Aerodynamics of a Flapping-Perturbed Revolving Wing

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At low Reynolds numbers, revolving wings become less efficient in generating lift for hovering flight due to the increasing adverse viscous effects. Flying insects use reciprocating revolving wings that exploit unsteady aerodynamic mechanisms for lift augmentation. Here, the aerodynamics of an alternative that introduces unsteadiness to the revolving wings through vertical flapping and its potential to improve aerodynamic performance are investigated. The force production and the flow pattern of such flapping-perturbed revolving wing are analyzed via combined experimental and computational investigations. The results show that drag reduction can be produced consistently by a flapping-perturbed revolving wing at zero angle of attack. The reduction is linearly dependent on Strouhal number, and the critical Strouhal number at equilibrium rotating state is similar to that of two-dimensional heaving plates. At positive angles of attack, the flapping perturbation leads to substantial lift augmentation, accompanied by relatively small increase of drag or even minor drag reduction, depending on the Strouhal number and flapping amplitude. Though slightly less efficient at these angles of attack in terms of power loading, flapping perturbations can be used to improve the maximum lift coefficients attainable by revolving wings and thus have potential applications in micro air vehicle designs.

I. Introduction

In the last two decades, the development of micro air vehicles (MAVs) flying at low Reynolds number similar to those of insects and small birds has received wide interest [1–7]. Although comparative studies on the aerodynamic efficiency of three possible configurations of MAVs (i.e., fixed, rotary, and flapping wings) are still scarce and arguably difficult to perform in trackable settings, it has been widely recognized that realizing efficient flight at low Reynolds number hinges on successfully exploiting the complex unsteady aerodynamics from vortex-dominated flow when neither viscous nor inertial forces dominate [8,9]. This is fundamentally different from conventional flight at high Reynolds number, where steady rotary flight using high aspect ratio ($\mathcal{R}$) and low angle of attack ($\alpha_0$) revolving wings, for example, is undoubtedly the most efficient and popular configuration to achieve hovering [10].

Researchers have examined insect flight in great detail for understanding nature’s solutions of low Reynolds number flight, where various unsteady aerodynamic mechanisms for lift augmentation have been observed, quantified, or modeled [11–17]. One of the most prominent mechanisms is the existence of stable leading-edge vortex (LEV) attached to insect wings revolving either in reciprocal or unidirectional fashion [11,16,17]. Unlike helicopters or turbines, the lift generated by insect wings increases up to an angle of attack of 45 deg [16–18]. The existence of stable LEV produces a “Kutta-like” condition around the leading edge and produces a suction force normal to the upper surface, instead of the conventional leading-edge suction [19]. The cause of this stability, although still under

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**Nomenclature**

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Definition</th>
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<tbody>
<tr>
<td>$A^*$</td>
<td>$A, R_s/c$, normalized flapping amplitude</td>
</tr>
<tr>
<td>$A_f, A_p$</td>
<td>flapping amplitude and pitching amplitude, deg</td>
</tr>
<tr>
<td>$\mathcal{R}$</td>
<td>$b/c$, aspect ratio</td>
</tr>
<tr>
<td>$b$</td>
<td>span length, m</td>
</tr>
<tr>
<td>$C_L, C_D$</td>
<td>wind-frame lift and drag coefficients</td>
</tr>
<tr>
<td>$C_p$</td>
<td>aerodynamic power coefficients</td>
</tr>
<tr>
<td>$C_V, C_H$</td>
<td>vertical and horizontal force coefficients in inertial frame</td>
</tr>
<tr>
<td>$c$</td>
<td>chord length, m</td>
</tr>
<tr>
<td>$F, T$</td>
<td>aerodynamic force and torque vectors</td>
</tr>
<tr>
<td>$F_a$</td>
<td>added-mass force, N</td>
</tr>
<tr>
<td>$f$</td>
<td>flapping frequency, Hz</td>
</tr>
<tr>
<td>$h$</td>
<td>$\pi f/c$, reduced frequency</td>
</tr>
<tr>
<td>$k$</td>
<td>$\pi f/c$, normalized two-dimensional plunging amplitude</td>
</tr>
<tr>
<td>$P$</td>
<td>total aerodynamic power, W</td>
</tr>
<tr>
<td>PL</td>
<td>power loading</td>
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<tr>
<td>PL*, PT*</td>
<td>power efficiency of lift augmentation and drag reduction over revolving wing</td>
</tr>
<tr>
<td>$p$</td>
<td>static pressure</td>
</tr>
<tr>
<td>$Re$</td>
<td>$U_e c / \nu$, Reynolds number</td>
</tr>
<tr>
<td>$Ro$</td>
<td>Rossby number</td>
</tr>
<tr>
<td>$R_2, R_3$</td>
<td>radius of second and third moments of wing area</td>
</tr>
<tr>
<td>$S_l$</td>
<td>$2 R_2 R_3 / U_s$, Strouhal number</td>
</tr>
<tr>
<td>$t, T_{\text{ref}}$</td>
<td>$c / U_e$, real time, s, reference time, s</td>
</tr>
<tr>
<td>$\hat{t}$</td>
<td>$t / T_{\text{ref}}$, dimensionless time</td>
</tr>
<tr>
<td>$U_e$</td>
<td>revolving velocity at radius of gyration, m/s</td>
</tr>
<tr>
<td>$u$</td>
<td>velocity vector</td>
</tr>
<tr>
<td>$X, Y, Z$</td>
<td>axes in flapping coordinate system</td>
</tr>
<tr>
<td>$X_r, Y_r, Z_r$</td>
<td>axes in inertial coordinate system</td>
</tr>
<tr>
<td>$X_w, Y_w, Z_w$</td>
<td>axes in pitching coordinate system</td>
</tr>
<tr>
<td>$\Delta r$</td>
<td>angle of attack and effective angle of attack, deg</td>
</tr>
<tr>
<td>$\rho$</td>
<td>density, kg/m$^3$</td>
</tr>
<tr>
<td>$\tau$</td>
<td>$t / T_{\text{ref}}$, dimensionless time</td>
</tr>
<tr>
<td>$\nu$</td>
<td>kinematic viscosity, m$^2$/s</td>
</tr>
<tr>
<td>$\psi, \phi, \alpha$</td>
<td>angles of revolving, flapping, and pitching, deg</td>
</tr>
<tr>
<td>$\Omega$</td>
<td>revolving speed, rad/s</td>
</tr>
<tr>
<td>$\omega$</td>
<td>vector of angular velocity</td>
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In addition to the lift augmentation due to the stable attachment of LEV, insect flight further introduces unsteadiness into the aerodynamics through three-degree-of-freedom flapping wing motion. By periodically reversing the wing revolving direction coupled with rapid pitching, the insects take advantage of a number of unsteady aerodynamic mechanisms [9], such as rotational lift, added mass, and wake capture. Note that, although these unsteady mechanisms could further augment the lift created, from a quasi-steady point of view, flapping wings sacrifice part of the high lift from added mass, and wake capture. Apart from the LEV stability, within a wide range of $\alpha_0$, Reynolds number, and $\alpha$, the aerodynamic force and efficiency of a revolving insect wing were also examined extensively by experiments [17,18] or simulations [28,29].

