Fast Numerical Calculations of EHD Sliding Traction Forces; Application to Rolling Bearings

Based on analytical calculations assuming isothermal elastohydrodynamic (EHD) lubrication conditions, and on curve-fitting of a thermal correction factor, a new formula is proposed to calculate the sliding traction force developed in a concentrated contact, assuming a nonlinear, viscoelastic lubricant and unidirectional sliding. When sliding occurs in the rolling and transverse directions (bidimensional sliding), a simplified numerical method is outlined by which the components of the local shear stress are quickly calculated. The latter method is applied to the calculation of the shear stress and temperature distribution in the ball-raceway contacts of an angular-contact ball bearing. Because of the viscoelastic behavior of the lubricant, a nonzero lateral traction force is obtained.

Introduction

Rolling bearing users are interested in the accurate calculation of the shear stress in rolling contacts to be able to predict the performance of the bearing.

The calculation of traction forces has been treated many times in the literature; but, is still complex and requires much computer time. It has therefore been considered necessary to define simplified numerical methods to solve the mathematics involved. This paper describes two such methods as well as an example of their application.

It is now well accepted that the rheological behavior of a lubricant in an EHD lubrication contact is described by the Maxwell model, which superposes an elastic shear rate $\gamma_E$ and a viscous shear rate $\gamma_v$ [1-6].

\[
\gamma = \gamma_E + \gamma_v = \frac{\tau}{G} + \frac{\tau_0}{\eta} \sinh \left( \frac{\tau}{\tau_0} \right) \quad (1)
\]

$\gamma_v$ can be described by Johnson and Tevaarwerk’s nonlinear viscous relationship as used in equation (1), (reference [1]) and equation (2). At high pressure, large rolling speed or low temperature, elastic effects in the lubricant film can be important and equation (1) has to be used.

At low pressure, low rolling speed, high temperature, or for large elastic shear modulus $G$, $\gamma_E$ becomes negligible compared to $\gamma_v$, and equation (1) is reduced to equation (2), i.e., the lubricant behavior is simply viscous nonlinear:

\[
\gamma_v = \frac{\tau_0}{\eta} \sinh \left( \frac{\tau}{\tau_0} \right) \quad (2)
\]

Thermal effects on the “contacting” surfaces are described by equation (3), (reference [7]).

\[
\Delta \theta_j = \theta_s - \theta_0 =
\]

Integration of the energy equation through the film gives the temperature increase in the film:

\[
\frac{\partial^2 \theta}{\partial x^2} = \frac{-q}{k + \frac{pcu}{k} \frac{\partial \theta}{\partial x}} \quad (4)
\]

where $q$ is the dissipated power per unit volume (usually taken as $q = \tau \gamma_v$). The second term of the right-hand part of equation (4) is the convection term.

Equations (2), (3), and (4) without the convection effect have been solved by Johnson and Greenwood, who showed results in reference [8]. A curve-fitted formula of Johnson’s results is proposed in this paper. This paper also gives an analytical, isothermal average solution of equation (1), which will have to be modified by a thermal correction factor proposed herein.

When sliding occurs in two directions, equation (1) has to be replaced by the system of equation (5).

\[
\begin{align*}
\frac{\partial \gamma_s}{\partial x} &= \frac{\tau_s}{G} + \frac{\tau_0}{\eta} \sinh \left( \frac{\tau_0}{\tau_0} \right) \\
\frac{\partial \gamma_y}{\partial y} &= \frac{\tau_y}{G} + \frac{\tau_0}{\eta} \sinh \left( \frac{\tau_0}{\tau_0} \right) \\
\tau_0 &= \sqrt{\tau_s^2 + \tau_y^2} 
\end{align*} \quad (5a, 5b, 5c)
\]

A simple and fast numerical method to solve equations (3), (4), and (5) is outlined.

These methods are applied in the calculation of the shear stress distribution in an angular-contact ball bearing ball-
raceway contact, where a complex sliding speed distribution is found.

