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PAPER

Hovering efficiency comparison of rotary and flapping flight for rigid rectangular wings via dimensionless multi-objective optimization

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Abstract

In this work, a multi-objective optimization framework is developed for optimizing low Reynolds number (Re) hovering flight. This framework is then applied to compare the efficiency of rigid revolving and flapping wings with rectangular shape under varying Re and Rossby number (Ro, or aspect ratio). The proposed framework is capable of generating sets of optimal solutions and Pareto fronts for maximizing the lift coefficient and minimizing the power coefficient in dimensionless space, explicitly revealing the trade-off between lift generation and power consumption. The results indicate that revolving wings are more efficient when the required average lift coefficient $C_L$ is low ($<1$ for $Re = 100$ and $<1.6$ for $Re = 8000$), while flapping wings are more efficient in achieving higher $C_L$. With the dimensionless power loading as the single-objective performance measure to be maximized, rotary flight is more efficient than flapping wings for $Re > 100$ regardless of the amount of energy storage assumed in the flapping wing actuation mechanism, while flapping flight is more efficient for $Re < 100$. It is observed that wings with low $Ro$ perform better when higher $C_L$ is needed, whereas higher $Ro$ cases are more efficient at $C_L < 0.9$ regions. However, for the selected geometry and $Re$, the efficiency is weakly dependent on $Ro$ when the dimensionless power loading is maximized.

1. Introduction

Since the discovery of leading-edge vortices (LEVs) in insect flight (Maxworthy 1981, Dickinson 1994, Ellington et al 1996), flapping wings of insects have become a common design paradigm for micro air vehicles (MAVs) with hovering capabilities, as flapping flight is often considered as a superior form of flight over fixed or rotary flight at low Reynolds number (Re) regimes. In recent years, a variety of hovering-capable flapping-wing MAVs (e.g. Madangopal et al 2006, Wood (2008), Keennon et al 2012, Ma et al 2013, Karásek et al 2014, Coleman et al 2015, Roll et al 2015, Zhang et al 2016) ranging in wing length from 1.5 mm (Bronson et al 2009) to 70 mm (Hines et al 2014) have been developed.

Flapping flight is ubiquitous in nature and it has apparent advantages for maneuverability and arguably for stability. However, it is still unclear how its efficiency compares with that of rotary-wing air vehicles, which represent the most common engineering solution for hovering flight across a wide range of scales (Leishman 2006, Floreano and Wood 2015). Whether insects with flapping wings or quadcopters with revolving wings, flying at low $Re$ regimes (e.g. $10^3$), small fliers unavoidably suffer from low aerodynamic efficiency due to increased viscous effects (Shyy et al 2007, Spedding and McArthur 2010). For example, when $Re$ is lower than 100, the lift-to-drag ratio ($C_L / C_D$) of an airfoil drops to almost two orders of magnitude lower than that of a large aircraft (Shyy et al 2007, Lentink and Dickinson 2009) which corresponds to large profile power losses. Still, which one of these two locomotion strategies is more efficient remains a debate in the literature.

Despite both suffering from low efficiency, the aerodynamics of flapping wings are fundamentally different from those of revolving wings in two aspects. First, insect wings use a higher angle of attack, which requires a sufficiently low wing aspect ratio ($AR$) (or Rossby number ($Ro$)) to prevent stalling and to maintain a stable LEV structure (Lentink and Dickinson 2009, Cheng et al 2013, Kruyt et al 2015) for large wing strokes. Rotary wings, on the other hand, typically have a higher $AR$ and use a lower angle of attack. Second, the flapping wings of insects undergo rapid wing...
reversal and pitching motions that create unsteady forces and vortex shedding. It has been shown that there exist a variety of unsteady mechanisms in insect flight that could create additional lift forces (compared with steady revolving wings), such as clap and fling (Sane 2001), rotational lift (Sane and Dickinson 2002), added mass (Ellington 1984b, Sane and Dickinson 2002, Sane 2003) and wake capture (Dickinson et al. 1999, Birch and Dickinson 2003, Lehmann 2008). Importantly, these unsteady mechanisms may potentially improve the aerodynamic efficiency of flapping wings by producing lift more efficiently than the quasi-steady aerodynamics of revolving wings. However, lift produced by flapping wings may deviate from the maximum ‘quasi-steady’ lift produced by rotary wings with a constant optimal angle of attack (Pesavento and Wang 2009); there is a trade-off between leveraging unsteady lift and maintaining the optimal steady lift, which needs to be investigated systematically. Moreover, it is expected that such a trade-off depends critically on the Re, Re, AR, and patterns of wing trajectories, which together form a large parameter space to be investigated.

Notably, several attempts have been made to compare the efficiency of flapping and rotary flight (Lentink and Dickinson 2009, Mayo and Leishman 2010, Zheng et al. 2013). However these studies are somewhat limited in terms of the range of parameters investigated (e.g. Re and AR), often did not optimize the shapes of the wing flapping trajectories for efficiency, and used different metrics for efficiency. As a result, they often reached different conclusions. For example, (Lentink and Dickinson 2009) shows that a sinusoidally flapping fruit-fly wing has significantly lower efficiency than steady rotation for 110 < Re < 14,000; however, (Zheng et al. 2013) shows that a flapping hawkmoth wing achieves more than twice the efficiency of a revolving wing when Re = 100. In addition, a review (Mayo and Leishman 2010) of published power measurements for various hover-capable fliers in nature and engineering shows that flapping-wing insects consistently achieve higher efficiencies than revolving wings. Nonetheless, there is no study providing a complete answer to which locomotion strategy is more efficient for MAVs.