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In this work, motivated by the preceding hypothesis, we investigate the effects of flapping perturbation applied to a revolving wing with $\alpha R = 3$ and $Re = 1500$ using a robotic wing operating in mineral oil. In particular, we aim to identify the condition for potential lift augmentation and drag reduction as well as their dependence on Strouhal number and the corresponding flow patterns. The remainder of this paper is organized as follows. In Sec. II, we describe the experimental setup, coordinate system, and wing kinematics in detail, followed by the description of data processing and numerical methods used. In Sec. III.A, we present the results for the aerodynamic performance of a steadily revolving wing, which serves a benchmark case for the rest of analysis. Then, flapping perturbations, modeled by Strouhal number and normalized flapping amplitude $A^*$, are applied to a revolving wing at three critical $\alpha_0$ (0, 20, and 45 deg), and the resulting aerodynamics are presented in Secs. III.B–III.E. We gain further insights into the flow patterns and the vortex dynamics of the flapping-perturbed revolving wing (FP-RW) by means of computational fluid dynamics (CFD) method, based on which the effects of flapping perturbations are explained in detail. Then, in Secs. III.F–III.H, we compare the results of FP-RW with the symmetric breaking phenomenon of a heaving plate and the fluttering behavior of a flexible plate. More discussion on the improvement of aerodynamic performance due to the unsteadiness introduced by flapping is provided. Finally, conclusions drawn from the study are presented in Sec. IV.

II. Materials and Methods

A. Experimental Setup

The complete experimental setup including the robotic wing, motion control, and data acquisition are described in Fig. 1. The robotic wing is servo-driven and has three degrees of freedom (DOFs), i.e., revolving, flapping, and pitching, which are controlled by servos 1, 2, and 3, respectively. Each servo (XM450-W350-R, Robotics) consists of a DC motor, microcontroller, magnetic encoder, and gear system, and it uses an embedded proportional–integral–

![Fig. 1 Schematics of experimental setup: a) robotic wing, b) schematic setup, and c) a flowchart for motion control and data acquisition.](image-url)
derivative (PID) controller for position, velocity, or acceleration tracking. The errors in kinematic angles are calculated according to an encoder resolution of 1 deg/4096. A six-axis force/torque sensor (Nano17-IP65, ATI) is mounted between the output shaft of servo 3 and the wing root, and it measures the total fluid and inertial force/torque acting on the wing. The model was immersed into a 0.8 × 0.8 × 0.8 m acrylic tank filled with mineral oil (white mineral oil, Tulco, density 826 kg/m³, viscosity 6 cSt). The Reynolds number, defined as $U_g c/\nu$, is 1500. Here, $U_g = 0.15$ m/s is the revolving velocity at radius of gyration. The temperature of mineral oil was maintained within 20 ± 2°C, resulting in a narrow deviation of Reynolds number within ±100. The relative error due to boundary effect was estimated to be less than 1% in terms of the equations introduced by Dickinson et al. [12].

The motion control was implemented using a real-time target machine (performance real-time target machine, Speedgoat). As shown in Fig. 1c, the motion control model was first developed in Simulink (The MathWorks) and then downloaded into the target machine. RS-485 serial communication at a rate of 500 Hz was used between the target machine and servos via IO-323 interfacing board (Speedgoat). The instantaneous position of each servo was measured by its internal magnetic encoder and then used for positional tracking, with the reference trajectory given by the target machine. The analog signals from the Nano17 at each sampling instant were acquired by the target machine and converted into digital signals using the analog-to-digital converter on the IO-323.

The rectangular wing model is made of a rigid flat acrylic plate with chord $c = 0.06$ m, span $b = 0.18$ m, and thickness about 0.025$c$. The offset ($\Delta r$) between wing root and the axis of rotation was 0.9$c$. The base of the wing was mounted on the tool side of the Nano 17 at its midchord position. To ensure the rigidity of the wing during motion, a 2 mm half-round carbon-fiber rod was attached to both sides of the wing surface along the span at midchord position. The radius of the second and third moments of the wing area are given by the following equations:

$$R_2 = \frac{1}{bc} \int_0^{\Delta r+b} r^2 c(r) \, dr$$  

$$R_3 = \frac{1}{bc} \int_0^{\Delta r+b} r^3 c(r) \, dr$$

B. Coordinate System and Kinematics

A coordinate system with five coordinate frames is used to describe the wing kinematics, as shown in Fig. 2. An inertial frame ($OX,Y,Z$) is first introduced, followed by three origin-shared consecutive rotating intermediate frames, i.e., revolving frame ($OX,Y,Z_r$), flapping frame ($OX,Y,Z_f$), and pitching frame ($OX,Y,Z_p$). The angles of revolving, flapping, and pitching ($\psi, \varphi, \alpha$) are defined about $Y_r$, $Z_f$, and $X_f$, respectively. Within a closer slice in the flapping frame, an additional wing-fixed frame ($OX_a,Y_a,Z_a$) is attached at the wing root by offsetting the pitching frame $\Delta r$ along $X_f$. Based on this coordinate system, the wing kinematics is prescribed as follows:

$$\psi = \Omega t$$  

$$\varphi = -A_f \cos(2 \pi f t)$$  

$$\alpha = \alpha_0$$

The wing revolves at a constant angular speed $\Omega$, whereas the flapping motion is sinusoidal. The flapping frequency and amplitude are $f$ and $A_f$, respectively. $\alpha_0$ represents the pitching angle (or angle of attack), which is fixed for each trial. The total revolving angle is set at 1440 deg (four revolutions) to ensure a fully developed wake. A
ramp-function transit from the rest to the prescribed revolving and flapping motion [Eqs. (2–5)] was applied within the first 120 deg revolving angle to eliminate any abrupt change in wing velocity. Nonetheless, only 1.5 to 3.5 revolutions were considered in our analysis when the wake was fully developed. (Details please refer to Fig. 3a in Sec. III.A.)

The following dimensionless numbers are used in our analysis:

\[ \mathcal{R} = \frac{b f}{c} \]  
\[ R e = \frac{U_g c}{u} \]  
\[ S t = \frac{2 f A_f R_2}{U_g} \]  
\[ k = \frac{\pi f c}{U_g} \]  
\[ A^* = \frac{A_f R_2}{c} \]  

The frequency \( f \) and magnitude \( A_f \) of flapping perturbation are calculated based on specified values of \( k \) and \( A^* \). The parameters and their values used in this study are summarized in Table 1. Note that our definition of Reynolds number only includes the revolving velocity for both revolving wings and flapping-perturbed revolving wings. The Reynolds number of a FP-RW can be up to 3000 if the flapping velocity is considered. However, based on the studies of Lentink and Dickinson [17] and Wu et al. [29], the Reynolds number dependence of lift and drag coefficients of flapping or revolving wings is less significant when the Reynolds number is over 1000. Thus, we believe the increase in Reynolds number due to flapping motion is less significant when the Reynolds number is over 1000.

Fig. 3 in Sec. III.A.)

\[ \alpha_e \text{ is defined as the angle between the apparent flow velocity } U_f \text{ and } U_\infty. \]

Similar to previous studies [29,44], the effective wing angle of attack \( \alpha_e \) is defined as the angle between the apparent flow velocity and the chord. As shown in Fig. 2b, \( \alpha_e \) can be defined as

\[ \alpha_e = -\arctan \left( \frac{U_f(r)}{U_\infty(r)} \right) + \alpha_0 \]  

where \( U_f(r) \) and \( U_\infty(r) \) are the flapping and revolving velocity at a spanwise location \( r \). According to the prescribed wing kinematics, \( \alpha_e \) can be calculated as follows:

\[ U_f(r) = 2 \pi f r A_f \sin(2\pi f t) \]  
\[ U_\infty(r) = \Omega r \cos(\phi t) = \Omega r \cos(-A_f \cos(2\pi f t)) \]  
\[ \alpha_e(t) = -\arctan \left( \frac{2 \pi f A_f \sin(2\pi f t)}{\Omega \cos(-A_f \cos(2\pi f t))} \right) + \alpha_0 \]

Based on the definition of \( \alpha_e \), the orthogonal decomposition of resultant force coefficient \( C_f \) in the flapping frame is illustrated in Fig. 2b. The decomposition at zero \( \alpha_0 \) is symmetric during upstrokes and downstrokes, whereas two possible conditions appear during upstrokes when \( \alpha_0 \) increases, as shown in Fig. 2b. A negative \( \alpha_e \) can be generated when the magnitude of \(-\arctan(U_f(r)/U_\infty(r))\) is larger than \( \alpha_0 \) (upstroke scenario 1 in Fig. 2b). Given a smaller \(-\arctan(U_f(r)/U_\infty(r)), \alpha_e \) can also be positive (upstroke scenario 2 in Fig. 2b). Here, lift and drag coefficient \( (C_L, C_D) \) is referred to the apparent velocity \( U_f(r) \), whereas \( (C_{L_f}, C_{D_f}) \) denotes the force coefficients projected to \( Y_f \) and \( Z_f \).