**Method 1: Curve-Fitting of Johnson and Greenwood's Results (reference [8])**

Using average isothermal values of rheological parameter \( \eta_{0, P, \text{iso}} \) and \( \tau_{0, P, \text{iso}} \), Johnson describes graphically the average shear stress \( \tau \) as a function of \( X \), \( \xi \), and \( \chi \) (Fig. 1). The index \( P \) means that the average values with respect to pressure are used. \( X \) is the dimensionless sliding speed

\[
X = \frac{\tau}{
\left( \frac{\eta}{\tau_{0, P, \text{iso}}} \right) \left( \frac{\tau_{0, P, \text{iso}}}{\eta} \right)^{1/2}}
\]

\( \xi \) is associated with the temperature increase on the surface, and contains the half length \( L \) of the contact.

\[
\xi = \frac{k}{h_c} \left( \frac{L}{\pi k_m \rho_m c_m u} \right)^{1/2}
\]

\( \chi \) is associated with the temperature increase through the lubricant film, and contains the temperature-viscosity parameter \( \delta \).

\[
\chi = \frac{1}{\tau_0 h_c} \left( \frac{2k(\eta_{P, \text{iso}})}{\tau_{0, P, \text{iso}}} \right)^{1/2}
\]

where

\[
\eta = \eta_{\text{iso}} \exp(-\delta \Delta \theta)
\]

The isothermal solution is given using Johnson and Tevaarwerks' relationship:

\[
\frac{(\tau_{P, \text{iso}})}{(\tau_{0, P, \text{iso}})} = s h^{-1}(X)
\]

The thermal solution (including the thermal effect on the "contacting" surfaces and in the lubricant film) can be calculated by solving a system of two coupled nonlinear equations (equations (19) and (20) in reference [8]). This is time-consuming and it was found attractive to express the thermal solution \( \tau \) using the isothermal solution with a thermal correction factor \( c_1 \), the latter obtained by curve-fitting.

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**Nomenclature**

- \( a \) = half length of the contact ellipse in the lateral direction for an EHD point contact (m)
- \( b \) = half length of the contact ellipse in the rolling direction for an EHD point contact (m)
- \( c \) = specific heat \((J/kg\cdot°C)\)
- \( c_h \) = thermal correction factor
- \( G \) = shear modulus of the lubricant \((N/m^2)\)
- \( h_c \) = central lubricant film thickness \((m)\)
- \( k \) = thermal conductivity of the lubricant \((W/m\cdot°C)\)
- \( L \) = length of an EHD slice in the rolling direction
- \( P \) = pressure \((N/m^2)\)
- \( Q \) = dissipated power \((W/m^3)\)
- \( t \) = time of transit \((s)\)
- \( T \) = ratio \( \dot{\gamma}/\dot{\gamma} \)
- \( \dot{T} \) = ratio \( \dot{\gamma}/\dot{\gamma} \)
- \( u \) = average rolling velocity \((m/s)\)
- \( u = \frac{U_1 + U_2}{2} \)
- \( U_1, U_2 \) = velocity of each surface \((m/s)\)
- \( X \) = sliding parameter
- \( X = \frac{\eta_{\text{iso}}}{\tau_0} \)
- \( \chi \) = rolling axis
- \( y \) = lateral axis
- \( z \) = normal direction axis
- \( \dot{\gamma} \) = shear rate \((s^{-1})\)
- \( \dot{\gamma}_c \) = shear rate \((s^{-1})\)
- \( \dot{\gamma}_c = \frac{\Delta \dot{\gamma}}{h_c} \)
- \( \dot{\gamma}_s \) = shear rate \((s^{-1})\)
- \( \dot{\gamma}_s = \frac{\Delta \dot{\gamma}}{h_c} \)
- \( \delta \) = thermoviscous parameter \((°K^{-1})\)
- \( \delta' \) = displacement of the centre of pressure \((m/s)\)
- \( \Delta \theta_1 \) = surface temperature increase \((°K)\)
- \( \Delta \theta_2 \) = surface temperature increase \((°K)\)
- \( \Delta \theta_c \) = temperature increase at the middle of the film \((°K)\)
- \( \theta \) = temperature \((°K)\)
- \( \eta \) = dynamic viscosity \((Ns/m^2)\)
- \( \eta_0 \) = dynamic viscosity at zero pressure and temperature \((Ns/m^2)\)
- \( \eta_{\text{app}} \) = apparent dynamic viscosity
- \( \eta_{\text{app}} = \lim_{\gamma \rightarrow 0} \frac{\eta}{\gamma} \)
- \( \theta_0 \) = ambient temperature \((°K)\)
- \( \theta_1 \) = contact temperature \((°K)\)
- \( \theta_c \) = maximum temperature in the middle of the film
- \( \tau \) = lubricant shear stress \((N/m^2)\)
- \( \tau_0 \) = characteristic lubricant shear stress \((N/m^2)\)
- \( \omega_b \) = rotational speed of the ball \((rad/s)\)
- \( \omega_i \) = rotational speed of the inner ring \((rad/s)\)
- \( \omega_c \) = rotational speed of the cage \((rad/s)\)

