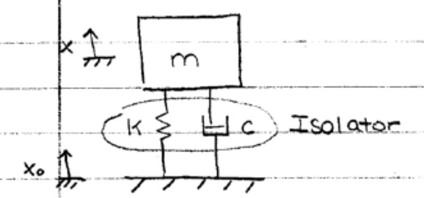
Solution worked out by a former student

(9.27) Vibration Isolation

An electronic instrument is to be isolated from a panel that vibrates at frequencies ranging from 25 Hz to 35 Hz. It is estimated that at least 80% vibration isolation must be achieved to prevent damage to the instrument. If the instrument weighs 85 N, find the necessary static deflection of the isolator.




$$m = \frac{85\,N}{9.81\,m/s^2}\left(\frac{kg\,m}{N s^2}\right) = 8.665\ kg$$

Frequency range: $25\ Hz \Rightarrow \omega = 2\pi f = 157.08\ rad/s$

$35\ Hz \Rightarrow \omega = 2\pi f = 219.91\ rad/s$

Governing Equation: $\ddot{x} + 2\xi\omega_n\dot{x} + \omega_n^2 x = F_0\sin(\omega t)$

Response Amplitude: $X = \dfrac{F_0/\omega_n^2}{[(1-r^2)^2+(2\xi r)^2]^{1/2}}$

$$\frac{X}{X_0} = TR = \frac{[1+(2\xi r)^2]^{\frac{1}{2}}}{[(1-r^2)^2+(2\xi r)^2]^{\frac{1}{2}}}$$ = ratio of magnitude of force transmitted to that of the exciting force $= 1-0.8 = 0.2$

For $\xi$ small $TR \approx \dfrac{1}{|(1-r^2)|}$ where $r = \dfrac{\omega}{\omega_n}$ $\quad \omega_n = \sqrt{\dfrac{k}{m}}$

$\therefore TR = 0.2 = \dfrac{1}{|(1-r^2)|} \rightarrow \dfrac{1}{0.2} = |(1-r^2)| \rightarrow \pm 5 = 1-r^2 \rightarrow r^2 = 6$, $\cancel{r^2 = -4}$

$\therefore r^2 = 6$ or $r = \sqrt{6} = 2.449$

For $f = 25\ Hz = 157.08\ rad/s$: $r = \dfrac{\omega}{\omega_n} \rightarrow \omega_n = \dfrac{\omega}{r} = \dfrac{157.08\ \frac{rad}{s}}{\sqrt{6}} = 64.127\ rad/s$

and $\omega_n = \sqrt{\dfrac{k}{m}} \rightarrow K = \omega_n^2 m = (64.127\ rad/s)^2(8.665\ kg) = 35633.4\ N/m$

For $f = 35\text{ Hz} = 219.91 \text{ rad/s}$ : $r = \frac{\omega}{\omega_n} \rightarrow \omega_n = \frac{\omega}{r} = \frac{219.91 \frac{rad}{s}}{\sqrt{6}} = 89.778 \text{ rad/s}$

and $\omega_n = \sqrt{\frac{k}{m}} \rightarrow k = \omega_n^2 m = (89.778 \text{ rad/s})^2 (8.6605\text{ kg}) = 69{,}841.4 \text{ N/m}$

Static deflection $\delta_{st} = \frac{W}{k}$

For $f = 25\text{ Hz}$, $\delta_{st} = \frac{85\text{ N}}{35633.4 \text{ N/m}} = 0.002385\text{ m} = 2.385\text{ mm}$

For $f = 35\text{ Hz}$, $\delta_{st} = \frac{85\text{ N}}{69{,}841.4 \text{ N/m}} = 0.001217\text{ m} = 1.217\text{ mm}$

$\therefore$ choose the <u>larger</u> static deflection: $\boxed{2.385\text{ mm}}$

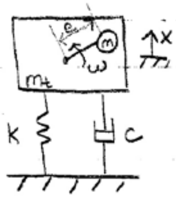
(Choice of the smaller static deflection/smaller stiffness @ $f = 35\text{ Hz}$ would not satisfy requirement at $f = 25\text{ Hz}$)

So, k=35633.4 N/m and $\delta_{st} \geq 2.385$ mm will satisfy the design requirement.

