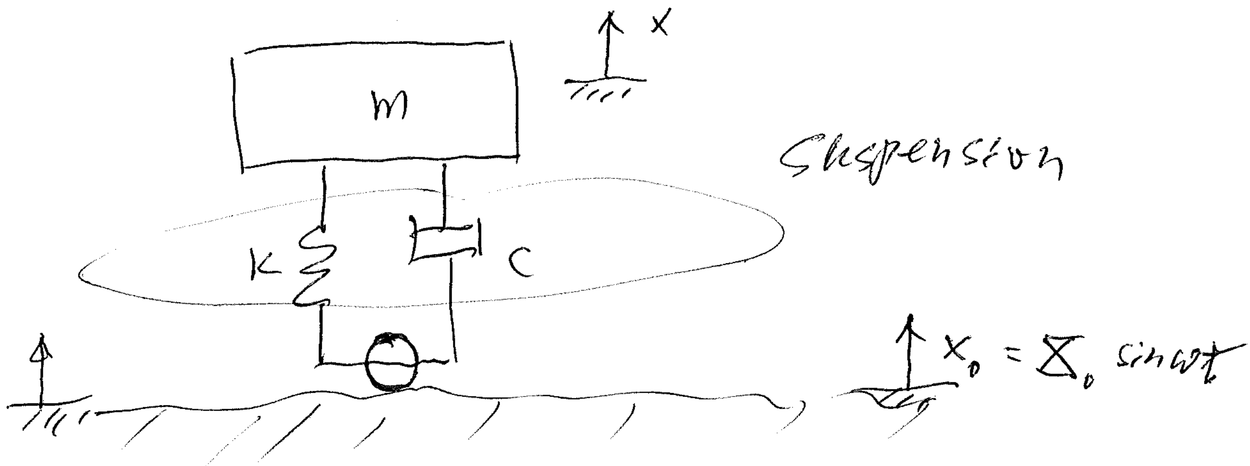
Example: vehicle suspension design



Model:

$$m\ddot{x} + c\dot{x} + kx = (k^2 + c^2\omega^2)^{1/2} X_0 \sin(\omega t + \alpha)$$

Or

$$\ddot{x} + 2\xi\omega_n\dot{x} + \omega_n^2 x = E \sin(\omega t + \alpha)$$

$$E = \frac{(k^2 + c^2\omega^2)^{1/2} X_0}{m} = \left(\frac{k^2}{m^2} + \frac{c^2}{m^2}\omega^2\right)^{1/2} X_0$$

$$= \left(\omega_n^4 + (2\xi\omega_n)^2\omega^2\right)^{1/2} X_0$$

$$= \left(1 + (2\xi r)^2\right)^{1/2} \omega_n^2 X_0$$

Follow-up response is:

$$x_p(t) = X \sin(\omega t + \alpha + \psi)$$

Eq. (m) $\Rightarrow$

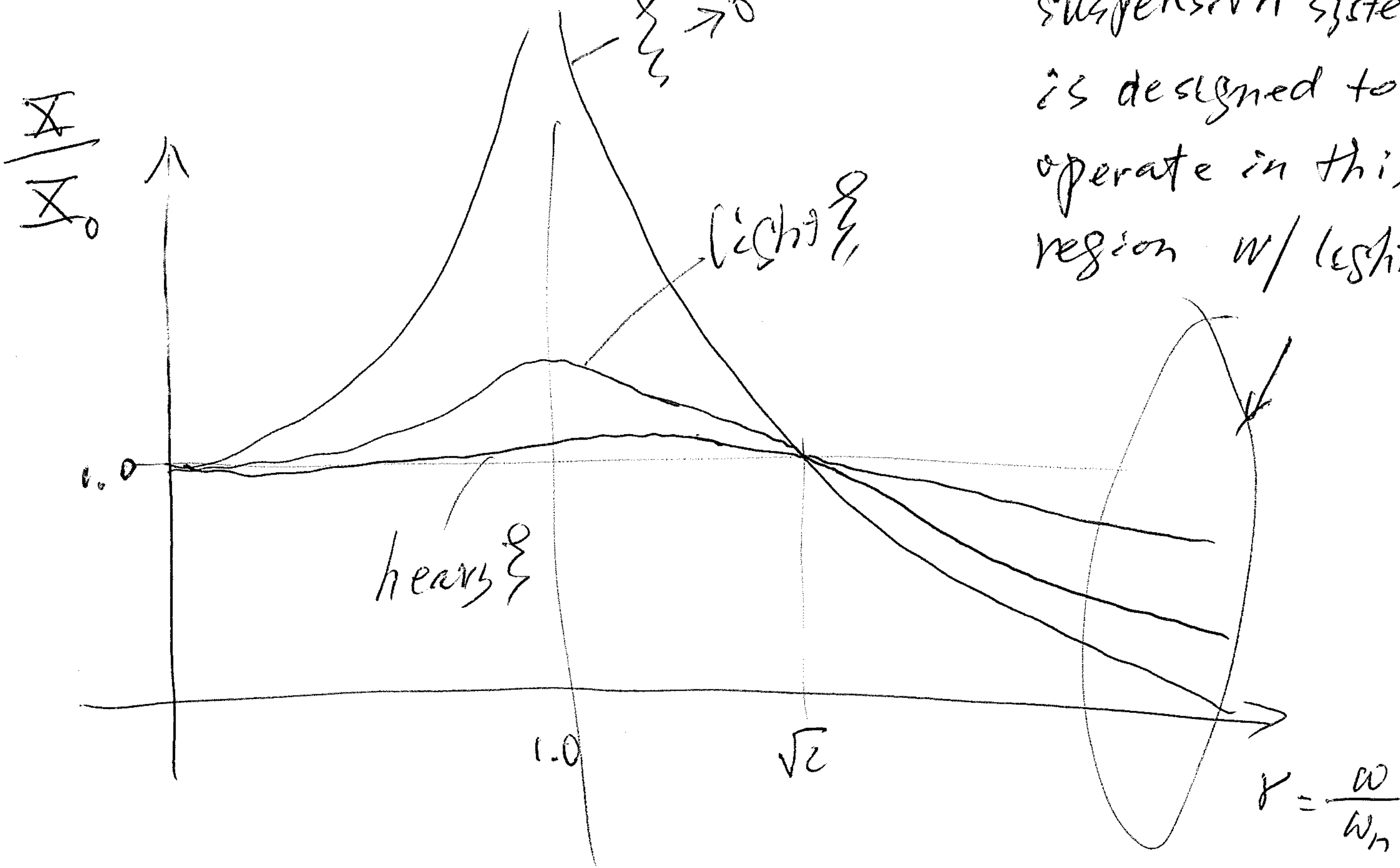
$$\boxed{X = \frac{[1 + (2\xi r)^2]^{1/2}}{[(1-r^2)^2 + (2\xi r)^2]^{1/2}} X_0}$$

Define

$$\frac{X}{X_0} = \text{motion transmissibility.}$$

$$\boxed{\frac{X}{X_0} = \frac{[1 + (2\xi r)^2]^{1/2}}{[(1-r^2)^2 + (2\xi r)^2]^{1/2}} = T_R} \quad (B)$$

— relate to r and ξ by the same expression as the force transmissibility in previous rotating machinery problem

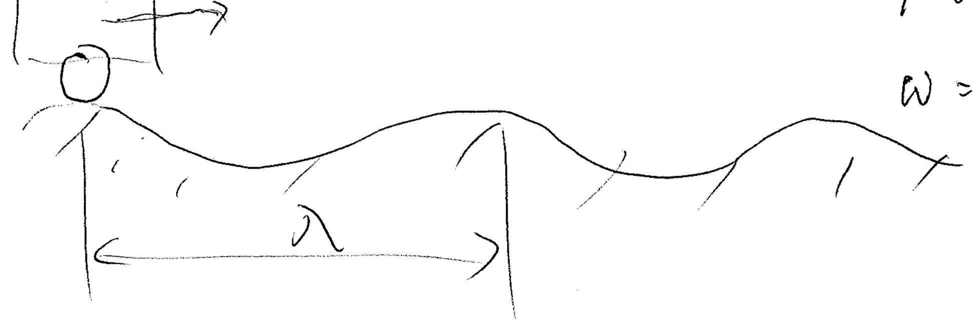


suspension system is designed to operate in this region w/ light ξ.