**Lower index**
- \( c \) = central
- \( E \) = elastic
- \( eq \) = equivalent
- \( iso \) = isothermal
- \( m \) = solid
- \( P \) = average pressure dependent value
- \( V \) = nonlinear viscous

**Upper index**
- \( \ast \) = value at the previous abscissa \((x - \Delta x)\)

---

\[
\frac{(\tau)_p}{(\tau_0)_{p,iso}} = c_1 \cdot sh^{-1}(X)
\]  
(11)

with
\[
c_1 = \exp[-(0.13887 + 1.04415 \xi) X^{-1.43117 - 0.408644}, X^{0.739629 + 0.149129(1)}]
\]  
(12)

The curve-fitted solution can be compared to the exact numerical one on Fig. 1, (dashed line).

The curve-fitting provides a fast computer solution, and can also be used for large sliding speeds, for a visco-elastic lubricant.

It should be noticed that Method 1 (and Method 2 in the following section) is based on the assumption that \( \tau_0 \) does not vary across the lubricant film and is therefore temperature independent. For many lubricants, this assumption has been verified experimentally. For some lubricants, a small temperature dependency has been observed, and in these cases a constant value of \( \tau_0 \) is fixed for each operating temperature. For some lubricants, a small temperature dependency has been observed, and in these cases a constant value of \( f_0 \) does not vary across the lubricant film and is therefore temperature independent. For many lubricants, this assumption has been verified experimentally. For some lubricants, a small temperature dependency has been observed, and in these cases a constant value of \( f_0 \) is fixed for each operating temperature.

Method 2: Analytical Solution of the Visco-Elastic Model, and Curve-Fitting of the Thermal Effects

Isothermal Solution in the Case of Unidirectional Sliding. In the case of longitudinal sliding only, equation (1) can be integrated analytically using the constant parameter \( X \) as defined in equation (6). The solution, developed in reference [9], is a function of \( X \) and the dimensionless time parameter \( T \).

\[
T = t / (\eta / G)
\]
(13)

\[
\frac{(\tau)_p}{(\tau_0)_{p,iso}} = \ln \left( \frac{\sqrt{1 + X^2} + (1 + X)D}{\sqrt{1 + X^2} - (1 - X)D} \right)
\]
(14)

where
\[
D = \frac{1 - \exp(-\sqrt{1 + X^2})}{1 + \exp(-\sqrt{1 + X^2})}
\]
(15)

Expansion series of equations (14) and (15) gives the limiting expression of \( \tau/\tau_0 \) for large and small values of \( T\sqrt{1 + X^2} \).

- for \( T\sqrt{1 + X^2} > 10 \):
  \[
  \frac{(\tau)_p}{(\tau_0)_{p,iso}} = sh^{-1}(X)
  \]
(16)

i.e., the solution is purely viscous.

- for \( T\sqrt{1 + X^2} < 10^{-3} \):
  \[
  \frac{(\tau)_p}{(\tau_0)_{p,iso}} = XT
  \]
(17)

i.e., the lubricant behavior is purely elastic.

Thermal Correction Factor \( c_2 \). The thermal solution is found applying a thermal correction factor \( c_2 \) to the previous isothermal solution (equation 14). The expression \( c_2 \) is similar to that of \( c_1 \) (equation 12), except that the sliding parameter \( X \) is replaced by an equivalent sliding parameter \( X_{eq} \).

\[
X_{eq} = \left( \frac{\tau_{v,E}}{\tau_v} \right) \times X
\]
(18)

where \( \tau_{v,E} \) is the viscoelastic solution described by equation (14) and \( \tau_v \) the viscous solution described by (10). \( X_{eq} \) is introduced in order to prevent \( c_2 \) from being small when elastic effects are large, i.e., when \( \tau_{v,E} < \tau_v \). In this case, \( X_{eq} \) is small, leading to a value of \( c_2 \) close to 1 which is expected for elastic nondissipative behavior. \( c_2 \) has to be considered as a numerical tool only. For viscous behavior, \( c_2 \) is equal to \( c_1 \).

Method 3: Fast Numerical Solution For the Bidimensional Sliding Case

When sliding occurs in the rolling and lateral directions (\( \Delta u \) and \( \Delta v \), respectively), equations (5) have to solved simultaneously with equations (3) and (4).

At any ordinate \( y \), the shear stresses \( \tau_x \) and \( \tau_y \) are calculated along the x-axis.