On the other hand, the wing kinematics for MAVs have been optimized in several studies (Berman and Wang 2007, Kurdi et al. 2010, Taha et al. 2013, Yan et al. 2015, Ke et al. 2017) which cover specific cases and do not report a comparison of efficiency for flapping and rotary wings. For instance, (Berman and Wang 2007) minimizes the energy consumption for three insect species using genetic algorithms while the generated lift is constrained to achieve hovering. A similar problem is solved by Kurdi et al. (2010) using a gradient descent algorithm whereas (Taha et al. 2013) provides a calculus-of-variations approach to it. Finally, (Yan et al. 2015) analyzes the effects of aerodynamic modeling on the optimization problem. However, these studies are of specific cases and provide neither efficiency comparisons nor solutions suitable for a rigorous comparison.

Therefore, in this paper, we aim to perform a systematic comparison of the optimal hovering efficiency between flapping and rotary flight (or flapping and revolving wings; ‘revolving’ and ‘rotary’ are used interchangeably), for a range of Re and Ro pertaining to insect flight and MAV’s of similar sizes. The specific objective of this paper is twofold: first, to develop a dimensionless multi-objective optimization framework for low Re hovering flight; and second, to analyze the hovering efficiency of flapping and rotary flight where the advantages of the proposed optimization framework are demonstrated. This study only focuses on comparing flapping and revolving wings on an equal footing, i.e. both having the same wing design of a rectangular thin rigid plate, in order to solely identify the effects of the flight pattern/mode per se (revolving versus flapping) on aerodynamic efficiency.

2. Methods

2.1. Dimensionless multi-objective optimization framework

Performing optimization in a dimensionless parameter space is particularly important for investigations on bio-inspired fluid mechanics with highly complex forms, which require explorations in a large parameter space. Dimensionless analysis allows us to investigate the fundamental properties of physical problems by isolating the effects of dimension and reducing the number of parameters in the optimization; as a result, the solutions of the optimization problem, i.e. dimensionless wing angle trajectories, will embody a wide range of solutions for specific physical problems and design parameters. Additionally, dimensionless optimization enables us to draw more general conclusions on low Re flight compared to those used in the literature. Therefore, the dimensionless solutions obtained from the current study may be applied subsequently to guide the design of physical systems given specific physical requirements for different MAV applications.

The overall dimensionless optimization framework is depicted in figure 2. It starts with choosing the dimensionless parameters Re, AR, Ro etc to set the properties of the aerodynamic model. This model then calculates the lift force acting on the wing, F_L, and the corresponding power consumption, P. In this framework, the dimensionless parameters determine the solution of the optimization problem and are kept fixed throughout the optimization. Then, the outputs of the aerodynamic model, i.e. F_L and P, are also normalized and their averages over one flapping or revolving cycle are used as the objective functions in the optimization. In the literature, the commonly used form of normalization is based on a quasi-steady assumption that yields to instantaneous lift and power.
coefficients, \( C_L(t) \) and \( C_P(t) \). Thus, in this work the two objective functions for the optimization are defined as the cycle averages of these two coefficients \( C_L \) and \( C_P \).

Although the problem can be reduced to a single objective function in the form of a performance measure (Berman and Wang 2007, Kurdi et al 2010, Ke et al 2017), the trade-off behavior of the two objectives cannot be fully revealed with such an approach. In particular, an optimal solution found from a single objective function combining \( C_L \) and \( C_P \) is limited to that certain performance measure and does not explore other possible combinations of \( C_L \) and \( C_P \). Therefore, optimization is performed based on a multi-objective approach to overcome this limitation. The optimization framework and how to compute \( C_L \) and \( C_P \) are further explained in the following sections.

The optimization yields a Pareto front (Deb 2014) and a set of optimal wing trajectories for varying values of \( C_L \) and \( C_P \). The resulting trajectories are given as kinematic variables \( \varphi(t) \) and \( \alpha(t) \) in figure 2, which are defined in section 2.4.2. The resulting angles determine the shape of the trajectories without specifying the frequency of flapping. These solutions in dimensionless space can be converted to those in the physical domain with known dimensions and known constraints for a specific application, e.g. flapping frequency, maximum velocity, power consumption, lift generation, etc. The proper set of \( C_L \) and \( C_P \) couples can be found from these Pareto front plots and the rest of the physical parameters can be matched to that application. In this manner, an iterative approach can be used to find the required \( C_L \) and \( C_P \) combination or to account for design/analysis limitations with optimal wing trajectories.

Aiming to demonstrate how to apply the proposed optimization scheme and to provide some insight into the paramount question of whether flapping wings or revolving wings are more efficient for low \( Re \) hovering flight, we apply the framework to a specific case with \( Re \) and \( Ro \) selected as control parameters and analyze their effects on the optimal solutions. These two parameters are selected because they determine the effects of viscosity and the rotational acceleration (related to three-dimensional effects) (Lentink and Dickinson 2009, Chin and Lentink 2016). Thus, they are the two most critical design factors determining the lift and power of the flight. The rest of this section is devoted to the details of this specific case.

2.2. Wing characteristics and key optimization parameters

As a preliminary to the details of the framework, the wing geometry and the key parameters should be established, which are summarized in figure 3. A rectangular wing with a rotation axis parallel to the leading edge is chosen for the optimization and the wing trajectory analysis. The rectangular geometry is one of the benchmark geometries, commonly used in flapping wing aerodynamics and design literature, and is thus selected in our study.

The wing is assumed to be a thin rigid plate and the physical parameters to define the wing geometry are described in figure 3. The total length from wing root to wing tip is defined as \( R \), the wing platform span is \( b \), and the constant chord length is denoted as \( \hat{c} \). Note that \( R \) and \( b \) can be different if there is an offset from the wing root to the platform. The distance of the rotation axis from the leading edge is kept constant and defined as \( x_{rot} \).