Typical design:

$$\omega_n = \sqrt{\frac{k}{m}} \sim 2.0\ Hz$$ (lower for luxurious car)

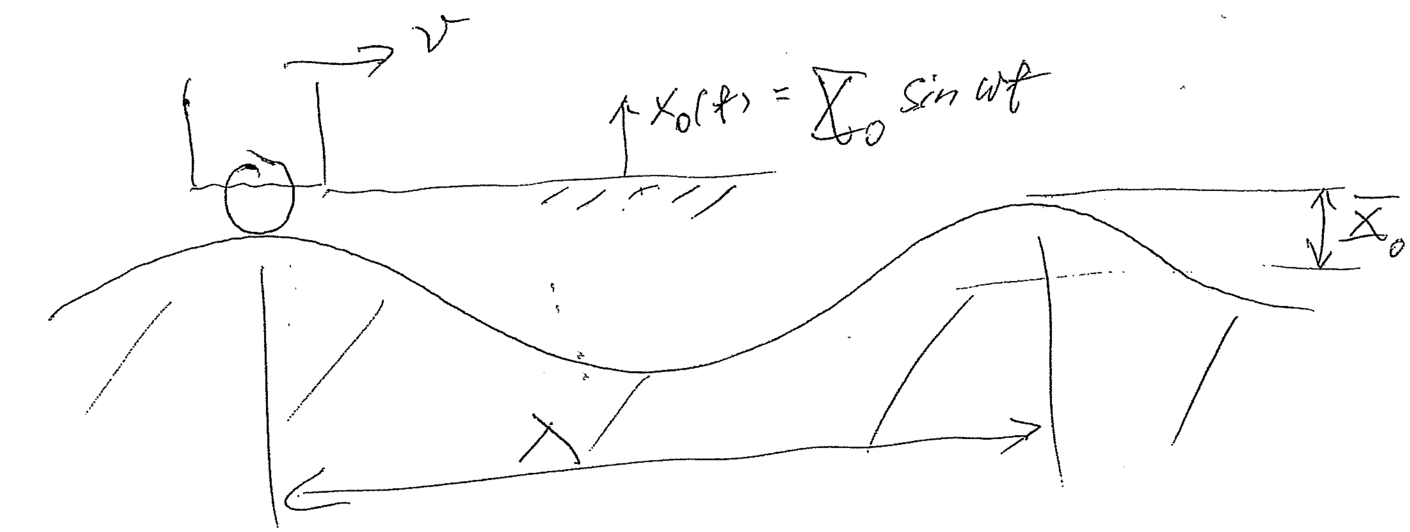
$$\xi \sim 0.5.$$



$$Tv = \lambda$$

$$\omega = \frac{2\pi}{T} = \frac{2\pi v}{\lambda}$$

Application to road pavement assessment



$\because$ $vT = \lambda$ and $T = \dfrac{2\pi}{\omega}$

So, $\boxed{\omega = \dfrac{2\pi v}{\lambda}}$ rad/s

Let $\omega_n = 2\,\text{Hz} = 4\pi$ rad/s

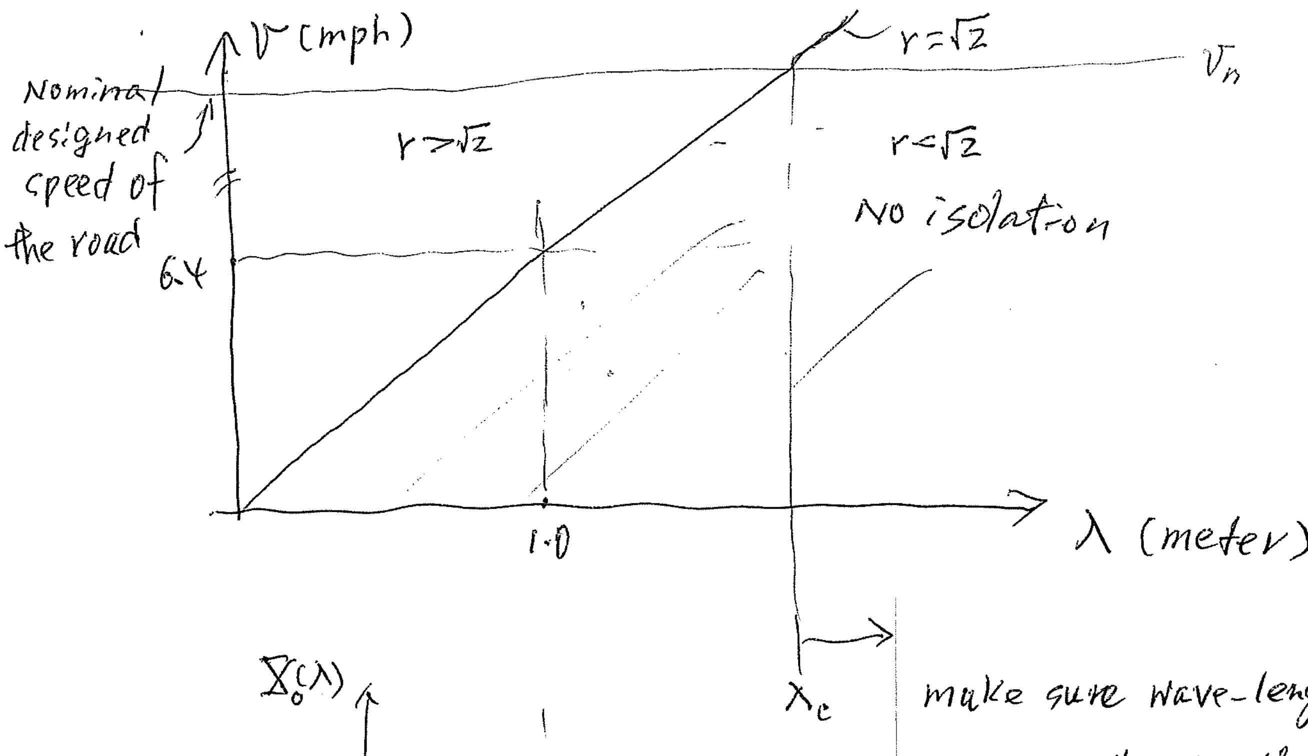
and $r = \sqrt{2}$

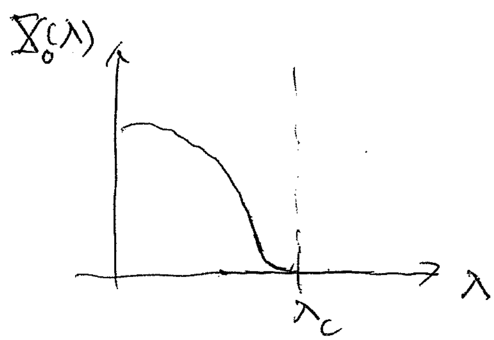
Then

$$\frac{\omega}{\omega_n} = \frac{2\pi v/\lambda}{4\pi} = \sqrt{2}$$

$\Rightarrow$ $v = 2\sqrt{2}\,\lambda = 2.83\lambda \ \frac{m}{s} = 6.4\lambda$ mph (λ in meter)

So, need to drive faster than 6.4λ mph to produce $r > \sqrt{2}$ or $X < X_0$





make sure wave-length components in the road longer than λ_c are insignificant.

Highway of $v_n = 65$ mph $\longrightarrow$ $\lambda_c = 10$ m

Parking lot of $v_n = 5$ mph $\longrightarrow$ $\lambda_c = 0.7$ m