Appendix 1 shows how the solution of the system of the differential equations (5) can be reduced to find the root \( \tau_v \) of the function \( F \).

\[
F \left( \frac{\tau_x}{(\tau_0)_h} \right) = A \cdot \frac{\tau_x}{(\tau_0)_h} + B \cdot sh \left( \sqrt{1 + F^2} \frac{\tau_x}{(\tau_0)_h} \right) - C = 0
\]
(19)

In equation (19), \( A, B, C, \) and \( F \) are defined as follows:

\[
A = \left( \frac{u(\tau_0)_h}{(\tau_0)_h} \right) \Delta x \gamma_x
\]
\[
B = \left( \frac{\tau_0}{\gamma_x \sqrt{1 + F^2}} \right)
\]
\[
C = 1 + A \cdot \frac{\tau_x}{(\tau_0)_h}
\]
\[
F = \frac{\gamma_x}{\gamma_y} + A \cdot \frac{\tau_y}{(\tau_0)_h}
\]
\[
1 + A \cdot \frac{\tau_y}{(\tau_0)_h}
\]
(20)

where \( \gamma_x \) and \( \gamma_y \) are calculated as \( \Delta u/h_c \) and \( \Delta v/h_c \), respectively.

\( (\eta)_s \) means that the average valued viscosity with respect to temperature through the film has to be used; the pressure being the one calculated at the point of coordinate \((x,y)\). The same is applied for \( (G)_s \) and \( (\tau_0)_h \).

The average values are found by calculating the maximum increase of temperature \( \Delta \theta_c \) through the film. \( \Delta \theta_c \) is defined as the increase of temperature in the film from the surface temperature \( \theta_s \). \( \Delta \theta_c \) is the increase of temperature obtained by equation (3). For example can be calculated assuming a triangular temperature profile through the lubricant film.

\[
(\eta)_s = \eta_{iso} \exp \left[ -\Delta \theta_c \right] \frac{1}{b \Delta \theta_c} (1 - \exp(-\Delta \theta_c))
\]
(21)

In reference [9], equation (4) has been integrated numerically and it was found that the exact solution of the temperature profile through the film often approximated a triangular shape. Equation (4) was therefore also integrated analytically in reference [9], assuming a triangular temperature profile and a constant dissipated power \( q \) per unit volume. \( q \) is expressed as:

\[
q = \tau_v
\]
(22)

From reference [9]:

\[
\Delta \theta_c = \left\{ \frac{q}{k} - \rho c u \left( \frac{-\Delta \theta_t - \Delta \theta_s}{\Delta x} \right) \right\} \frac{h_t^2}{8 + E \Delta \theta_t^2}
\]
(23)

where

\[
E = \frac{\rho c u h_t^2}{k 12 \Delta x}
\]

Since a triangular temperature profile through the lubricant film is assumed, equation (3) is used with \( \partial \theta / \partial z \) on surface = \( 2 \Delta \theta_c / h_c \).
Fig. 2  Comparisons between the solution obtained using method 1, 2 and 3 and experimental results (reference [12]).

![Diagram](http://tribology.asmedigitalcollection.asme.org/)

Fig. 3  The angular ball bearing contact studied; coordinate system of the inner ring elliptical contact. Data in Table 1.

Iterations of equation (3), equations (19) and (23) give a fast numerical solution for $\tau_x$.

To summarize, the solution $\tau_x$ at any point is found by iteration of equations (19) and (23), after having calculated $\Delta \phi$, by equation (3). Average values of $\eta$, $G$, $r_0$ are used in equation (19) as proposed in equation (21).

$$\tau_x = F r_g$$  \hspace{1cm} (24)

Once the solution at point $(x,y)$ is obtained, an increment $\Delta x$ is performed on $x$.

The solution, which can be called a full point-by-point solution, can also be used in the unidirectional sliding case ($F=0$) and is tested in the next section against experimental results. The average solution obtained with Method 2 is also tested.

**Comparisons Between the Different Methods [1-3] and Experimental Results**

The three methods described are compared in Fig. 2 using published data, reference [12]. A cylinder of radius 71.4 mm is in contact with a crowned disc of radii 33 mm and 22.57 mm such that the contact area is a circle. The applied load of 206 N gives rise to a pressure of 1 GPa.