For this work, it is crucial to define the wing geometry with only one physical, characteristic length and other dimensionless parameters to enable optimization in dimensionless space. Therefore, the offset from the wing root to the start of the wing platform is defined as \( \Delta R = (R - b)/R \); the wing platform geometry is represented by the aspect ratio \( AR = R/\hat{c} \) which also determines the wing chord length; the position of the rotation axis is normalized as...
\( \hat{x}_{rot} = x_{rot}/\hat{c} \); and finally, the local spanwise coordinate \( r \) and the chord length \( c \) are normalized as \( \hat{r} = r/R \) and \( \hat{c} = c/\hat{c} \) (equal to 1 in this work), respectively. Accordingly, the dimensionless radius of the second moment of area, \( \hat{r}_2 = r_2/R \), can be calculated from

\[
\hat{r}_2 = \sqrt{\frac{1}{1 - \Delta \hat{R}} \int_0^1 \hat{r}^2 \hat{c} \, d\hat{r}},
\]

where the term \( 1/(1 - \Delta \hat{R}) \) results from the nonzero offset from the wing root, i.e. \( R - b \).

In addition to the geometrical parameters, we consider two dimensionless parameters of fluid motion, the primary determining factors of the optimized wing trajectories, i.e. \( Re \) and \( Ro \). Although there are several definitions of \( Re \) using different characteristic velocities, the one using the mean velocity at \( \hat{r}_2 \) (or the radius of gyration), i.e.

\[
Re = \frac{\hat{c} U_{\overline{\lambda}}}{\nu},
\]

is adopted here for consistency with that in the aerodynamic model used in the study in (Lee et al. 2016), where \( \nu \) is the kinematic viscosity of the fluid and \( U_{\overline{\lambda}} \) is the (stroke) averaged velocity at \( \hat{r}_2 \). Additionally, \( Ro \) is calculated by

\[
Ro = \hat{r}_2 \, AR.
\]

In this study, \( Ro \) is controlled by the aspect ratio \( AR \) and thus the offset ratio of the wing \( \Delta \hat{R} \) is kept constant. Since \( \Delta \hat{R} \) does not change, \( \hat{r}_2 \) is a constant value.

We also consider the power consumption due to wing inertia in the optimization, so the wing moment of inertia should be normalized for the same reason stated above,

\[
I_{ab} = I_{ab}/R^2 \rho,
\]

where \( \rho \) is the density of the fluid.

The dimensionless parameters for the cases studied here are listed in table 1. All the parameters other than \( Re \) and \( Ro \) are kept constant. It is assumed that there is no offset from the wing root (\( \Delta \hat{R} = 0 \)) while the rotation axis is aligned with the leading edge (\( \hat{x}_{rot} = 0 \)). To calculate the dimensionless moment of inertia, we assume that the wing density to air density and wing thickness to wing length ratios are identical to those of the fruit fly (\( Drosophila melanogaster \)). The wing mass per unit area and wing length data for the fruit fly are taken from (Ray et al. 2016). The effects of \( Re \) are analyzed for \( AR = 3 \) as commonly observed in nature and corresponding to \( Ro = 1.73 \). The optimization is performed for \( Re = 100, 300, 900, 3000 \) and 8000. On the other hand, to observe the dependence of the optimization on \( Ro, Re = 500 \) is kept constant and the problem is solved for \( AR = 2, 4, 6 \) and 8 which results in \( Ro = 1.15, 2.31, 3.46 \) and 4.62.

### 2.3. Wing kinematics and parameterization

The wing kinematics are described by three Euler angles, namely the stroke angle \( \phi \), the position of the wing in the horizontal stroke plane; the deviation angle \( \theta \), the vertical wing deviation from the horizontal stroke plane; and the pitch angle \( \alpha \), the angle about its radial axis, as shown in figure 4. In nature, most insect wing trajectories only have a small amount of out-of-plane motion from the stroke plane (Weis-Fogh 1973, Ellington 1984a). Moreover, it has been shown in the literature that the effect of the deviation angle \( \theta \) is secondary for lift and drag generation and power consumption compared to the other two angles and other fluid/wing properties (Sane and Dickinson 2001, Qin et al. 2014). Therefore, in this work, we only consider the wing kinematics in the horizontal plane (i.e. as described by the stroke angle and pitch angle). On the other hand, it should be noted that the optimal flapping wing trajectories and performance we report here can be potentially suboptimal compared with those considering the deviation angle. Therefore, our results may slightly underestimate the maximal efficiency achievable by flapping wings.

#### 2.3.1. Coordinate transformations and equation of motion

The rotation matrix from the fixed reference frame to the wing frame \( R_0^1 \) can be written as

\[
R_0^1 = R_0^2 \cdot R_0^3,
\]
where $R^i_j$ is represented in figure 4(a) and is a rotation around the $z$ axis whereas $R^2_1$ is shown in figure 4(b) and is a rotation around the $\hat{x}$ axis. The angular velocity vector of the wing expressed in the reference frame, $\omega^{0,0}_{2}$, can be determined by

$$ (\omega^{0,0}_{2}) = R^0_2 \cdot R^0_0, \tag{6} $$

where the $(\cdot)$ operator converts a vector to a $3 \times 3$ skew symmetric matrix which transforms the cross product operation of vectors to matrix multiplication (Murray et al. 2017). Then, the angular velocity $\omega^{0,0}_{2}$ is found as

$$ \omega^{0,0}_{2} = \begin{bmatrix} \cos(\phi)\dot{\alpha} \\ \sin(\phi)\dot{\phi} \\ \dot{\theta} \end{bmatrix}. \tag{7} $$

