III Multiple DOF systems

A SDOF system is described by one 2nd-order differential eq. $(a_2\ddot{y} + a_1\dot{y} + a_0 y = RHS)$

An NDOF system is described by N 2nd-order differential eqs.

SDOF model captures the fundamental mode (FM) of vibration of the underline system

NDOF model captures N modes of the vibration.

MDOF modeling features:

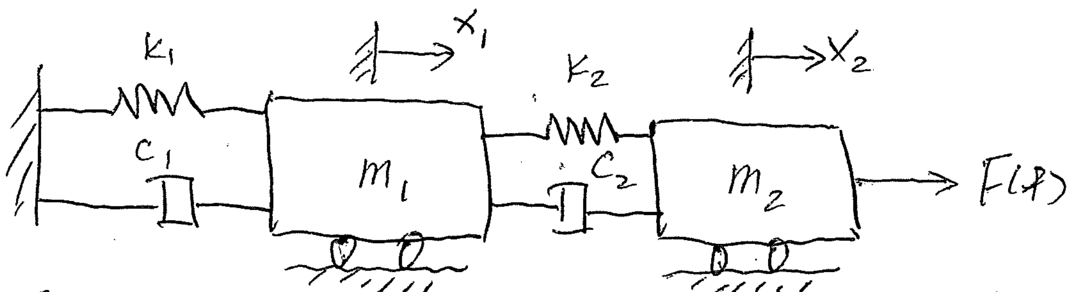
— topics of advanced vibration problems

— concept of vibration absorption of one mode by another.

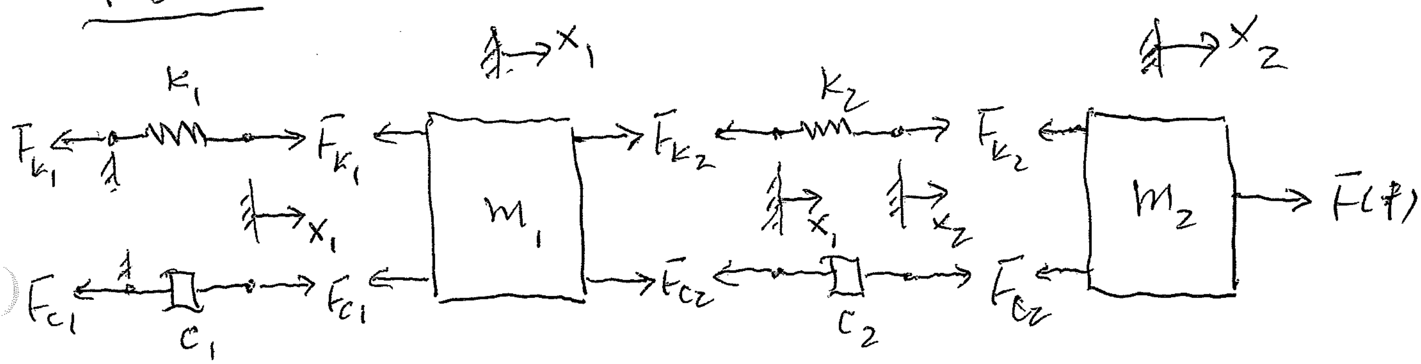
— advanced design/analysis tools.

3.1 Math model

Example: mass-spring-damper system



FBDs



Ele. eqs

k_1 — $F_{k_1} = k_1 x_1$ ①

c_1 — $F_{c1} = c_1 \dot{x}_1$ ②

m_1 — $m_1 \ddot{x}_1 = -F_{k_1} - F_{c2} + F_{k2} + F_{c2}$ ③

k_2 — $F_{k_2} = k_2 (x_2 - x_1)$ ④

c_2 — $F_{c2} = c_2 (\dot{x}_2 - \dot{x}_1)$ ⑤

m_2 — $m_2 \ddot{x}_2 = F(t) - F_{k_2} - F_{c2}$ ⑥

Gov. eqs (DOF vars: x_1 and x_2, expect 2 eqs)

Eq. (3) $\Rightarrow$

$$m_1 \ddot{x}_1 = -k_1 x_1 - c_1 \dot{x}_1 + k_2(x_2 - x_1) + c_2(\dot{x}_2 - \dot{x}_1)$$

$$\Rightarrow \boxed{m_1 \ddot{x}_1 + (c_1 + c_2)\dot{x}_1 - c_2 \dot{x}_2 + (k_1 + k_2) x_1 - k_2 x_2 = 0} \quad (7)$$

Eq. (6) $\Rightarrow$

$$m_2 \ddot{x}_2 = F(t) - k_2(x_2 - x_1) - c_2(\dot{x}_2 - \dot{x}_1)$$

$$\Rightarrow \boxed{m_2 \ddot{x}_2 - c_2 \dot{x}_1 + c_2 \dot{x}_2 - k_2 x_1 + k_2 x_2 = F(t)} \quad (8)$$

Eqs. (7) and (8) can be arranged in a matrix:

$$\begin{bmatrix} m_1 & 0 \\ 0 & m_2 \end{bmatrix} \begin{bmatrix} \ddot{x}_1 \\ \ddot{x}_2 \end{bmatrix} + \begin{bmatrix} (c_1 + c_2) & -c_2 \\ -c_2 & c_2 \end{bmatrix} \begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} + \begin{bmatrix} (k_1 + k_2) & -k_2 \\ -k_2 & k_2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

$$= \begin{bmatrix} 0 \\ F(t) \end{bmatrix} \quad (9)$$

Or

$$[M]\ddot{\bar{x}} + [C]\dot{\bar{x}} + [K]\bar{x} = \bar{F}(t) \qquad (9)$$

$[M], [C], [K]$ = mass, damping, stiffness matrices ($N \times N$ square matrice)

$\ddot{\bar{x}}, \dot{\bar{x}}, \bar{x}$ = acceleration, velocity, displacement vectors

$\bar{F}(t)$ = excitation vector

Eq. (9) is the standard form of the math model for a MDOF system.

Recall for a SDOF system,

$a_2\ddot{x} + a_1\dot{x} + a_0 x = F$ — a special case of Eq. (9)