The above results are obtained with $\xi = 0$. The same calculation procedures may be followed for $\xi \neq 0$. Some damping is always needed such as $\xi = 0.1 \sim 0.5$ depending on applications.
You may try it with say $\xi = 0.1$.

Vibration Isolation

(9.35) A compressor of mass 120 kg has a rotating unbalance of 0.2 kg-m. If an isolator of stiffness 0.5 MN/m and damping ratio 0.06 is used, find the range of operating speeds of the compressor over which the force transmitted to the foundation will be less than 2500 N




Given: $me = 0.2$ kg-m

$m_t = 120$ kg

$k = 0.5 \times 10^6$ N/m

$\xi = 0.06$

$F_t < 2500$ N

$$TR = \frac{F_t}{F_e} = \frac{[1+(2\xi r)^2]^{\frac{1}{2}}}{[(1-r^2)^2+(2\xi r)^2]^{\frac{1}{2}}} \quad \text{and } F_e = me\omega^2$$

$$\frac{F_t}{me\omega^2} = \frac{[1+(2\xi r)^2]^{\frac{1}{2}}}{[(1-r^2)^2+(2\xi r)^2]^{\frac{1}{2}}} \quad \text{and } r = \frac{\omega}{\omega_n} \rightarrow \omega^2 = r^2\omega_n^2$$

$$\frac{F_t}{mer^2\omega_n^2} = \frac{[1+(2\xi r)^2]^{\frac{1}{2}}}{[(1-r^2)^2+(2\xi r)^2]^{\frac{1}{2}}} \quad \text{OR} \quad \frac{F_t}{me\omega_n^2} = \frac{r^2[1+(2\xi r)^2]^{\frac{1}{2}}}{[(1-r^2)^2+(2\xi r)^2]^{\frac{1}{2}}} \quad (1)$$

and $\omega_n^2 = \frac{k}{m} = \frac{0.5\times10^6 \text{ N/m}}{120 \text{ kg}} = 4166.67 \text{ rad/s}^2$, $\omega_n = 64.55$ rad/s

∴ from eq (1) above, setting $F_t = me\omega_n^2 r^2 \frac{[1+(2\xi r)^2]^{\frac{1}{2}}}{[(1-r^2)^2+(2\xi r)^2]^{\frac{1}{2}}} < 2500$ N

or $(0.2 \text{ kgm})(4166.67 \text{ rad/s}^2)\, r^2 \frac{[1+(2\cdot0.06r)^2]^{\frac{1}{2}}}{[(1-r^2)^2+(2\cdot0.06r)^2]^{\frac{1}{2}}} < 2500$ N

$$r^2 \frac{[1+0.0144r^2]^{\frac{1}{2}}}{[1-2r^2+r^4+0.0144r^2]^{\frac{1}{2}}} < 3$$

Squaring both sides

$$\frac{r^4(1+0.0144r^2)}{r^4-1.9856r^2+1} < 9 \quad \rightarrow \text{Setting the LHS} = 8 < 9$$