Using the angular velocity vector, the equation of motion for a single wing can be derived by using Lagrangian mechanics. Note that, since the wing thickness ($h_{wing}$) is much smaller than $R$, $I_y = I_{xx} = I_z = I_y = 0$ is a valid approximation, and from the perpendicular axis theorem $I_z = I_{xx} + I_y$. The simplified equations of motion with dimensionless inertia are as follows:

$$ R^5 \rho \left(I_{xx}\ddot{\alpha} + \dot{I}_{xx}\cos(\alpha)\dot{\phi} - \frac{1}{2}\dot{I}_{xx}\sin(2\alpha)\dot{\phi}^2\right) = M_{p\alpha}, \tag{8} $$

$$ R^5 \rho \left(I_{zz} + \dot{I}_{zz}\sin^2(\alpha)\right)\ddot{\phi} + \dot{I}_{zz}\cos(\alpha)\ddot{\alpha} + \dot{I}_{zz}\sin(2\alpha)\dot{\alpha}\dot{\phi} - \dot{I}_{zz}\sin(\alpha)\dot{\phi}^2 = M_{z\alpha}, \tag{9} $$

where $M_{p\alpha}$ and $M_{z\alpha}$ are the total external moments applied to the wing body in the $\hat{x}_2$ and $\hat{z}_2$ axes respectively. In our model, these external moments consist of aerodynamic and actuation moments. Actuation moments are provided by flight muscles in biological fliers and mechanical actuators (electric motors, piezo-actuators, etc.) in robotic fliers.

### 2.3.2. Wing trajectory parameterization.

To solve the optimization efficiently, the wing trajectories need to be parameterized properly so the problem can be transcribed into a finite variable nonlinear optimization. As the optimized trajectory will be dimensionless and free of frequency effects, we only need to parameterize the time-scaled trajectories. We use two sets of wing parameterizations to compare the performance of revolving and horizontally flapping wings. For revolving wings, we use the following parameterization:

$$ \phi(t) = C_\phi \sin(t) \tag{10} $$

$$ \alpha(t) = a_\alpha \tag{11} $$

where $t$ is the dimensionless time ($t = \frac{t_f}{f}$), $f$ is the frequency scale (the flapping or revolving frequency) and $a_\alpha \in [0, \pi/2]$ is the wing pitch angle to be optimized. Note that, for a rotary wing, the generated lift and power only consist of translational components (section 2.4), so in the non-dimensionalization stage (section 2.5), the parameter $C$ is canceled out. In other words, if $Re$ and the wing geometry are kept constant, the performance of revolving wings only depends on the constant angle of attack whereas $C$ is an arbitrary constant for optimization purposes. For simplicity, we use a $C$ of 1 in this work.

To transcribe the flapping wing trajectory, we adopt the wing pattern proposed by Berman and Wang (2007), which is extensively used in the literature (Chang and Wang 2014, Elzinga et al. 2014, Nabawy and Crowther 2014b, Yan et al. 2015). The wing trajectory as a function of dimensionless time is given as follows:

$$ \phi(t) = \frac{\phi_m}{\sin^{-1}(K_\phi)} \sin^{-1} \left(K_\phi \sin(2\pi t) \right), \tag{12} $$

$$ \alpha(t) = \frac{\alpha_m}{\tanh^{-1}(C_\alpha)} \tanh^{-1} \left(C_\alpha \sin(2\pi t + \alpha_{ph} \phi) + \alpha_{off} \right), \tag{13} $$

where $\phi_m$ and $\alpha_m$ are the stroke and pitch angle amplitudes respectively, $\alpha_{ph}$ determines the phase between the stroke angle and the pitch angle, and $\alpha_{off}$ is the offset of the mean pitch angle from the horizontal position. In this function approximation, $K_\phi$ and $C_\alpha$ are the parameters determining the shape of the trajectory. The effects of these parameters can be
Table 2. Optimization variable limits and constraints.

<table>
<thead>
<tr>
<th>Variable/Constraint</th>
<th>Description</th>
<th>Lower Lim.</th>
<th>Upper Lim.</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\phi_m)</td>
<td>Stroke amplitude (rad)</td>
<td>(\pi/10)</td>
<td>(\pi)</td>
</tr>
<tr>
<td>(\alpha_m)</td>
<td>Wing pitch amplitude (rad)</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>(\alpha_{pha})</td>
<td>Phase angle for the wing</td>
<td>(-\pi)</td>
<td>(\pi)</td>
</tr>
<tr>
<td>(\alpha_{off})</td>
<td>Wing pitch offset (rad)</td>
<td>0</td>
<td>(\pi)</td>
</tr>
<tr>
<td>(K_\phi)</td>
<td>Affects the shape of (\phi)</td>
<td>0</td>
<td>0.97</td>
</tr>
<tr>
<td>(C_\alpha)</td>
<td>Affects the shape of (\alpha)</td>
<td>0</td>
<td>20</td>
</tr>
<tr>
<td>(\alpha_{eff} - \alpha_m)</td>
<td>Minimum pitch angle</td>
<td>0</td>
<td></td>
</tr>
<tr>
<td>(\alpha_m + \alpha_{eff})</td>
<td>Wing pitch amplitude (rad)</td>
<td></td>
<td>(\pi)</td>
</tr>
</tbody>
</table>

found in (Berman and Wang 2007). The dimensionless time derivatives of the wing angles are related to those according to

\[
\frac{d\phi}{dt} = \phi = \int f d\phi = f\hat{\phi}, \quad \frac{d^2\phi}{dt^2} = \hat{\phi} = f^2 \hat{\phi}, \quad \frac{d^2\phi}{dt^2} = \hat{\phi}, \quad (14)
\]

for \(\phi\) and

\[
\frac{d\alpha}{dt} = \alpha = \int f d\alpha = f\hat{\alpha}, \quad \frac{d^2\alpha}{dt^2} = \hat{\alpha} = f^2 \hat{\alpha}, \quad \frac{d^2\alpha}{dt^2} = \hat{\alpha}, \quad (15)
\]

for \(\alpha\). As seen more clearly in the aerodynamic model, \(f\) is irrelevant in the dimensionless optimization and is canceled out while calculating \(C_\alpha\) and \(C_\phi\) as \(Re\) is kept constant throughout the optimization. The flapping or revolving frequency is included in the selection of \(Re\) and also has significance when converting the results back to the physical domain.