C. Force Measurement

The sensor that measured the force and torque acting on the wing has sensing ranges for force and torque of \( \pm 12 \text{ N} \) and \( \pm 120 \text{ N} \cdot \text{mm} \), with resolutions of \( 1/320 \text{ N} \) and \( 1/64 \text{ N} \cdot \text{mm} \), respectively. The calibration and postprocessing of raw force and torque data were conducted in MATLAB. A fourth-order Butterworth low-pass filter was used to remove noise from raw data. The cutoff frequency was at least five times the fundamental frequency in each trial. The fundamental frequency of each trial is defined as the flapping frequency in flapping-perturbed revolving wings and revolving frequency in pure revolving cases, respectively. Samples of raw and filtered data for a pure revolving wing at 45 deg and a FP-RW (\( St = 0.33, A^* = 0.9, k = 0.58, \) and \( \alpha_0 = 45 \text{ deg} \)) are plotted in Fig. 4a. Note that our choice of filtering frequency may remove some fluid signals with higher frequencies, for example, that related to the breakup of the wing’s boundary layer, the physics of which are not investigated in the current study. Each trial was repeated three times, and good repeatability was observed among trials and among separate cycles within the same trial for the representative FP-RW (defined above), as shown in Figs. 4b and 4c.

To evaluate the uncertainty of force/torque measurements, we repeated a test revolving case (\( \alpha_0 = 45 \text{ deg} \)) for 12 times. The mean results of multimeasures were quickly converged over first three measurements. Assuming a Gaussian distribution, 99.7% statistic confidence intervals were \( \pm 0.05 \% \) for \( C_V \) and \( \pm 0.07 \% \) for \( C_H \). The force/torque measurements were also compared with those obtained from CFD simulation. (Details please refer to Fig. 3b in Sec. III.A.)
D. Data Processing

The total force/torque measured by the sensor contains gravity, buoyancy, inertial, and aerodynamic components. The resultant force/torque due to gravity and buoyancy \( F_G;T_G \) is constant in the fixed frame, first measured when \( \psi;\phi;\alpha = 0 \) and is denoted as \( F_G;0;T_G;0 \). It is then transformed to the wing-fixed frame \( F_G;0;Τ_G;0 \rightarrow F_G;0;Τ_G;0 \) using the rotation matrices \( R_{F,0\rightarrow t} \) and \( R_{T,0\rightarrow t} \). The subscripts 0 and \( t \) mean that the force and torque are at rest or during the motion, respectively:

\[
F_{G,t} = R_{F,0\rightarrow t} F_{G,0} \tag{15}
\]

\[
T_{G,t} = R_{T,0\rightarrow t} T_{G,0} \tag{16}
\]

\[
R_{F,0\rightarrow t} = \begin{bmatrix}
\sin \phi & 0 \\
0 & \cos \phi \cos \alpha \\
0 & \cos \phi \sin \alpha 
\end{bmatrix}, \quad
R_{T,0\rightarrow t} = \begin{bmatrix}
0 & 0 \\
0 & -\cos \phi \sin \alpha \\
0 & \cos \phi \cos \alpha 
\end{bmatrix}
\]

Ideally, the gravity and buoyancy were considered to mostly generate a force \( F_G \) in \( Y_i \) and a torque \( T_G \) in \( Z_i \) when \( \psi;\phi;\alpha = 0 \). In our experiments, the force and torque components in other inertial axes when \( \psi;\phi;\alpha = 0 \) were an order of magnitude smaller, and the effect of \( \Delta r \) during transformation was also ignored. Therefore, the gravity and buoyancy effects were excluded from the total force/torque and then the remaining force/torque in the wing-fixed frame \( F_{m,t};T_{m,t} \) and the remaining force/torque in the wing-fixed frame \( F_{m,t};T_{m,t} \) were translated to those with respect to the center of rotation (i.e., to the pitching frame) \( F_{p,t};T_{p,t} \):

\[
F_{w,t} = F_{m,t} - F_{G,t} \tag{17}
\]

\[
T_{w,t} = T_{m,t} - T_{G,t} \tag{18}
\]

\[
F_{p,t} = F_{w,t} \tag{19}
\]

\[
T_{p,t} = T_{w,t} + \Delta r_t \times F_{w,t} \tag{20}
\]

Finally, the force and torque in the inertial frame were obtained using a coordination transformation with a series of rotations \( (R_{p\rightarrow f,t};R_{f\rightarrow r,t};R_{r\rightarrow i,t}) \):

\[
(F_{f,t};T_{f,t}) = R_{p\rightarrow f,t}(F_{p,t};T_{p,t}) \tag{21}
\]
\[ (F_{r,t}, T_{r,t}) = R_{f \rightarrow r,i}(F_{f,t}, T_{f,t}) \]  
\[ (F_{i,t}, T_{i,t}) = R_{r \rightarrow i, f}(F_{r,t}, F_{p,t}) \]

where the subscripts \((p, f, r, i)\) indicate the pitching frame, flapping frame, revolving frame, and inertial frame, respectively.

The inertial force and torque were estimated by repeating the experiments in the air, and then the measured force and torque were subtracted from those obtained from experiments in fluids. The subtraction of gravity was also performed for the air measurements. The procedure of data processing is summarized in Fig. 4a. The instantaneous aerodynamic power is calculated as follows:

\[ P_t = T_{i,t} \cdot \omega_{i,t} \]  

where \(\omega_{i,t}\) is the angular velocity in the inertial frame, a transformation of revolving, flapping, and pitching velocity according to the orientation \((\psi, \phi, \alpha)\). Note that the reference angular velocity was used in the calculation because the difference between the actual and reference trajectories were negligible. Samples of reference and actual trajectories are shown in Fig. 4c:

\[ \begin{bmatrix} \dot{\psi}_i \\ \dot{\phi}_i \\ \dot{\alpha}_i \end{bmatrix} \]

where \(\omega_{i,t} = R_{w,t} \begin{bmatrix} \dot{\psi}_i \\ \dot{\phi}_i \\ \dot{\alpha}_i \end{bmatrix}\)

\[ R_{w,t} = \begin{bmatrix} 0 & \sin \psi & \cos \psi \sin \psi \\ 1 & 0 & \sin \phi \\ 0 & -\sin \psi & \cos \psi \sin \psi \end{bmatrix} \]

The following dimensionless scheme for vertical and horizontal force coefficients \((C_V, C_H)\) is used (similar to Usherwood and Ellington [16] and Kruyt et al. [18]). The y-axis components of force and torque in the inertial frame are used:

\[ C_V = \frac{2F_{i,y}(t)}{\rho \Omega^2 R_i^2 bc} \]  
\[ C_H = \frac{2T_{i,y}(t)}{\rho \Omega^2 R_i^3 bc} \]  
\[ C_F = \frac{2P_t}{\rho \Omega^2 R_i^3 bc} \]