or $0.0144r^6 + r^4 = 8(r^4 - 1.9856r^2 + 1)$

or $r^6 - 486.111r^4 + 1103.111r^2 - 555.556 = 0$

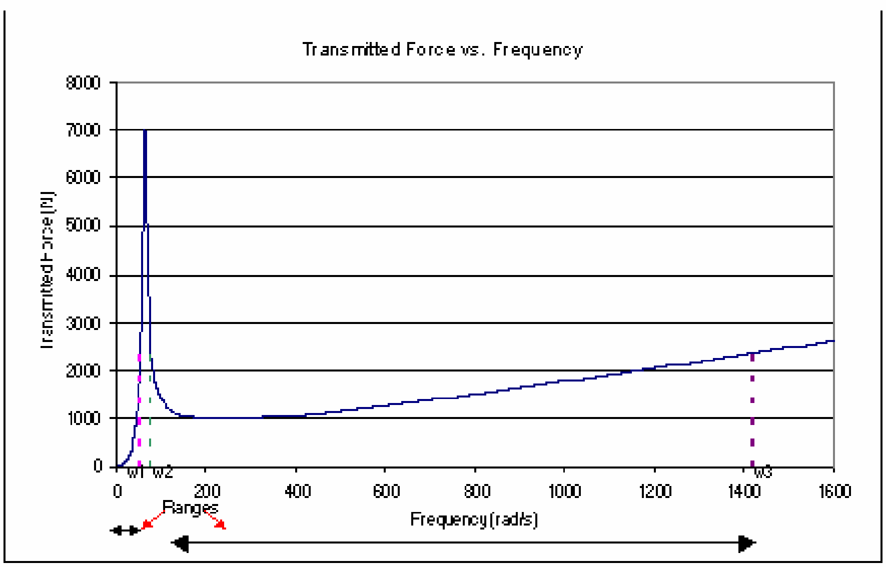
Solving (using a graphic calculator) for the roots of the eq

$r_1 = 0.867$ or $r = \omega/\omega_n \rightarrow \omega_1 = r_1\omega_n$ $\quad \omega_1 = 55.96$ rad/s

$r_2 = 1.235$ or $\omega_2 = r_2\omega_n$ $\quad \omega_2 = 79.72$ rad/s

$r_3 = 21.996$ or $\omega_3 = r_3\omega_n$ $\quad \omega_3 = 1419.84$ rad/s

See plot of $F_t = me\omega_n^2 \dfrac{r^2[1+(2\xi r)^2]^{\frac{1}{2}}}{[(1-r^2)^2+(2\xi r)^2]^{\frac{1}{2}}}$



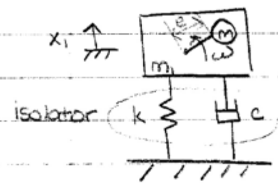

Choose ranges: $0$-$\omega_1$ (0- 55.96 rad/s) and $\omega_2$-$\omega_3$ (79.72-1419.84 rad/s)

The above plot gives the two ranges of operating speeds within which the force transmitted to foundation will be less than 2500 N.

(9.43) When a washing machine, of mass 200 kg and an unbalance

oration 0.02 kg-m, is mounted on an isolator, the isolator deflects by

olation 5 mm under the static load. Find (a) the amplitude of the washing machine and (b) the force transmitted to the foundation at the operating speed of 1200 rpm




Given: $me = 0.02\ \text{kgm}$ $\quad \omega = 1200\ \text{rpm} = 125.66\ \frac{\text{rad}}{\text{s}}$

$$m_1 = 200\ \text{kg}$$

$$\delta_{st} = \frac{mg}{k} = 5\times10^{-3}\ \text{m}$$

$$\text{or } k = \frac{mg}{\delta_{st}} = \frac{(200\ \text{kg})(9.81\ \text{m/s}^2)}{(5\times10^{-3}\ \text{m})} = 392400\ \text{N/m}$$

$$\omega_n = \sqrt{\frac{k}{m_1}} = 44.29\ \text{rad/s}$$

Recall: Response amplitude $X = \dfrac{E/\omega_n^2}{[(1-r^2)^2 + (2\zeta r)^2]^{\frac{1}{2}}}$, $r = \omega/\omega_n$ $\quad F_0 = me\omega^2$, $E = F_0/m_1$

Assume $\zeta$ small or zero.

$$X = \frac{me\omega^2/\omega_n^2 m_1}{|(1-r^2)|} = \frac{\frac{me}{m_1}r^2}{|(1-r^2)|} \quad \text{where } r = \frac{125.66\ \text{rad/s}}{44.29\ \text{rad/s}} = 2.837$$

$$X = \frac{mer^2}{m_1|(1-r^2)|} = \frac{(0.02\ \text{kgm})(2.837)^2}{(200\text{kg})|(1-2.837^2)|} = 1.1419\times10^{-4}\ \text{m}$$