For the flapping wing trajectory we employ, limits and linear constraints are applied on the optimization variables to ensure a variable space that only includes physically feasible trajectories. In particular, it is guaranteed that \(\alpha(t) \in [0, \pi]\) and the total stroke amplitude is smaller than 180°. Also, the shape parameters \(K_\phi\) and \(C_\alpha\) are limited to exclude unphysical high-velocity peaks in the trajectory. The limits and constraints used are listed in table 2. Note that, in that table, \(\phi_m\) has a minimum bound of \(\pi/10\). The reason for this limitation is explained in section 2.4.2.

2.4. Aerodynamic model, force and power calculation

The optimization procedure (section 2.6) requires a time-efficient calculation of aerodynamic forces acting on the wing and the corresponding power consumption, which as performance measures have to be evaluated many times to obtain optimal solutions, thus precluding any computationally expensive methods. Therefore, here we use a semi-empirical quasi-steady model, which estimates forces and moments using analytical models with their parameters estimated from empirical data. However, most existing quasi-steady models (e.g. Dudley and Ellington 1990), Dickinson et al 1999, Sane and Dickinson 2002, Usherwood and Ellington 2002, Berman and Wang (2007), Nabawy and Crowther (2014a) are derived either based only on data collected from a limited range of parameters (e.g. \(Re, Re\), \(AR\)), or based on analytical treatment using classical airfoil theories with substantial simplifications, therefore only capturing local fluid behavior and lacking generality for the current investigation.

In this work we adopt a recently developed aerodynamic model (Lee et al 2016), which predicts the aerodynamic force and power coefficients for a wide range of critical non-dimensional parameters, including \(Re, AR\) and \(Ro\). Therefore, this model enables us to optimize wing trajectories for a large parameter space and compare the performance of flapping and revolving wings systematically.

2.4.1. Force calculation

The following set of equations provides forces acting on the wing based on the aerodynamic model from Lee et al (2016) which is reformulated based on the coordinate frame, variable definitions and normalization schemes used in the current study. Note that, in this work, \(\alpha(t)\) is treated as the wing pitch angle instead of the angle of attack as defined in Lee et al (2016), and it does not switch signs as the wing trajectory changes from upstroke to downstroke or vice versa,

\[
F_L = F_{L,\text{tr}} + \cos(\alpha) (F_{N,\text{rot1}} + F_{N,\text{rot2}} + F_{N,\text{add}}), \quad (16)
\]

\[
F_D = F_{D,\text{tr}} + \sin(\alpha) (F_{N,\text{rot1}} + F_{N,\text{rot2}} + F_{N,\text{add}}), \quad (17)
\]

where \(F_{L,\text{tr}}\) and \(F_{D,\text{tr}}\) are the translational lift and drag forces respectively, \(F_{N,\text{rot1}}\) and \(F_{N,\text{rot2}}\) are the rotational lift force components and \(F_{N,\text{add}}\) is an added mass force component. In addition to the conventional force components proposed in Dickinson et al (1999) and Lee et al (2016) we introduce a secondary rotational lift component \(F_{N,\text{rot2}}\) that solely depends on the rotational velocity \(\hat{\alpha}\). The detailed expressions for the force components are given below:

\[
F_{L,\text{tr}} = \rho R f^2 \left( f_{\text{rot1}} f_{AR,\text{tr}} C_\lambda(\alpha) \frac{1}{2AR} \hat{\phi} \int_0^T 1^2 \tilde{C} d\tilde{r} \right), \quad (18)
\]

\[
F_{D,\text{tr}} = \rho R f^2 \left( f_{\text{rot1}} f_{AR,\text{tr}} C_\lambda(\alpha) \frac{1}{2AR} \hat{\phi} \int_0^T 1^2 \tilde{C} d\tilde{r} \right), \quad (19)
\]

\[
F_{N,\text{rot1}} = \rho R f^2 \left( f_{\text{rot1}} f_{\text{rot1}} C_{\text{rot1}} \frac{1}{AR} \hat{\phi} \int_0^T 1^2 \tilde{C}^2 d\tilde{r} \right), \quad (20)
\]

\[
F_{N,\text{rot2}} = \rho R f^2 \left( C_{\text{rot2}} \frac{1}{AR} \hat{\alpha} \|\hat{\alpha}\| \int_{TE} x^2 \tilde{C}^2 d\tilde{x} \right), \quad (21)
\]

\[
F_{N,\text{add}} = \rho R f^2 \left( f_{AR,\text{add}} C_{\text{add}} \frac{\pi}{4} \left( \sin(\alpha) \frac{1}{AR} \int_0^T 1^2 \tilde{C}^2 d\tilde{r} + \hat{\alpha} \frac{1}{AR^2} \int_0^T (\tilde{C} - \tilde{C}_{\text{rot}}) d\tilde{r} \right) \right). \quad (22)
\]
In this model, $C_L$ and $C_D$ are the translational lift and drag coefficients whereas $C_{rot1}$ and $C_{rot2}$ are the coefficients of the two components of the rotational lift $F_{N,rot1}$ and $F_{N,rot2}$, respectively. Similarly, $C_{add}$ is the corresponding coefficient of the added mass. Note that these coefficients are multiplied by several correction factors denoted by $f_{\cdot \cdot}$. These factors adjust these coefficients for wings with different geometries and flight conditions. In particular, $f_{Ro,rot}$ and $f_{AR,rot}$ correct the translational forces for wing geometries with different $Re$ and $AR$ while $f_{rots}$ and $f_{rots}$ adjust $F_{N,rot1}$ for different instantaneous $\alpha$ and the rotation axis location $\dot{x}_{rot}$ respectively. Finally, $f_{AR,add}$ denotes the $AR$ correction factor for the added mass. The explicit expressions for these coefficients and factors are given in (23)–(32).