Aerodynamic efficiency was measured based on power loading (PL), a dimensionless ratio defined as \(PL = C_V / C_F\). The coefficients with an overbar denote the cycle-averaged values:

\[ PL = \frac{\bar{C}_V}{\bar{C}_F} = \frac{\bar{F}_{z,y} U_g R_i^3}{\bar{P}} \]

To further measure the performance of the flapping-perturbed wing, we define two additional efficiency parameters: PL* and PT*. PL* measures the augmentation of \(\bar{C}_V\) relative to the changes in \(\bar{C}_p\) [Eq. (30)] and therefore the power efficiency of lift augmentation with respect to the revolving wing. PT* measures the reduction of \(\bar{C}_H\) relative to the changes in \(\bar{C}_p\) [Eq. (31)] and therefore the power efficiency of drag reduction with respect to the revolving wing:

\[ PL^* = \frac{\bar{C}_V - \bar{C}_V^r}{\bar{C}_p - \bar{C}_p^r} \]  
\[ PT^* = \frac{\bar{C}_H - \bar{C}_H^r}{\bar{C}_p - \bar{C}_p^r} \]

where \(\bar{C}_V^r, \bar{C}_H^r,\) and \(\bar{C}_p^r\) denote the coefficients of benchmark revolving cases. Note that, for a better comparison in aerodynamics, the reference velocity \(U_g\) is constant in obtaining dimensionless coefficients for both a pure revolving wing and a flapping-perturbed revolving wing.

### E. Numerical Method

In addition to the experiments, numerical simulation was used to simulate the flowfield generated by a revolving wing and a flapping-perturbed revolving wing, given the geometry and kinematics in experiments. Based on the artificial compressibility method developed by Rogers et al. [46,47], an in-house numerical solver was employed to calculate the velocity and pressure field within the domain. Details on the solver can be found in a previous study [44]. The governing equations for the flow are the three-dimensional (3-D) incompressible unsteady Navier–Stokes equations, as written in the following dimensionless form:

\[ \nabla \cdot \mathbf{u} = 0 \]

\[ \frac{\partial \mathbf{u}}{\partial t} + (\mathbf{u} \cdot \nabla)\mathbf{u} + \nabla \mathbf{P} - \frac{1}{\rho} \nabla \nabla^2 \mathbf{u} = 0 \]

where \(\mathbf{u}\) is velocity vector, and \(p\) is static pressure. The reference time \(T_{\text{ref}}\) is defined as \(c / U_g\), and the dimensionless time \(\tau = t / T_{\text{ref}}\), where \(t\) is the dimensionless time. Another dimensionless time \(\tau\) is introduced to better describe the time course of wing motion over a flapping cycle, as \(\tau = 0\) and \(\tau = 1\) indicate the start of upstroke and the end of downstroke, respectively.

A O-H grid is widely used in 3-D airfoil CFD studies, with the grid size, outer boundary, first grid distance, and time step all well validated in a previous study [44] and summarized in Fig. 5. Specifically, a refined mesh with 81 x 81 x 91 nodes (in normal, chordwise and spanwise directions) and a 0.001 first grid distance is selected. The computational domain size and time step are selected to be 30c and 400, respectively. The numerical method used here has been validated in previous studies for flapping wings, revolving wings, and flapping rotary wings [23,29,44]. For the current study,

\[ \text{O-H Grid} \]

Nodes: 81x81x91
First grid distance: 0.001c
Domain size: 30c
we performed additional validation with our experiments on both revolving wing (Fig. 3) and flapping-perturbed revolving wings (Figs. 7, 10, and 13). Because good agreements of force and torque trajectories were achieved between the experiments and simulations, we used the flowfield obtained from numerical simulations to interpret the evolution of the fluid structure and its correlation with the measured or the simulated force/torque.

III. Results and Discussions

A. Revolving Wing

The aerodynamic performance of the wing undergoing steady revolving motion is first evaluated and will be used as a benchmark for the analysis of flapping-perturbed revolving wings (FP-RWs). The results are summarized in Fig. 3. Both instantaneous vertical and horizontal forces ($C_V$ and $C_H$) are characterized by initial steep increases in the first 120 deg revolution, leading to a peak followed by a slight drop (Fig. 3a). The drop of force after the first revolution is a result of downwash, as the wing sweeps into its wake developed in the former revolution. The wake and the effect of downwash effect become steady after the first 1.5 revolutions, and thereafter $C_V$ and $C_H$ approach steady-state values. The force coefficients for steady revolving wings are calculated based on the averaged forces within the gray block (540 to 1260 deg) and are plotted as functions of $\alpha_0$ as shown in Fig. 3b. Also included are the numerical results of the force coefficients, which agree well with experimental results.

As shown in Fig. 3b, the dependence of both averaged vertical and horizontal forces ($C_V$ and $C_H$) on $\alpha_0$ are similar to those reported in previous studies [18]. The $C_V$ peaks around $\alpha_0 = 45$ deg, with a maximum value about 1.43. $C_H$ increases gradually for the first 15 deg and thereafter keeps a linear increase until 60 deg. As the aerodynamic power is all spent to overcome drag (horizontal force), the $C_V$ shares similar trend with $C_H$. The power loading also exhibits a steep increase before it peaks at 15 deg, indicating that $\alpha_0$ for maximal efficiency is significantly lower than $\alpha_0$ for peak lift.

B. Effects of Flapping Perturbations on Averaged Force Production

The aerodynamics of the flapping-perturbed revolving wing was evaluated at three angles of attack, subjected to flapping perturbation using 14 combinations of normalized flapping amplitude $A^*$ and Strouhal number $St$; the results, together with the revolving benchmark cases, are summarized in Fig. 6. The error bars are shown in vertical and horizontal forces ($C_V$ and $C_H$) representing the standard deviation at each data point. Notably, some error bars in the plot are small and thus blocked by the data points. As shown in Fig. 6a, a reduction of drag is observed consistently by introducing flapping perturbations at $\alpha_0 = 45$ deg. The drag shows a linearly reducing trend over Strouhal number $St$, and no drag exists when Strouhal number is larger than 0.32. The flapping perturbation also results in a small negative lift at $\alpha_0 = 45$ deg, which is possibly due to a slight asymmetry in the setup and the initial condition of wing flapping. The power loading (PL) of

Fig. 6  Cycle-averaged force coefficients and aerodynamic efficiency for flapping-perturbed revolving wings over Strouhal number at various $A^*$ and $\alpha_0$: a) $\alpha_0 = 0$ deg, b) $\alpha_0 = 20$ deg, and c) $\alpha_0 = 45$ deg.
the FP-RW, as well as that of the revolving benchmark, are all close to zero (Fig. 6a). The efficiency of drag reduction is described by $PT^*$ and is also shown in Fig. 6a. $PT^*$ decreases with Strouhal number nearly asymptotically Fig. 6a, as the extra aerodynamic power of a FP-RW over revolving benchmarks significantly increases with Strouhal number. This indicates that the drag reduction through flapping perturbation is the most efficient at an infinitesimally small Strouhal number, which has no practical significance. To better understand the efficiency, we define an alternative efficiency factor for drag reduction:

$$PT^* = \frac{C_H - C_{H0}}{C_P}$$ (33)

Here, we consider the ratio of drag change and the total power, instead of the change of power. The result shows that this efficiency factor peaks at $St = 0.1$. Note that this measure is similar to that used commonly in the pitching–flapping wing studies [32,48]. It is known that the regime of Strouhal number for efficient propulsion of a pitching–flapping wing is 0.2–0.4 [34]. However, for flapping-only airfoils [48], a numerical study shows that the peak efficiency of thrust generation is 0.3 when $kh = 0.35$ ($St \sim 0.06$). Here, $h$ is the normalized flapping amplitude. Therefore, the higher Strouhal number for the peak efficiency of thrust generation by a pitching–flapping wing can be explained by the pitching effect. For FP-RWs considered in the current study, which lacks wing pitching motion, the peak efficiency for thrust generation and the corresponding Strouhal number are comparable to those of flapping-only airfoils (yellow line in Fig. 6a). The discrepancy in peak value and Strouhal number might be a result of 3-D effects.

