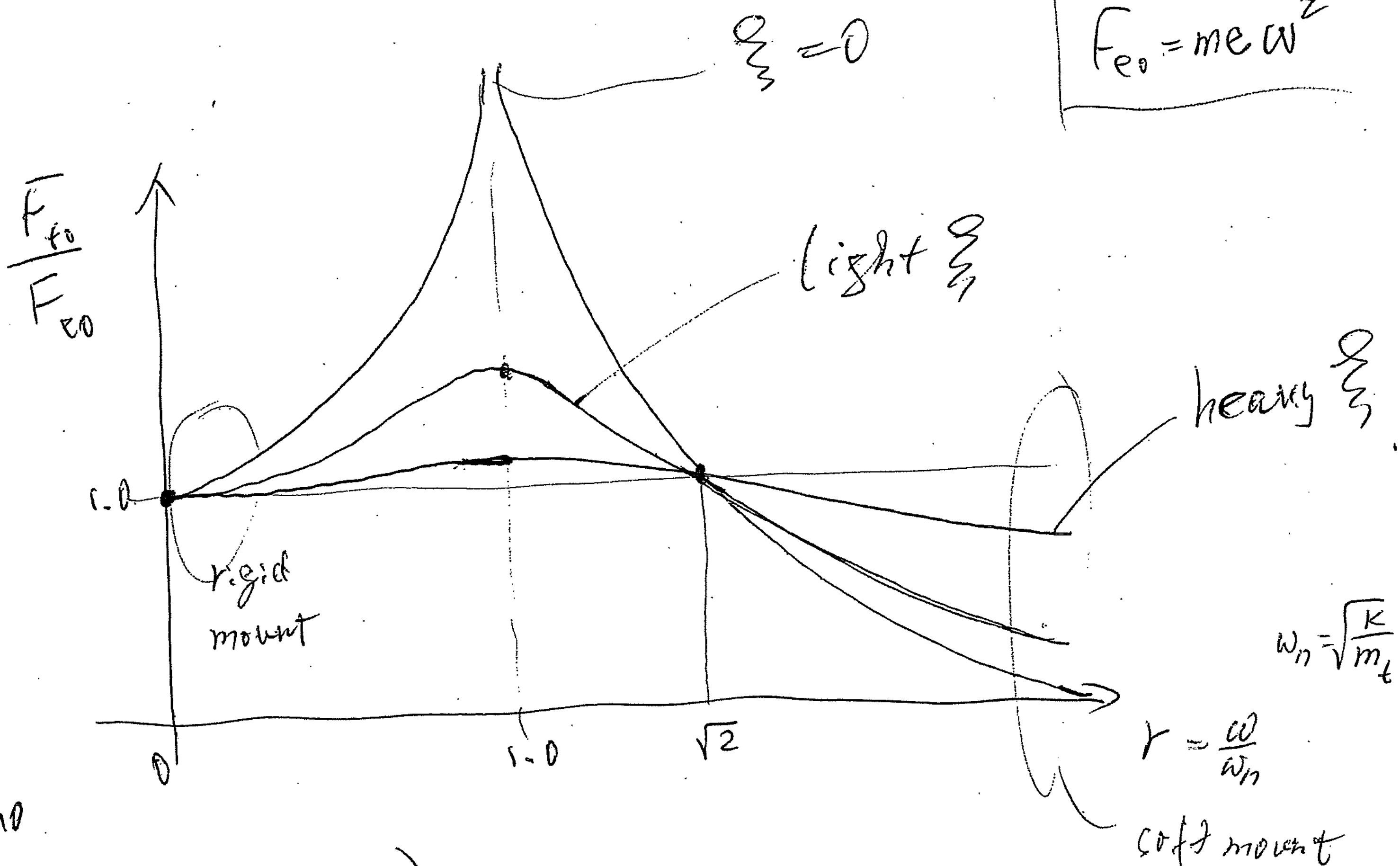
$$TR = \frac{F_{to}}{F_{eo}}$$ — a measure of F_{to} in the scale of F_{eo}

$$\Rightarrow \quad \boxed{TR = \frac{\left[1+(2\xi r)^2\right]^{1/2}}{\left[(1-r^2)^2+(2\xi r)^2\right]^{1/2}}} \qquad (2)$$

Eq. (2) measures how large the force of machine operation is transmitted to its foundation or bearings. at various ~~operating speed~~ r. and ξ.