The rolling speed is 1.2 m/s and the lubricant, Shell “Turbo 33,” used at 35°C has the following rheological properties:

$\eta = 0.084 \exp(1.97 \times 10^{-6} P - 4.76 \times 10^{-2}(\theta - 303 \text{°K}))$ Pa.s

$G = 8.46 \times 10^7 \text{Pa}$; $r_0 = 4.36 \times 10^6 \text{Pa}$

A unidirectional sliding speed $\Delta w$ occurs in the rolling direction. The calculated traction coefficients of friction $\mu$ versus the slip/roll ratio for the three described methods are plotted in Fig. 2. $\mu$ is defined as the ratio between the traction force and the load. The traction force was calculated as the integral over the circular point contact of the shear stress $\tau_x$.

<table>
<thead>
<tr>
<th>$R_{1g}$</th>
<th>$R_{2g}$</th>
<th>$D_w$</th>
<th>$R_{og}$</th>
<th>$R_{od}$</th>
<th>$\alpha$</th>
</tr>
</thead>
<tbody>
<tr>
<td>(10^-m)</td>
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<td>(m)</td>
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<td>rad</td>
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<tr>
<td>27.57</td>
<td>44.93</td>
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</tr>
<tr>
<td>$\omega_f$</td>
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<td>rad/s</td>
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<td>628.3185</td>
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<td>1246.987</td>
<td>8007</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Experimental points obtained by Tevaarwerk in reference [12] are also plotted in Fig. 2, although these values of $\mu$ were obtained under conditions of lateral side slip:

- It can be observed in Fig. 2 that the calculated values of $\mu$ using Method 3 are very close to the experimental values of $\mu$, which puts confidence in the full point by point solution. This good agreement is to be expected because the above values of $G$ and $r_0$ have been obtained by curve-fitting of experimental results in reference [12] or reference [1].

- Methods 1 and 2 give identical results at large sliding speed/roll ratios because elastic effects are negligible at large $X$ values, as described in equation (16). Furthermore, these results are within 20 percent (in this example) of the results obtained by the full point-by-point solution of Method 3, which can be considered as the reference. In some other cases, a better correlation was found between the results of Methods 1 and 3 in the pure nonlinear viscous cases (reference [8]).

At small sliding speed/rolling speed ratios, elastic effects in the lubricant are important and a reduction in friction is calculated using Methods 2 and 3. Method 1, ignoring elastic effects, cannot be used in this case as the calculated friction coefficient is too large. Results obtained using Method 2 are of a correct order of magnitude, although friction is slightly too large compared to the results of Method 3. When using a shear modulus which varied locally with the pressure in the contact, the discrepancies observed in the elastic zone of the traction curves are smaller.

**Application to Rolling Bearings**

A computer program has been written to calculate the traction forces and power losses in roller bearings. Although the traction forces calculation has been considerably simplified using the method 3, their full calculation is rather complex. This is because the kinematics of each rolling element have to be individually defined by solving the equilibrium of forces and moments about the rolling element.

The last refinements concerning film thickness calculations and lubrication regime criteria, reference [10], are included. In the following example, the full solution is used to describe the shear stress and temperature distribution in an angular-contact ball bearing inner raceway/ball contact. The data concerning the geometry, kinematics and load are given in Fig. 3 and Table 1.

As mentioned above, the kinematics have been chosen such that the traction forces acting on the ball are close to equilibrium, assuming only ball/raceway contacts. The rolling speeds is 10.8 m/s.

The load gives rise to a maximum Hertzian pressure of 3 GPa. At this high pressure, the rheology of the lubricant has not been verified. The previous values of $G$ and $r_0$ are nevertheless used because the calculated local maximum traction coefficient in the inner raceway contact is 8 percent, which is acceptable as an illustrative example. The sliding speed varies at each point of the contact, and has a complex distribution because of different superimposed effects:

- the Hertzian contact is not flat, but curved in the rolling and lateral directions. The sliding speed therefore varies parabolically in the two directions; and especially along the $y$ direction, positive and negative values of $\Delta w$ are found.

**Table 1 Geometrical, kinematic and loading data applicable to Fig. 3**

<table>
<thead>
<tr>
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the contact angle creates a sliding speed distribution identical to the one obtained by spinning.

- EHD lubrication effects shift the position of the centre of pressure slightly towards the inlet zone by a small distance shown in Fig. 3 as described in reference [3] and [13]. The position of the spin axis is therefore not exactly the centre of the elliptical contact.

Figures 4, 5, and 6 (a and b) give the sliding speed distribution Δu and Δv along the main directions x and y at different positions: y = 0, x = 0, y = −0.33a.

Figures 4, 5, and 6 (c and d) give the corresponding shear stresses T_x and T_y distribution.