When $\alpha_0$ increases to 20 deg, as shown in Fig. 6b, augmentation of average lift is apparent when Strouhal number is higher than a critical value, beyond which $C_L$ increases linearly with Strouhal number. A higher $A^*$ corresponds to a larger critical Strouhal number, whereas the slopes of $\Delta C_L / \Delta St$ for all $A^*$ are close to 2. The maximum $C_L$ for an FP-RW is almost twice as that of the revolving wing. However, in most cases, the FP-RW also generates higher drag than that of the revolving wing, except at $A^* = 1.35$ and $St = 0.17$ or 0.33 ($k = 0.19$ or 0.39, enclosed by black circles in Fig. 6b). The increase of drag is also strongly affected by $A^*$ because a larger $A^*$ corresponds to a slower increase of drag with Strouhal number (Fig. 6b). For $A^*= 0.45$ and 0.90, the drag increases linearly with Strouhal number, whereas for $A^*= 1.35$, the drag first reduces slightly and then increases with Strouhal number. Notably, the amount of lift augmentation is more significant than the increment of drag, especially for higher $A^*$. Within the range of Strouhal number, the power loading (PL) of the FP-RW drops rapidly from that of the revolving wing. The relative power loading ($PL^*$) shows an exponential decreasing trend with Strouhal number, which indicates more efficient lift augmentation at lower Strouhal number. Besides, a lower $A^*$ is beneficial for more efficient lift augmentation, which is opposite of that for drag reduction. Overall, the results suggest that, if a lift augmentation with minor increase of drag (or high drag reduction efficiency, Fig. 6b) is required, a flapping perturbation with a higher $A^*$ is preferred. On the other hand, if more efficient lift augmentation instead of higher drag reduction efficiency is needed, a perturbation with a small Strouhal number and $A^*$ is preferred.

The increase in both lift and drag is also found at $\alpha_0 = 45$ deg, and the effect of $A^*$ on lift augmentation is not obvious. The critical Strouhal number for the onset of $C_L$ and $C_{H0}$ increase is around 0.18 (Fig. 6c). The slopes for both $C_L$ and $C_{H0}$ with respect to Strouhal number are about 1.5, which indicate identical increment for both. Compared with the revolving wing, the power loading of the FP-RW in this case linearly decreases over Strouhal number. The absolute values of all $PL^*$ and $PT^*$ within the Strouhal number investigated are slightly greater than 0, except for FP-RWs at $k = 0.19$ and $A^* = 0.9$ and 1.35 ($St = 0.11$ and 0.17), where there appears to be a slight decrease in $C_L$ and $C_{H0}$, as shown in black circles.

C. Instantaneous Force and Flow Pattern at 0 Degrees Angle of Attack

To further understand the effects on fluid dynamics due to flapping perturbation, here we analyze the instantaneous forces and the corresponding flow pattern. As an example, only representative cases with $St = 0.56$ and $A^* = 0.9$ ($k = 0.97$) for all $\alpha_0$ are analyzed here because the drag reduction and lift augmentation are more significant at higher Strouhal number $St$. In Fig. 7a, apart from a slight phase offset, the instantaneous lift $C_L$ is mostly negative during upstrokes and positive during downstrokes. As expected, the downward and upward force peak at midupstrokes and middownstrokes, respectively, and the magnitude of each peak is comparable, therefore resulting in a near-zero cycle-averaged lift. The numerical results agree well with those of experiments, except for a small delay at the downward force peak.

However, compared to a revolving wing, an increase of instantaneous horizontal force $C_H$, which represents a reduction of drag or generation of thrust during both up and downstrokes, is an apparent result of flapping perturbation (Fig. 7b). The alteration of $C_H$ due to flapping is consistent in both experiment and simulation. The aerodynamic power $C_P$ peaks around each midstroke, which follows the trend of flapping velocity. This is not surprising because most aerodynamic power is consumed by flapping perturbations, whereas that due to revolving is negligible. The magnitude and orientation of force vectors (derived from $C_L$ and $C_H$) are plotted in Fig. 7g. The diagram is stretched horizontally to clearly distinguish the wing chord section and force vector at each time step, whereas the angles between the apparent velocity and the wing chord are preserved.

A wing revolving with $\alpha_0 = 0$ deg only subjects to drag without creating any lift. With symmetrically vertical flapping perturbation, the wing generates thrust, which can be partly explained by examining the effective angle of attack ($\alpha_e$). When a flapping is introduced, $\alpha_e$ and the resultant forces are altered substantially by the change of apparent velocity. Specifically, a nonzero $\alpha_e$ leads to a lift component of force that points forward for thrust generation (similar to Knoller–Betz effect [49]), and the drag, which is opposite to the apparent wing velocity, is also deflected toward the vertical direction. As a result, the geometric change of $\alpha_e$ and the apparent velocity cause a smaller negative horizontal force projection (i.e., reducing drag) or a positive horizontal force projection (increasing thrust).

Note that the force acting on a wing undergoing unsteady motion can be divided into two components, i.e., circulatory forces (quasi-steady and unsteady) and noncirculatory forces. The quasi-steady circulatory force is determined by the aforementioned variations in the effective angle of attack. In addition, noncirculatory mechanisms (i.e., added mass) also play an indispensable role in the thrust generation. This can be shown by comparing the wind-frame lift and drag coefficients to those predicted by quasi-steady circulatory theory with and without added-mass terms (Figs. 7e and 7f, respectively), similar to the method introduced by Dickinson et al. [12]. The quasi-steady (QS) model only includes the quasi-steady circulatory forces acting on a revolving wing at the identical $\alpha_e$, whereas the quasi-steady added-mass model (QSAM) also includes the added-mass term (noncirculatory). The added-mass force $F_a$ can be derived based on the unsteady accelerations of the wing [15] and formulated as

$$F_a = -(f_e)(f_e) \rho \sigma \hat{\psi} \cos(\alpha) \int_{\text{wing}} c^*_r \, dr$$ (34)

$$\begin{align*}
 f_e &= 1.294 - 0.590 R_e^{-0.662} \\
 f_e &= 0.776 + 1.911 R_e^{-0.487}
\end{align*}$$ (35)

Here, $f_e$ and $f_R$ are the correction factors for a finite plate and Reynolds number, respectively. Note that, according to kinematics used in the current study, the added-mass force is only produced by the flapping acceleration. For comparability, the wind-frame drag $C_D$ is calculated from the wind-frame drag force normalized by the total apparent velocity. In general, the QS predictions only capture the overall trend, leaving a large discrepancy, especially around each stroke reversal. However, this discrepancy can be explained by the added-mass effect (QSAM model). When the wing passes midupstroke and starts to decelerate, due to the fluid inertia, the fluid elements below the lower surface can "push" the wing upward, resulting...
in an upward added-mass force. The upward \( F_a \) is expected to be perpendicular to the wing surface, and thus the downward wind-frame force \( (C_L \text{ and } C_D) \) can be reduced by \( F_a \). During the beginning of the following downstroke, the added-mass force \( F_a \) is generated by the fluid elements below the wing, resulting in an upward increase for the upward \( C_L \) and \( C_D \). The added-mass effect during downstroke reversal is similar to those during upstrokes. Note that the slight asymmetry in the measured force during upstroke and downstroke can be a result of the setup asymmetry and the initial condition of wing flapping. Nevertheless, the QSAM model still underestimates the force generation after the onset of each stroke (white circles in Figs. 7e and 7f). This can be explained by the unsteady circulatory forces (wake capture) such as those due to the LEV impingements shown in Fig. 8, which are not included in the QSAM model.