\begin{align}
C_L(\alpha) &= (1.966 - 3.94 Re^{-0.425}) \sin(\alpha), \quad (23) \\
C_D(\alpha) &= (0.031 + 10.48 Re^{-0.761}) \\
&\quad + (1.873 - 3.14 Re^{-0.396})(1 - \cos(2\alpha)), \quad (24) \\
C_{rot1} &= 0.842 - 0.507 Re^{-0.1577}, \quad (25) \\
C_{rot2} &= 2.67, \quad (26) \\
C_{add} &= 0.776 + 1.911 Re^{-0.687}, \quad (27) \\
f_{Ro,rot} &= -0.205 \arctan(0.587(Ro - 3.105)) + 0.870, \quad (28) \\
f_{AR,rot} &= 32.9 - 32.0 AR^{-0.00361} \quad (29) \\
f_{rots} &= \begin{cases} 1 & \text{if } 0 < \alpha < 45^\circ \\
-1 & \text{if } 135^\circ < \alpha < 180^\circ \\
\cos(\alpha) & \text{otherwise} \end{cases} \quad (30) \\
f_{rots} &= 1.570 - 1.239 \ddot{x}_{rot} \quad (31) \\
f_{AR,add} &= 1.294 - 0.590 AR^{-0.662} \quad (32)
\end{align}

Note that the force terms and the coefficients presented here are simplified for the problem at hand. Therefore, the correction factors, such as the ones corresponding to the effects of wing taper, are omitted. The derivation of this model and further analysis of the force expressions are provided in Lee et al (2016).

2.4.2. Power calculation.

The aerodynamic power consumption by a wing trajectory is calculated by the dot product of the angular velocity vector and the aerodynamic moment acting on the wing. Note that Lee et al (2016) also provides improved correlations for the centers of pressure for aerodynamic moment calculation for their model. The following set of equations presents the aerodynamic moments acting on the wing $M_{\dot{x},aero}$ and $M_{\dot{z},aero}$ about the $\dot{x}$ and $\dot{z}$ axes respectively:

\begin{align}
M_{\dot{x},aero} &= -R \dot{\alpha} F_{\dot{x},rot} + \dot{d}_x \sin(\alpha) F_{\dot{y},rot} \\
&\quad + \dot{\alpha} (\dot{F}_{N,rot} + F_{N,rot2}) \dot{d}_{add} + T_{add}, \quad (33)
\end{align}

The added torque term $T_{add}$ is defined as

\begin{equation}
T_{add} = -\rho R^2 \left( (1.114 + 7.89 Re^{-0.885}) \frac{1}{128} \frac{1}{AR} \ddot{x} \right). \quad (35)
\end{equation}

Furthermore, $\dot{d}_x$ and $\dot{d}_z$ denote the moment arms for the translational forces, $d_{rot}$ and $\dot{\rho}_rot$ are the moment arms for the rotational forces, and $\dot{d}_{add}$ and $\dot{\rho}_{add}$ are the moment arms for the added mass term. These moment arms are given in Lee et al (2016) as follows:

\begin{align}
\dot{d}_x &= \frac{1}{AR} (-0.0799 \cos(2\alpha) + 0.377 - \dot{x}_{rot}), \quad (36) \\
\dot{d}_z &= \frac{1}{AR} (0.398 - \dot{x}_{rot}), \quad (37) \\
\dot{d}_{rot} &= \frac{1}{AR} (0.5 - \dot{x}_{rot}), \quad (38) \\
\dot{\rho}_x &= \dot{r}_2 (0.0784 \cos(2\alpha) + 1.088), \quad (39) \\
\dot{\rho}_z &= \dot{r}_2 0.993, \quad (40) \\
\dot{\rho}_{add} &= \dot{r}_2 1.078. \quad (41)
\end{align}

Then, the resulting moment equations can be simply written as

\begin{align}
M_{\dot{x},act} &= M_{\dot{x}} - M_{\dot{x},aero} \quad (42) \\
M_{\dot{z},act} &= M_{\dot{z}} - M_{\dot{z},aero} \quad (43)
\end{align}

where $M_{\dot{x},act}$ and $M_{\dot{z},act}$ are the total actuation moments (including both inertial moments, $M_{\dot{x}}$ (8) and $M_{\dot{z}}$ (9), and aerodynamic components, $M_{\dot{x},aero}$ (33) and $M_{\dot{z},aero}$ (34)) required to drive the wing about the $\dot{x}$ and $\dot{z}$ axes respectively. Then the resulting mechanical power to actuate the wing with the given trajectory $\dot{P}(t)$ can be calculated by

\begin{equation}
\dot{P}(t) = f \left( (M_{\dot{x}} - M_{\dot{x},aero}) \ddot{x} + (M_{\dot{z}} - M_{\dot{z},aero}) \ddot{z} \right). \quad (44)
\end{equation}

It is clear from this notation that when $\dot{P}(t)$ is positive, the power is consumed to accelerate or decelerate the wing while overcoming the inertial and aerodynamic moments, whereas for negative values of $\dot{P}(t)$ the actuator absorbs the power, which can be either dissipated from the system or partially stored in an elastic element in the actuator. Therefore, as the net power consumption (or $CP$) is one of the objective functions, the amount of energy stored in the elastic component (if any) will play a major role in the results of optimization, especially for the comparison of flapping and revolving wings. The effects of energy storage are modeled according to the following.