Figures 4, 5 (e) give the calculated increase of temperature Δt on the "contacting" surfaces and through the film (Δt_c).

Along the center slice, at position y = 0 (Fig. 4) the following comments apply:

- Figure 4 (a) the sliding speed Δu in the x direction is almost constant because curvature effects in the rolling direction are small.
- Figure 4(b) shows typical sliding speed distributions Δu in the y direction obtained by spinning. Because of the effects described above, the sliding speed is not zero at x = 0 but equal to 0.000233 m/s.
- the symmetry of Fig. 4(c) shows that the lubricant behavior is almost viscous nonlinear when calculating T_x.
- but Fig. 4(d) shows us that elastic effects have to be included when evaluating the lateral shear stress T_y. Effectively, the lateral shear stress is almost always positive, although the lateral sliding speed Δv varies almost symmetrically from positive to negative values. For a viscous nonlinear behavior of the lubricant, at x = 0, T_y should be equal to 2.16 10^5 Pa instead of 7.10^6, calculated with the viscoelastic relationship.

- Figure 4(e) gives the temperature increases on the surfaces and through the film of 5°C and 31°C.
At position $x=0$ (Fig. 5), along the $y$ direction, the following comments apply:
- Figure 5(a) shows large sliding speed variations of $\Delta u$ because of curvature effects. Zero sliding speeds are found around the position $y = -0.31\alpha$ and $y = 0.45\alpha$.
- Figure 5(b) shows very small sliding speeds $\Delta u$ due to $\delta'$. This confirms the theoretical and experimental viscoelastic behavior of the lubricant gives a nonzero lateral traction force. This would be due to the symmetrical positive and negative lateral sliding speed distribution. A lateral traction force. This is calculated when spin occurs.
- Figure 5(c) shows that relatively large shear stress components $\tau_y$ are calculated near the rolling lines although the sliding speeds $\Delta u$ are very low; i.e., the elastic effects in the $y$ direction are important. The large values of $\tau_y$ are also explained when interpreting Fig. 6.
- Figure 5(d) shows temperature increases on the surface and through the film of $8^\circ C$ and $72^\circ C$. No increases are found near the rolling lines because sliding speeds are low. The dissymmetries observed in Fig. 5(e) are due to the dissymmetry in $\Delta u$ distribution (see Fig. 5(a)). For negative values of $y$, the sliding speeds $\Delta u$ are calculated and are responsible for the large temperature increases calculated in Fig. 5(e).
- Figure 5(e) shows temperature increases on the surface and through the film of $8^\circ C$ and $72^\circ C$. No increases are found near the rolling lines because sliding speeds are low. The dissymmetries observed in Fig. 5(e) are due to the dissymmetry in $\Delta u$ distribution (see Fig. 5(a)). For negative values of $y$, the sliding speeds $\Delta u$ are calculated and are responsible for the large temperature increases calculated in Fig. 5(e).
- Figure 5(f) shows that the shear stress follows the shear strain $\gamma_y$ rather than the shear rate $\dot{\gamma}_y$, therefore reaches its maximum value when $\gamma_y$ is maximal; i.e., at $x=0$. This is not the case when the behavior is viscoelastic or viscous, as drawn on Fig. 4(d).

Conclusions
Based on the use of average properties of the lubricant, a formula is proposed to obtain directly the thermal, nonlinear viscoelastic sliding traction force due to unidirectional sliding speed, in a line contact or in a slice of a point contact. This formula provides very fast and reasonably accurate results compared to experimental ones.

A simple numerical procedure is also outlined to obtain, at any position of the contact, the two shear stress components, the temperature increases on the surface and the temperature increases through the film. This procedure may be used in a general sliding contact, for example one combining longitudinal, lateral and spinning sliding speeds. In a simple slip configuration, the last method has been tested against experimental points and proven to be very accurate.

For elliptical Hertzian contacts such as those found in angular-contact ball bearings, elastic effects in the lubricant film are important near the rolling lines in the rolling and lateral directions. Elastic effects must be included for accurate calculation of the lateral traction forces when spin occurs.

A viscous nonlinear rheological model, or the use of a constant coefficient of friction, would give rise to a zero lateral traction force. This would be due to the symmetrical positive and negative lateral sliding speed distribution. A viscoelastic behavior of the lubricant gives a nonzero lateral traction force. This confirms the theoretical and experimental conclusions of Gentle and Boness[11], who used an elastic lubricant behavior to explain their experimental observations.

In the simulation described herein, significant temperature increases in the film and on the surface can be determined.

References