The flow pattern was further inspected to make clear the vortex behavior of a flapping-perturbed revolving wing. Based on the numerical results, we calculated the dimensionless Q-value to identify the vortex core [50], which is shown in Fig. 8. The dimensionless pressure on the surface is also included. Generally, the vortex behavior and the corresponding pressure distribution are symmetric between upstrokes and downstrokes. A trailing-edge shear layer is first formed after the onset of each stroke, which rolls into a trailing-edge vortex (TEV) and is then shed (Figs. 8a and 8b, \( t = 0-0.25 \) and \( 0.5-0.75 \)). A leading-edge vortex (LEV) and tip vortex (TV) are generated during each stroke. The LEV is initially pinned along the leading edge, but then lifts off as an archlike structure and still stays attached to the wing (Figs. 8a and 8b, \( t = 0.25 \) and 0.75). A closed vortex ring (VR) then takes shape as a coherent structure of the root vortex, TV, LEV, and TEV. As the next stroke begins, the previously formed LEV moves downstream from the wing, is absorbed into the newly formed TEV, and is finally shed from the trailing edge (Figs. 8a and 8b, \( t = 0-0.25 \) and \( 0.5-0.75 \)). To further show the shedding of LEV and TEV and interactions of vortices, the two-dimensional (2-D) contour of dimensionless vorticity in \( Z_w \) direction at 50% wing span is plotted in Fig. 9 for the entire flapping cycle. Although there is a strong dissipation, a reversed Kármán vortex street is apparent in the near wake, especially before each midstroke. A deflected jet is induced by the reversed Kármán vortex street, contributing to the thrust generation. The timing of thrust peak coincides with the peak of jet strength. The LEV collides with the wing surface while moving downstream at each reversal, as it merges with the TEV by the combined compression of inflow and moving surface. This collision of LEV and the wing surface (i.e., a wake capture phenomenon) is an evidence of the unsteady effects at each reversal.

D. Instantaneous Force and Flow Pattern at 20 Degrees Angle of Attack

As is shown in Fig. 10a, the general trend for instantaneous lift \( C_V \) at \( \alpha_0 = 20 \) deg is similar to that for \( \alpha_0 = 0 \) deg. However, force magnitude is lower in upstrokes, and the period of positive \( C_V \) is extended from downstroke to upstroke, which corresponds to a lift augmentation. The instantaneous horizontal force \( C_H \) exhibits a reversed trend compared with \( C_V \) (i.e., the period of drag is extended, whereas that of thrust is reduced), resulting in an increased average drag over the cycle. Additionally, the aerodynamic power consumption is greater during downstrokes mainly due to the higher \( \alpha_e \), which has a peak around 90 deg (Fig. 10d).

Because of the positive \( \alpha_e \), the added-mass effect due to symmetrically flapping on wind-frame \( C_L \) and \( C_D \) is no longer symmetric, resulting in a higher \( C_D \) augment during upstroke reversal.
and the following downstroke (Fig. 10f, \( \dot{t} = 0.5 \)–0.75, difference between QS and QSAM model). The effective angle of attack (\( \alpha_e \)) at each middownstroke exceeds 90 deg (Fig. 10d, \( \dot{t}/.0136 \)0.75); thus, a slight loss in \( \text{CL} \) is observed in both model predictions and experimental values (Fig. 10e, \( \dot{t}/.0136 \)0.75). The relationship among resultant force and wing motion is shown in Fig. 10g. The orientation of resultant force is shifted from pointing up-backward to pointing down-forward in upstroke, whereas the force is mostly pointing down-forward during downstroke. This follows the change of \( \alpha_e \) because a higher flapping velocity during midupstroke orients the wing apparent velocity above the leading edge, generating a negative \( \alpha_e \) (Fig. 2b).

As shown in Fig. 11, the behavior of vortices is no longer symmetric between up- and downstrokes due to the nonzero \( \alpha_e \), nor is the lift force alteration (despite the flapping perturbation per se is symmetric). At the beginning of the upstroke, the vortex ring generated in the previous downstroke shed downstream (VR\(_d\)) maintains integrity until midupstroke (Fig. 11a, \( \dot{t} = 0.125 \)). For simplicity, subscripts \( u \) and \( d \) indicate upstroke and downstroke, respectively. The impingement of leading-edge vortex (LEV\(_d\), shed in the previous downstroke) upon the upper surface is observed during \( \dot{t} = 0.125 \) to \( \dot{t} = 0.25 \). At \( \dot{t} = 0.25 \), the trailing-edge vortex (TEV\(_u\)) is first generated, followed by the leading-edge vortex (LEV\(_u\)) and tip vortex (TV\(_u\)). The TEV\(_u\) is detached from the wing root, and thus the vortex ring (VR\(_u\)) is not closed, whereas the LEV\(_u\) remains attached along the leading edge with increasing strength toward the tip. The low-pressure region (LPR) on the upper surface is within the attached LEV\(_u\). As shown in Fig. 11b, during downstrokes, a stronger VR\(_d\) is formed, including a tilted TEV\(_d\) and LEV\(_d\) (\( \dot{t} = 0.75 \)). The TEV\(_d\) takes shape in a more obvious archlike structure yet pinned at wing root and tip, connecting root vortex and tip vortex as a closed ring (VR\(_d\)), unlike the open VR\(_u\). Unlike the upstroke, the impingement of LEV\(_u\) upon the lower surface is less significant during \( \dot{t} = 0.5 \) to \( \dot{t} = 0.625 \). The LPR on the upper surface is related to the attached LEV\(_d\) and TV\(_d\), excluding the area where the vortex is tilted away from the surface. Generally, as shown in Fig. 11a (Top view, T-View) and Fig. 11b (Bottom View, B-View), the upper surface in upstrokes and lower surface in downstrokes are mostly a high-pressure region (HPR). However, a significantly weakened HPR is observed during upstrokes due to the impingement of the previous leading-edge vortex (\( \dot{t} = 0 \) and 0.125). This reduces the downward pressure differential force generated during upstrokes and thus lowers the peak of lift and drag (Figs. 11a and 11b).
The merge of the previously formed LEV and the new TEV is also observed at $\alpha_t = 20\degree$, but a more obvious merge is found in the upstroke (Fig. 12, $i = 0.125$ and 0.25). Additionally, TEV$_d$ and TV$_d$ are almost horizontally aligned at $i = 0.25$; thus, no obvious reversed Kármán vortex street is formed. At $i = 0.75$, as shown in Fig. 12, though the TEV$_d$ remains strong (with a minor slit into a weaker part), the TV$_d$ is almost dissipated. Thus, the jet effect is not as obvious as that when $\alpha_t = 0\degree$. Despite the fact that a weaker LEV$_d$ collides with the surface during downstrokes (Fig. 12, $i = 0.5$ and 0.625), the added-mass force is, however, more significant on wind-frame drag $C_D$. This can be explained by an up to 90 deg $\alpha_t$ during downstrokes (Fig. 10d), which means that the added-mass force $F_a$ is mainly projected into $C_D$ direction.