In an ideal case, all the power generated is stored in the elastic element, the wing is assumed to be perfectly elastic and no energy is dissipated within the actuator.
Then we equate $\tilde{P}(t)$ to the actuator power consumption $P(t)$:

$$P(t) = \tilde{P}(t).$$

Additionally, for perfectly elastic energy storage, it can be easily shown that the average of the inertial power terms, $M_k \dot{\alpha}$ and $M_\phi \dot{\phi}$, goes to 0 since the trajectory is periodic. However, in either animals or robots, there is no perfectly elastic storage. The other extreme is to assume all the energy is dissipated as heat or by other means (Berman and Wang 2007). Then the actual power consumption becomes

$$P(t) = \begin{cases} \tilde{P}(t) & \text{for } \tilde{P}(t) > 0 \\ 0 & \text{otherwise} \end{cases},$$

which overestimates the power consumption of a flapping wing, as it has been shown that even the lowest power stored ($10\% - 15\%$) can significantly improve the total power consumption for a flapping animal (Lehmann and Dickinson 1997). Therefore if partial storage (Kurdi et al 2010) is available, the actual power consumption becomes

$$P(t) = \begin{cases} \tilde{P}(t) & \text{for } \tilde{P}(t) > 0 \\ \beta \tilde{P}(t) & \text{otherwise} \end{cases},$$

where $\beta \in [0, 1]$ is a scalar that represents the ratio of power storage.

In this work, dimensionless optimization is performed only with $\beta = 0, 0.25$ and $0.5$ settings and cases with more energy storage are neglected. This is partly due to the fact that only a limited amount of energy storage is used by flapping-wing fliers from nature and near-perfect energy storage is also difficult to achieve in engineered flapping-wing fliers. In addition, the limitations of the quasi-steady aerodynamic model prevent us from using high-energy storage assumptions. Specifically, the rotational force term $F_{\text{rot}}$ in the current and other quasi-steady models (Berman and Wang 2007, Nabawy and Crowther 2014a) is non-dissipative, although it yields a good estimation of force locally. While performing the optimization, the non-dissipative property is exploited by the algorithm because it can generate force with zero or negative energy consumption. Through trial-and-error, it is found that, with sufficient energy dissipation introduced to the system, this problem can be avoided. In particular, if $\beta$ is selected to be sufficiently small (e.g. $< 60\%$), the energy dissipation will keep the rotational lift term from being exploited. Here only partial energy storage cases ($\beta = 0, 0.25$ and $0.5$) are selected for optimization and the corresponding results are used in the subsequent analysis.

In this study, it is also observed that the optimizer exploits the rotational lift term with an extremely small stroke amplitude. For such trajectories with small stroke amplitudes ($\approx 10^\circ$) and high pitching velocity, the quasi-steady aerodynamic model predicts a considerable amount of lift with almost no power consumption. This issue is also reported in Yan et al (2015) where the effects of aerodynamic models on optimization are investigated. A common remedy to solve this problem is the addition of a rotational dissipation term to the quasi-steady model (Berman and Wang 2007, Whitney and Wood 2010, Yan et al 2015). However, this remedy is not used in this work because the force coefficients and the correction factors in the aerodynamic model used here (section 2.4) are fitted from computational fluid dynamics (CFD) simulations in the absence of a rotational damping term, the effects of which are partly included in the other terms, and one cannot add such a term without redeveloping a complete quasi-steady model. Instead, a minimum amplitude for the stroke angle is specified ($\pi/10 \leq \phi_m$) to limit the possible wing trajectories in the region where the quasi-steady model is relatively accurate, and to remove the possibility of generating small-amplitude flapping flight which also has not been observed in flapping-wing fliers from nature.

### 2.5 Non-dimensionalization and reference velocity

The selection of the normalization method has critical effects on the results of dimensionless optimization when unsteady flapping flight is considered. The classical way to define these coefficients is as follows:

$$C_L(t) = \frac{F_L(t)}{0.5 \rho R U_{\text{ref}}^2},$$

$$C_P(t) = \frac{P(t)}{0.5 \rho R U_{\text{ref}}^3},$$

where $U_{\text{ref}}$ is the reference velocity and it is usually consistent with the reference velocity used in the definition of $Re$ (thus, in this case $U_{\text{ref}} = U_2$). Then for optimization purposes the cycle-averaged values of these coefficients can be computed as

$$\bar{C}_L = \frac{1}{T} \int_0^T F_L(t) \, dt$$

$$\bar{C}_P = \frac{1}{T} \int_0^T P(t) \, dt,$$

where $T$ is the period of each wing flapping or revolving cycle. Although this normalization is used frequently in the literature, it is not ideal for the purposes of comparing rotary and flapping flight which is explained in the following.