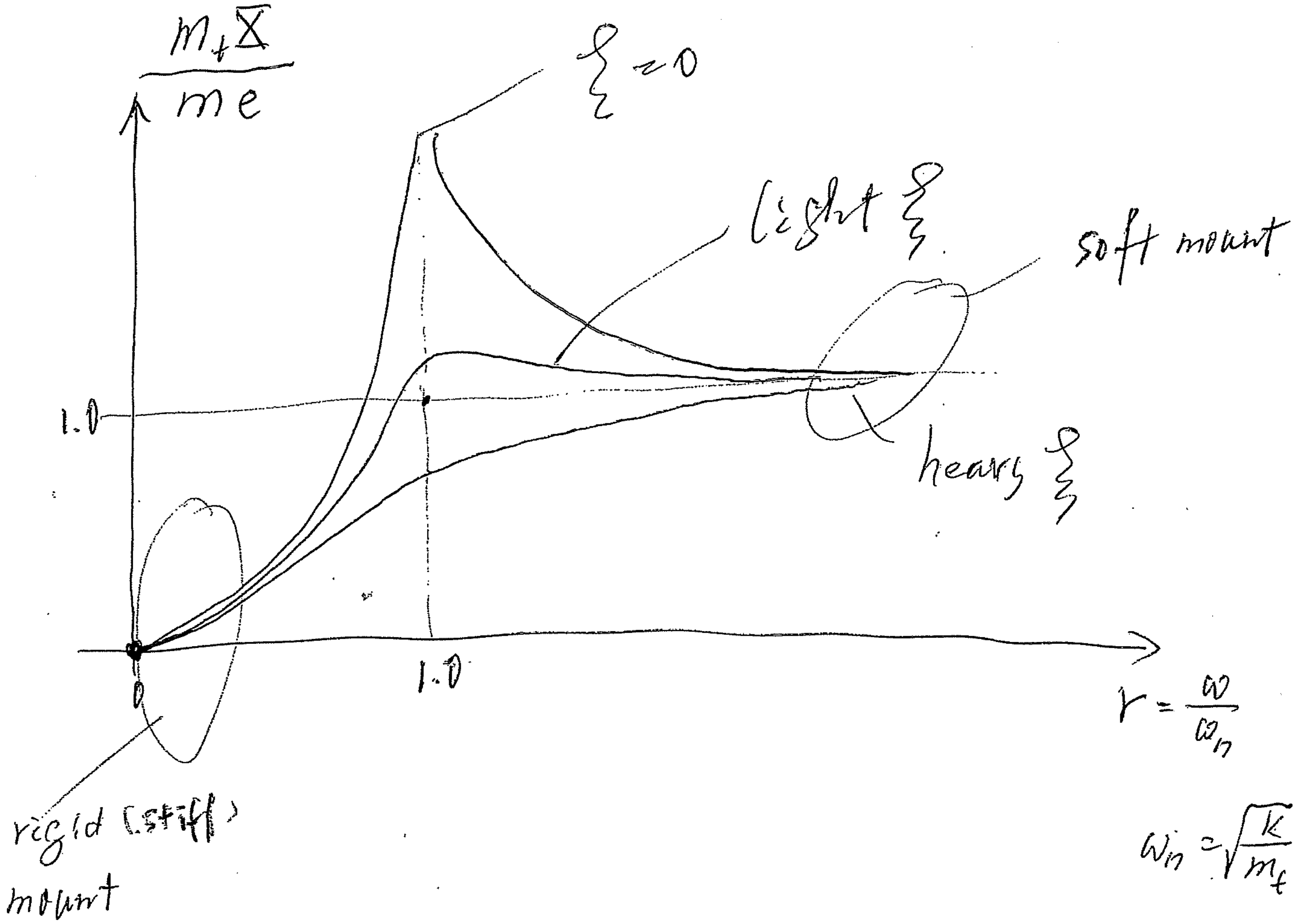
is for Eq(1), It may be presented in a chart of a family of curves to reveal some insights for machine design:

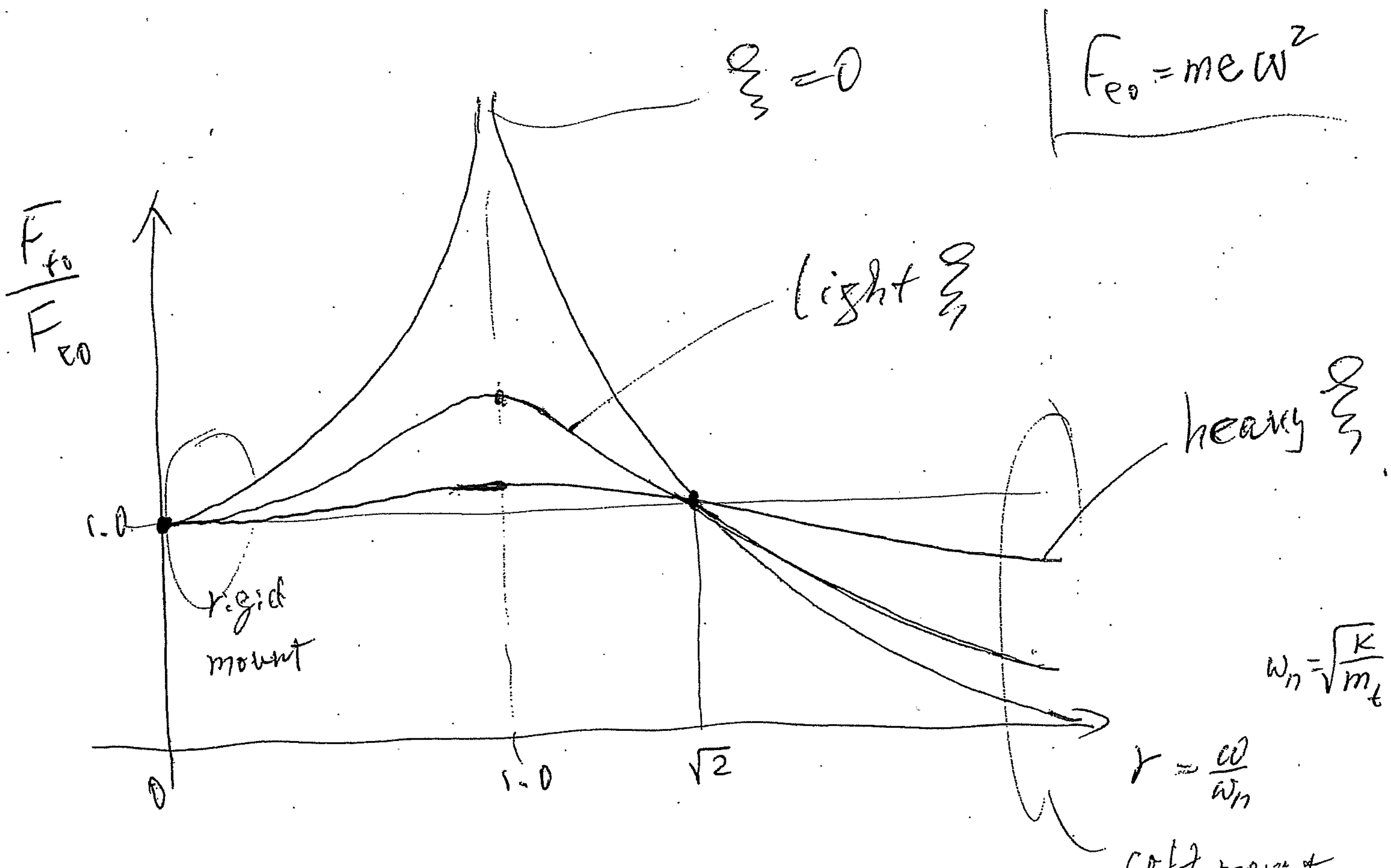
$F_{e0} = me\omega^2$



Demo

(Both charts ...)


$\frac{m_t X}{me}$
$\xi = 0$
light ξ
soft mount
1.0
heavy ξ
0
1.0
$r = \frac{\omega}{\omega_n}$
rigid (stiff) mount
$\omega_n = \sqrt{\frac{k}{m_t}}$


$\xi = 0$
$F_{e0} = me\omega^2$
$\frac{F_{t0}}{F_{e0}}$
light ξ
heavy ξ
1.0
rigid mount
$\omega_n = \sqrt{\frac{k}{m_t}}$
0
1.0
$\sqrt{2}$
$r = \frac{\omega}{\omega_n}$

From the charts of Eqs. (1) and (2), we see

1. System design should avoid operating near resonance, $\omega = \omega_n$ ($r = 1.0$), especially for systems of light damping.

2. Rigid mount may reduce machine vibration but may transmit large forces to foundation to generate high noise and high bearing stresses and to induce surrounding vibration, especially for high-speed machines.

3. Soft mount with light damping can largely reduce force transmitted to foundation and reasonably reduce machine vibration.

4. Both machine vibration and force transmitted to foundation can be reduced by reducing "e" --- ie. balancing the rotating mass of the machine, especially in high-speed operations.