When $\alpha_t = 45\degree$, $\alpha_t$ is significantly varied from $-10$ to over 100 deg (Fig. 13d); thus, the vorticity dynamics is significantly different between upstrokes and downstrokes, as shown in Fig. 14. Specifically, there is no vortex ring (VR$_e$) generated below the lower surface during upstrokes (Fig. 14a, $i = 0–0.5$). At the start of the upstroke ($i = 0$), the previously formed vortex ring (VR$_e$) detaches from the upper surface except at the wing root. The VR$_e$ then impinges onto the upper surface and interacts with the wing ($i = 0.125–0.25$). The leading-edge vortex (LEV$_e$) and tip vortex (TV$_e$) on the lower surface are much weaker compared with those at a smaller $\alpha_t$, and the LEV$_e$ is limited within a region near the wing tip. During downstrokes, the VR$_e$ is formed quickly on the upper surface ($i = 0.625–0.75$), and archlike leading-edge vortex (LEV$_d$) and trailing-edge vortex (TEV$_d$) are also observed ($i = 0.75$). The LEV$_d$, TEV$_d$, and TV$_d$ start to peel off from the wing tip at mid downstrokes while keeping connected with each other. As mentioned in Sec. III.B, the increases in lift and drag are almost identical at $\alpha_t = 45\degree$, indicating that the force alteration mainly results from the pressure difference. Because of the detachment of VR$_d$ in downstrokes, the low-pressure region (LPR) on the upper surface is reduced ($i = 0.75–0.875$), which slightly decrease the pressure difference during downstrokes. However, during upstrokes, the LPR on the lower surface and high pressure region (HPR) on the upper surface are both weakened ($i = 0–0.25$), as a result of the weaker LEV$_e$ and VR$_e$ impingement, respectively. These cause more reduction on the total pressure difference than downstrokes. Thus, the cycle-averaged pressure difference is upward, leading to an identical increase in both lift and drag.

A more explicit display of vortex dynamics can be found in Fig. 15. At $i = 0–0.375$, instead of a TEV$_d$, a strong shear layer (SL$_w$) is generated during upstrokes and interacts with previously formed.

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**Fig. 10** Instantaneous aerodynamic coefficients for a FP-RW at $\alpha_t = 20\degree$ ($St = 0.56$, $A^* = 0.9$, and $k = 0.97$). Definitions of subplots are identical to those described in Fig. 7.

**E. Instantaneous Force and Flow Pattern at 45 Degrees Angle of Attack**

For $\alpha_t = 45\degree$, the magnitude of instantaneous lift $C_L$ and horizontal force $C_D$ in downstrokes is significantly greater compared to those of upstrokes, leading to an increase in cycle-averaged lift and drag. Most power is consumed over downstrokes because the effective angle of attack ($\alpha_e$) is mostly close to 90 deg during this period, whereas it is close to 0 deg in the upstrokes. The variation of wind-frame lift $C_L$ is more significant during upstrokes (Fig. 13e), whereas the wind-frame drag $C_D$ has an opposite trend (Fig. 13f). For $i = 0.1–0.4$, the plateau stage of $C_D$, which is approximately zero, is due to $\alpha_e$ around zero. It is similar for $C_L$ within $i = 0.6–0.9$ when $\alpha_e$ is close to 90 deg (Fig. 13d). Note that the unsteady induced flow from the wake (i.e., the previously formed LEV) can also affect the effective angle of attack by changing the oncoming flow velocity but is not considered in our study.
TEV\_d. No obvious LEV\_u is generated during upstrokes at 50% spanwise position. During the whole downstroke (\(\hat{t} = 0.5 \cdots 0.875\)), the LEV\_d and TEV\_d maintain their position until the onset of upstroke, when the LEV\_d is absorbed into the SL\_u and the TEV\_d is shed into the wake.

F. Flapping-Perturbed Revolving wings and Symmetrical Breaking

As shown in the results (Sec. III.B), drag reduction due to flapping is observed in a FP-RW when angle of attack (\(\alpha_0\)) is 0 deg. As shown in Fig. 6a, no drag opposite of revolving is generated by a FP-RW when Strouhal number is approximately greater than 0.32. When \(St = 0.32\), the wing reaches an equilibrium state in which no torque is required to maintain revolving, which is instead maintained by the thrust generated by flapping. This equilibrium state resembles a similar balanced rotating state resulting from a vertically 2-D symmetric heaving wing, which is described as symmetric breaking phenomena in Vandenberghe et al. [51]. In their experiments, the wing starts to rotate about an unactuated pivot when a critical flapping Reynolds number \(Re_f\) is exceeded, leading to a bifurcating stable rotation. The critical \(Re_f\) for symmetric breaking was further found to decrease linearly when the dynamic viscosity of the fluid increases or equivalently as the pivot friction decreases. Given an infinitesimally small pivot friction, the ratio of \(Re_f\) and the revolving-based Reynolds number \(Re_{\Omega}\), which is the definition of Strouhal number in their study, is about 0.26 independent of the values of \(Re_f\) for comparison, we recalculated flapping/revolving-based Reynolds number and Strouhal number for the balanced FP-RW using Vandenberghe’s definitions:

\[
Re_f = \frac{fA_f R_{tip} \rho C}{\nu} \quad (36)
\]

\[
Re_{\Omega} = \frac{\int_{A_{tip}}^{} C_{\rho} \rho c(r) r^2 dr}{\int_{A_{tip}}^{} C_{\eta} \eta c(r) r dr} \quad (37)
\]

\[
St^* = \frac{Re_f}{Re_{\Omega}} \quad (38)
\]

The tip flapping velocity, defined as \(U_{f,tip} = f A_f R_{tip}\), is selected for \(Re_f\), and similarly, we assumed that the inertial torque coefficient...
Fig. 13 Instantaneous aerodynamic coefficients for a FP-RW at $\alpha_0 = 45^\circ$ deg ($St = 0.56$, $A^* = 0.9$, and $k = 0.97$). Definitions of subplots are identical to those described in Fig. 7.

Fig. 14 Isosurface of dimensionless Q-value $= 15$ for a FP-RW at $\alpha_0 = 45^\circ$ deg ($St = 0.56$, $A^* = 0.9$, and $k = 0.97$), including dimensionless pressure on the surface. Definitions of subplots are identical to those described in Fig. 8.
C^v$ is identical to the viscous torque coefficient $C^v$. Note that the pivot friction is not included in our measurement. According to the preceding definitions, the balanced state is achieved by a FP-RW at $St^*=0.22$. The slightly lower balanced $St^*$ for the FP-RW might be a result of the differences in wing kinematics and wing geometry. Nonetheless, here we confirmed that the relationship of flapping Reynolds number and revolving Reynolds number at a balanced state can be further extended to 3-D flapping cases, suggesting a fundamental phenomenon arising from the interaction of orthogonal wing movements (i.e., vertical flapping versus horizontal revolving).

G. Flapping-Perturbed Revolving Wings and Fluttering Behavior

The increases of lift and drag on a flapping-perturbed revolving wing at angles of attack of 20 and 45 deg remind us of the fluttering behavior of a flexible plate (e.g., that described in Curet et al. [39]). For a flexible-wing model that permits passive camber and flapping, Curet et al. observed an onset of passive flapping when a critical wind velocity is maintained and the angle of attack is around 5 deg. Once the flapping is initiated, significant increases in lift and drag occur, and a stronger leading-edge vortex is found during downstrokes than those during upstrokes. Therefore, despite the difference in the wing kinematics and geometry and the cause of flapping (one is generated passively, and the other is enforced in a prescribed manner), the conclusions drawn on the increases of lift and drag as well as the asymmetric LEV behavior are quite similar between the FP-RW at nonzero angle of attack and the fluttering behavior.