$$\boxed{X = 1.1419\times10^{-4}\ \text{m}}$$

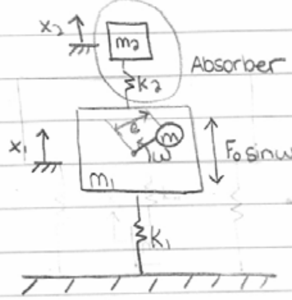
Recall: $TR = \dfrac{F_t}{F_0} = \dfrac{[1 + (2\zeta r)^2]^{\frac{1}{2}}}{[(1-r^2)^2 + (2\zeta r)^2]^{\frac{1}{2}}}$ $\quad$ assume $\zeta = 0$ or small, $F_0 = me\omega^2$

$$\text{or } F_t = \frac{me\omega^2}{|(1-r^2)|} = \frac{(0.02\ \text{kgm})(125.66\ \text{rad/s})^2}{|(1-2.837^2)|} \Rightarrow \boxed{F_t = 44.805\ \text{N}}$$

So, when a problem does not give info on damping, you assume it is zero.
Or you may assume any value that you feel is a good choice from practical point.

(9.51) An air compressor of mass 200 kg, with an unbalance of 0.01 kg-m, is found to have a large amplitude of vibration while running at 1200 rpm. Determine the mass and spring constant of the absorber to be added if the natural frequencies of the system are to be at least 20% from the impressed frequency

Vibration Absorption




Given: $m_1 = 200\,kg$

$me = 0.01$ kg-m

$\sqrt{\frac{k_1}{m_1}} = \omega = 1200\,rpm = 125.664\ rad/s$

$k_1 = \omega^2 m_1 = 3158273\ N/m$

$m_1, k_1$ = mass/stiffness of air compressor

$m_2, k_2$ = mass/stiffness of absorber

Equation of Motion

$$\begin{bmatrix} m_1 & 0 \\ 0 & m_2 \end{bmatrix}\begin{bmatrix} \ddot{x}_1 \\ \ddot{x}_2 \end{bmatrix} + \begin{bmatrix} (k_1+k_2) & -k_2 \\ -k_2 & k_2 \end{bmatrix}\begin{bmatrix} X_1 \\ X_2 \end{bmatrix} = \begin{bmatrix} \overset{me\omega^2}{F_0} \\ 0 \end{bmatrix} e^{i\omega t}$$

Design Objectives: Design absorber to:

1) Suppress vibration at 1200 rpm
2) Critical Natural frequencies 20% from impressed frequency

Amplitudes of Vibration

$$X_1 = \left| \frac{k_2 - \omega^2 m_2}{(k_1+k_2-\omega^2 m_1)(k_2-\omega^2 m_2) - k_2^2} F_0 \right| \quad (1) \qquad X_2 = \left| \frac{k_2}{(k_1+k_2-\omega^2 m_1)(k_2-\omega^2 m_2) - k_2^2} F_0 \right|$$

To reduce $X_1$ set $k_2 - \omega^2 m_2 = 0$ in Eq (1)

or $\omega = \sqrt{\frac{k_2}{m_2}} \rightarrow 125.664 = \sqrt{\frac{k_2}{m_2}} \rightarrow \frac{m_2}{k_2} = \left(\frac{1}{125.664}\right)^2$ (2)

Recall: $Det\,[-\omega_{cr}^2 M + K] = 0$

or $(k_1+k_2-\omega_{cr}^2 m_1)(k_2 - \omega_{cr}^2 m_2) - k_2^2 = 0$ (3)

Want $\omega_{cr} > 1.2\omega = 1.2(125.664) = 150.796 \text{ rad/s}$ ⓐ

and $\omega_{cr} < 0.8\omega = 0.8(125.664) = 100.531 \text{ rad/s}$ ⓑ

Use eq ③ twice with eq ② and conditions ⓐ and ⓑ

expand ↓

divide by $k_2$ ↓

ⓐ $(k_1 + k_2 - \omega_{cr}^2 m_1)(k_2 - \omega_{cr}^2 m_2) - k_2^2 = 0$

$k_1 k_2 - k_1 m_2 \omega_{cr}^2 + \cancel{k_2^2} - k_2 m_2 \omega_{cr}^2 - k_2 m_1 \omega_{cr}^2 + \omega_{cr}^4 m_1 m_2 - \cancel{k_2^2} = 0$