However, the effects of flapping and flapping on lift-to-drag ratio are somewhat different. In Curet et al. [39], a drop of lift-to-drag ratio was caused by fluttering because the drag increased more significantly than lift. However, as shown in Figs. 6b, an increase of lift-to-drag ratio is observed at $\alpha_0=20$ deg, especially when the flapping perturbation is subjected to a higher normalized flapping amplitude $A^\ast$, because the lift enjoys a more significant increase than drag over Strouhal number (blue line in Fig. 6b). In two distinct cases (labeled by black circles in Fig. 6b), a reduction in drag is observed, which leads to a significant increase of lift-to-drag ratio along with the increase in lift. When $\alpha_0$ further goes up to 45 deg, however, the increase of lift and drag on a FP-RW is almost identical at each Strouhal number and $A^\ast$ (Fig. 6c), thus indicating a reduction of lift-to-drag ratio.

H. Benefits of Unsteadiness Introduced by Flapping Perturbations

It is well accepted in the literature that unsteady effects can improve the aerodynamic performance of steadily revolving wings at low Reynolds number. One approach to introduce unsteadiness is to periodically switch the revolving directions of the wing accompanied by fast wing pitching (e.g., that used by insects), which can generate higher lift due to a number of unsteady aerodynamic mechanisms [9,12]. In the current study, we show that the unsteady effects introduced simply by symmetric vertical flapping can also increase the lift coefficient attainable by revolving wings at positive angle of attack (Figs. 6b and 6c). In addition to lift augmentation, as shown in Fig. 6b, a reduction of drag can be also achieved through flapping perturbation, given a high normalized flapping amplitude $A^\ast=1.35$ and a moderate angle of attack ($\alpha_0=20$ deg). This also results in an increase of the lift-to-drag ratio. However, both the lift augmentation and the drag reduction are at the cost of lower power loading and are limited to smaller angle of attack and proper Strouhal number, meaning that extra power is needed to improve the lift-to-drag ratio using flapping perturbation, and not all the revolving wings can benefit from flapping. For example, when $\alpha_0$ is increased to 45 deg (Fig. 6c), the lift augmentation still exists, but there is no increase of lift-to-drag ratio for all $A^\ast$.

The preceding results have practical implications because together they show that, although subjected to lower power loading, unsteady motion as simple as symmetric vertical flapping can lead to a more favorable lift-to-drag ratio unattainable by revolving wings. Note that dimensional lift can be increased simply by increasing the wing revolving velocity; however, it also increases the drag proportionally as the lift-to-drag ratio remains the same. Therefore, from this perspective, the aerodynamic performance can be improved fundamentally by flapping.

Note that, although the aerodynamic efficiency in terms of absolute power loading (PL in Figs. 6b and 6c) is reduced unavoidably by flapping perturbations, the ratio of lift augmentation over changes in power (PL* in Figs. 6b and 6c) due to flapping perturbations may give more insight on how to design flapping perturbation for lift augmentation from the perspectives of power constraints. At $\alpha_0=20$ deg, as shown in Fig. 6b, a flapping perturbation with a lower $A^\ast$ shows the highest PL*, indicating that less extra power is required using a low-$A^\ast$ perturbation than a high-$A^\ast$ one, given a similar lift augmentation. Note that a higher PL* generally means a lower lift augmentation within Strouhal number and $A^\ast$ at $\alpha_0=20$ deg. At $\alpha_0=45$ deg, the PL* of all cases are at similar low levels, and no obvious effect of $A^\ast$ is shown (Fig. 6c).

The aerodynamic performance of a FP-RW can be further compared to that of a flapping rotary wing (FRW), which are similar in terms of the combined revolving and flapping motion but are different is the cause of the revolving, and they become equivalent when the FP-RW is at the balanced rotating state if wing pitching is not considered in the FRW. An augmentation in lift and a drop in dimensional power loading are also observed in an FRW operating at $\alpha_0=20$ deg compared with those of revolving wings in Wu et al. [29]. An FRW is further regarded as one of the suitable MAV layouts for a high loading capability because the FRW can improve the maximum lift generated by a revolving wing. Using numerical simulation, Li et al. [45] calculated the cases beyond balanced FRWs (i.e., zero revolving drag) and found that the lift generated by a wing with combined flapping pitching and revolving increases over Strouhal number at $\alpha_0=15$ deg. These findings resemble those of the FP-RW derived from the current study.

IV. Conclusions

The nonlinear unsteady aerodynamics at low Reynolds number is fundamental for both insect flight in nature and micro air vehicles (MAVs) in engineering. The unsteady aerodynamic effects can be introduced to a revolving wing via reversing wing revolving direction similar to those of insect flight. Here, the unsteady aerodynamic
effects arising from vertical flapping perturbations applied to revolving wings (i.e., flapping-perturbed revolving wing, FP-RW) are investigated at $Re = 1500$. The aerodynamic force and efficiency of a FP-RW are measured experimentally within a wide range of Strouhal number $St$, normalized flapping amplitude $A^*$, and angle of attack ($\alpha_0$) and compared to those of a benchmark revolving wing. Further analyses of flow pattern are performed using the flowfield obtained from computational fluid dynamics (CFD) simulations.

At $\alpha_0 = 0$, significant drag reduction is observed, which decreases linearly over Strouhal number. When $St = 0.32$, the FP-RW reaches an equilibrium rotating state where the revolving is maintained by the thrust generated by flapping. The critical Strouhal number for equilibrium state is close between the FP-RW and a 2-D heaving plate. At positive $\alpha_0$, increases in lift and drag are found for the FP-RW. The lift augmentation linearly increases over Strouhal number beyond a critical Strouhal number. At $\alpha_0 = 20$ deg, the critical Strouhal number for lift augmentation is higher at a larger $A^*$, whereas the slope of lift augmentation over Strouhal number is about 2 for all $A^*$. At $\alpha_0 = 45$ deg, the effect of $A^*$ on lift augmentation is less significant. Apart from a lift augmentation, the power loading of a revolving wing is reduced once a flapping perturbation is introduced. At $\alpha_0 = 20$ deg, a slight drag reduction appears at the highest $A^*$ when $St = 0.17$ and 0.33 ($k = 0.19$ and 0.39), indicating an apparent increase of lift-to-drag ratio. However, the increase of drag is almost identical to the lift augmentation at $\alpha_0 = 45$ deg.

To further understand the cause of drag reduction at zero $\alpha_0$ and the force characteristics at positive $\alpha_0$, we examined the instantaneous force production together with the vortex behaviors for a typical FP-RW ($k = 0.97$, $A^* = 0.9$, and $St = 0.56$) at three critical angle of attack. At $\alpha_0 = 0$ deg, the drag reduction can be explained by the changes of effective angle of attack and the apparent velocity. Another contribution for the drag reduction is the generation of a reversed Kármán vortex street in the near wake. At positive $\alpha_0$, the increases in lift and drag are related to the changes of pressure difference between upper and lower surface. At $\alpha_0 = 20$ deg, the improvement of previously formed leading-edge vortex (wake capture) weakens the high-pressure region on the upper surface and therefore reduces the normal force during upstrokes, resulting in a higher increase of cycle-averaged lift than that of the cycle-averaged drag. At $\alpha_0 = 45$ deg, the pressure difference is reduced in both upstrokes and downstrokes, whereas the reduction in upstrokes is more significant and therefore results in an approximately equal increase of lift and drag due to an $\alpha_0$ of 45 deg.

The preceding findings provide a basic understanding of the aerodynamic effects of flapping perturbations on a steady revolving wing with various Strouhal number and $A^*$ at a low Reynolds number. The flapping perturbation is a viable way to reduce revolving drag and enhance lift generation of a revolving wing, which could bring some new insights to MAV design.

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