$$k_1 - k_1 \frac{m_2}{k_2}\omega_{cr}^2 - m_2\omega_{cr}^2 - m_1\omega_{cr}^2 + m_1\frac{m_2}{k_2}\omega_{cr}^4 = 0 \quad (4)$$

$$3158273 - 3158273\left(\frac{1}{125.664}\right)^2(150.796)^2 - m_2(150.796)^2 - (200)(150.796)^2 + (200)\left(\frac{1}{125.664}\right)^2(150.796)^4 = 0$$

or $\boxed{m_2 = 26.88 \text{ kg}}$

$\therefore$ Eq ② $\rightarrow$ $\boxed{k_2 = 424474 \text{ N/m}}$

ⓑ

$$k_1 - k_1\frac{m_2}{k_2}\omega_{cr}^2 - m_2\omega_{cr}^2 - m_1\omega_{cr}^2 + m_1\frac{m_2}{k_2}\omega_{cr}^4 = 0 \quad (4)$$

$$3158273 - 3158273\left(\frac{1}{125.664}\right)^2(100.531)^2 - m_2(100.531)^2 - (200)(100.531)^2 + (200)\left(\frac{1}{125.664}\right)^2(100.531)^4 = 0$$

$\boxed{m_2 = 40.5 \text{ kg}}$

Eq ② $\Rightarrow$ $\boxed{k_2 = 639553 \text{ N/m}}$

Since the values of $m_2$ and $k_2$ are larger for condition ⓑ we must use these values: $\boxed{m_2 = 40.5 \text{ kg} \quad k_2 = 639553 \text{ N/m}}$

Check

then Eq ④ $3158273 - 3158273\left(\frac{1}{125.664}\right)^2\omega_{cr}^2 - (40.5)\omega_{cr}^2 - (200)\omega_{cr}^2 + (200)\left(\frac{1}{125.664}\right)^2\omega_{cr}^4 = 0$

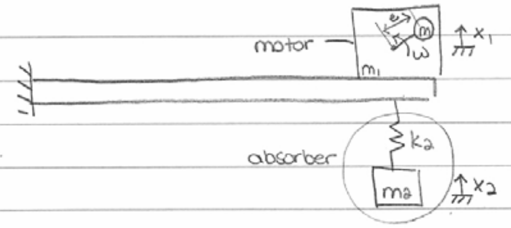
✓ $\omega_{cr1} = 157.1 \text{ rad/s} > 1.2\omega$ ✓ $\omega_{cr2} = 100.531 \text{ rad/s}$

In the previous problem, the true point is that one needs to examine results in both Part (a) and Part (b) to see which pair of $k_2$ and $m_2$ will satisfy the given design requirement of $\omega_n$ outside of $0.8\omega \sim 1.2\omega$.

Also, in the problem, because large vibration is developed at $\omega = 1200$ rpm, it has been assumed to be the critical speed (natural frequency) of the original system before the absorber is added. Based on this assumption, $k_1$ is calculated as listed early in the problem.

(9.52) An electric motor, having an unbalance of 2 kg-cm, is mounted at the end of a steel cantilever beam. The beam is observed to vibrate with large amplitudes at the operating speed of 1500 rpm of the motor. It is proposed to add a vibration absorber to reduce the vibration of the beam. Determine the ratio of the absorber mass to the mass of the motor needed in order to have the lower frequency of the resulting system equal to 75% of the operating speed of the motor. If the mass of the motor is 300 kg, determine the stiffness and mass of the absorber. Also find the amplitude of vibration of the absorber mass

Vibration Absorption




$me = 2$ kg-cm $= 0.02$ kg-m

$\omega = 1500$ rpm $= 157.08$ rad/s

$m_1 = 300$ kg

$k_1 = \omega^2 m_1 = 7.4022 \times 10^6$ N/m

Design Objectives: Design absorber to:

1) Suppress vibration at 1500 rpm
2) Lower critical frequency $= 75\%\,\omega = 0.75(157.08) = 117.81$ rad/s

Amplitudes of Vibration

$$X_1 = \left| \frac{k_2 - \omega^2 m_2}{(k_1 + k_2 - \omega^2 m_1)(k_2 - \omega^2 m_2) - k_2^2} F_0 \right| \quad (1)$$

$$X_2 = \left| \frac{k_2}{(k_1 + k_2 - \omega^2 m_1)(k_2 - \omega^2 m_2) - k_2^2} F_0 \right| \quad (2)$$

To reduce $X_1$ set $k_2 - \omega^2 m_2 = 0$ in Eq (1)

$$\omega = \sqrt{\frac{k_2}{m_2}} \quad \text{or} \quad 157.08 = \sqrt{\frac{k_2}{m_2}} \rightarrow \frac{m_2}{k_2} = \left(\frac{1}{157.08}\right)^2 \quad (3)$$

Then Eq (2) (divide by $k_2$)

$$X_2 = \left| \frac{1}{(k_1 + k_2 - \omega^2 m_1)\left(1 - \omega^2 \underbrace{\frac{m_2}{k_2}}_{=\frac{1}{\omega^2}}\right) - k_2} F_0 \right| = \frac{F_0}{k_2} \quad (4)$$

Recall: $\text{Det}\left[-\omega_{cr}^2 M + k\right] = 0$

or $(k_1 + k_2 - \omega_{cr}^2 m_1)(k_2 - \omega_{cr}^2 m_2) - k_2^2 = 0$ (5)

We want $\omega_{cr} = 0.75\,\omega = 0.75(157.08) = 117.81\ \text{rad/s}$

Use eq. (5) with eq. (3) and $\omega_{cr} = 117.81\ \text{rad/s}$

expand ↓ $(k_1 + k_2 - \omega_{cr}^2 m_1)(k_2 - \omega_{cr}^2 m_2) - k_2^2 = 0$

divide by $k_2$ ↓ $k_1 k_2 - k_1 m_2 \omega_{cr}^2 + \cancel{k_2^2} - k_2 m_2 \omega_{cr}^2 - k_2 m_1 \omega_{cr}^2 + m_1 m_2 \omega_{cr}^4 - \cancel{k_2^2} = 0$

$$k_1 - k_1 \frac{m_2}{k_2}\omega_{cr}^2 - m_2 \omega_{cr}^2 - m_1 \omega_{cr}^2 + m_1 \frac{m_2}{k_2}\omega_{cr}^4 = 0 \quad (6)$$

$$\therefore\ 7.4022\times10^6 - (7.4022\times10^6)\left(\frac{1}{157.08}\right)^2 (117.81)^2 - (m_2)(117.81)^2 - (300)(117.81)^2 + (300)\left(\frac{1}{157.08}\right)^2 (117.81)^4 = 0$$

$\boxed{m_2 = 102.08\ \text{kg}}$

then from Eq. (3) $k_2 = m_2(157.08)^2 \rightarrow \boxed{k_2 = 2.519\times10^6\ \text{N/m}}$

Ratio of absorber mass to motor mass: =

$$= \frac{m_2}{m_1} = \frac{102.08\ \text{kg}}{300\ \text{kg}} = \boxed{0.3403}$$

Amplitude of vibration of absorber mass $(X_2)$

Eq. (4) $X_2 = \dfrac{F_0}{k_2} = \dfrac{me\omega^2}{k_2} = \dfrac{(0.02\ \text{kg m})(157.08\ \text{rad/s})^2}{2.519\times10^6\ \text{N/m}}$

$X_2 = 1.959\times10^{-4}\ \text{m}$

$\boxed{X_2 = 0.1959\ \text{mm}}$

The problem asks to find the ratio of $m_2/m_1$. This may be done using Eq. (6) above without knowing the motor mass $m_1$. But it requires to assume that the critical speed is 1500 rpm of the motor system before an absorber is added. This assumption is reasonable as the problem states that large vibration is observed at this speed.