The instantaneous force and power terms are functions of the instantaneous velocity, and the notion of instantaneous $C_L$ and $C_P$ is better represented as follows:

$$F_L(t) \propto C_P(t) \, U_2^2 \, U(t),$$

$$P(t) \propto C_L(t) \, U_2^3 \, U(t),$$

where $U_2(t)$ is the instantaneous velocity at the spanwise location $r_2$. Therefore, if the normalization in
(50) and (51) is used, the cycle-averaged lift and power coefficients satisfy

\[
\bar{C}_L \propto \frac{\int_0^T C_L(t) U_{r_\alpha}^3(t) dt}{U_{r_\alpha}^2}, \tag{54}
\]

\[
\bar{C}_P \propto \frac{\int_0^T C_P(t) U_{r_\alpha}^3(t) dt}{U_{r_\alpha}^2}. \tag{55}
\]

Clearly, \(C_L\) and \(C_P\) are affected by the temporal profile of \(U_{r_\alpha}\), instead of fully revealing the underlying aerodynamic properties. Note that the problem only exists for flapping wings with varying \(U_{r_\alpha}\) especially when there are high peak velocities (so the numerator will be significantly greater than the denominator and this difference will increase as \(U_{r_\alpha}\) gets steeper peaks), and not for steadily revolving wings. Therefore, the above normalization may preclude a meaningful comparative study between the trajectories of flapping and rotary flight or even within flapping flight itself. A similar issue is also reported by Zheng et al (2013). To resolve this problem, we use an alternative normalization scheme as follows:

\[
\bar{C}_L = \frac{\int_0^T F_L(t) dt}{0.5 \rho Re \int_0^T |U_{r_\alpha}(t)|^2 dt}, \tag{56}
\]

\[
\bar{C}_P = \frac{\int_0^T P(t) dt}{0.5 \rho Re \int_0^T |U_{r_\alpha}(t)|^3 dt}, \tag{57}
\]

which eliminates the effects of the velocity profile and leads to a comparison of \(C_L\) and \(C_P\) solely based on the aerodynamic performance of the trajectories. This solution is similar to that used by Zheng et al (2013). One difference is Zheng et al (2013) used the integration of local fluid velocity over the wing surface as the reference velocity, whereas we use the stroke velocity at \(\bar{r}_2\), \(U_{r_\alpha}(t) = \bar{r}_2 R \dot{\phi}(t)\).

2.6. Multi-objective genetic algorithms
The current optimization problem with a highly nonlinear aerodynamic model is extremely challenging to solve analytically, requiring the use of heuristic optimization methods. Among these methods, evolutionary algorithms (EAs) are extensively used for multi-objective optimization because the use of a population in an iteration helps EAs to simultaneously find multiple non-dominated solutions (Deb 2014). Therefore, EAs are important for exploring the Pareto front of multi-objective optimization problems efficiently. Over time, several classes of EAs have been proposed in the literature and among these classes, genetic algorithms (GAs) are the most popular. The non-dominated sorting genetic algorithm (NSGA-II) (Deb et al 2002) is one of the benchmark algorithms in the literature and an efficient algorithm that guarantees diversity among solutions and elitism. The GA package of MATLAB uses a variant of this algorithm to perform multi-objective GA optimization. Since the scope of this work is to reliably find the optimal wing trajectories rather than compare algorithm speeds, NSGA-II is used in the optimization framework presented in this work.

For this problem, the population size selected is 800 and the number of generations chosen is 1000. Additionally, each optimization sequence is repeated eight times with random initial populations to better explore the variable space. Further refinement is not applied to the results of the GA as the solutions converge to results with negligible differences. Furthermore, the upper and lower bounds for the optimization variables are specified in the optimization as linear constraints and are described in previous sections and table 2.

3. Results

To compare the efficiency of flapping and revolving wings, the dimensionless optimization framework is applied to cases with specific wing geometrical properties and values of \(Re\) and \(Ro\), whose effects on the optimal solutions and the corresponding Pareto fronts are analyzed for both revolving and flapping wings. The sensitivities of these Pareto optimal solutions to the energy storage ratio \(\beta\) are also studied.

3.1. Effects of Reynolds number
Here we compare the aerodynamic efficiency between the optimized revolving and flapping trajectories and their dependence on \(Re\) ranging from 100 to 8000. The \(\bar{C}_L - \bar{C}_P\) Pareto fronts for \(Re = 100\), \(Re = 900\) and \(Re = 8000\) are presented in figure 5. The lower limit for the range of investigated \(Re\) is determined by the model used in (Lee et al 2016), since the model is not validated for \(Re < 100\). The efficiency increases as \(\bar{C}_L\) increases and \(\bar{C}_P\) decreases. Therefore, in the \(\bar{C}_L - \bar{C}_P\) plots, the further to the bottom right the Pareto fronts are located, the higher the efficiency they correspond to. Note that the commonly used single-objective measures of aerodynamic efficiency (e.g. dimensionless power factor \(\bar{C}_L^{1.5}/\bar{C}_P\) (Kruyt et al 2014), dimensionless power loading (PL) \(\bar{C}_L/\bar{C}_P\) (Zheng et al 2013), etc) can be found using these Pareto fronts.

From the Pareto fronts in figure 5, it can be seen that, for all energy storage cases, revolving wings are clearly more efficient than flapping wings at the low \(\bar{C}_L\) region while flapping wings are more efficient at higher \(\bar{C}_L\). The advantage of revolving wings at the low \(\bar{C}_L\) region diminishes as \(Re\) becomes smaller. In fact, at \(Re = 100\), the efficiency of flapping wings is almost the same as that of revolving wings for low \(\bar{C}_L\). This result suggests that at sufficiently low \(Re\), it is more efficient to use the unsteady forces introduced by flapping wings than to rely entirely on the steady forces of revolving wings for all \(\bar{C}_L\) regions. As some energy storage is introduced to the system (as \(\beta\) increases to 0.25 and 0.5), it is observed that flapping wings become more efficient at low \(\bar{C}_L\) regions, especially for high \(Re\) cases. Nonethe-
less, the Pareto fronts of different energy storage cases are observed to be almost identical for $C_L > 1$

The above results show that the efficiency strongly depends on the value of $C_L$ (figure 5). Therefore, in addition to $C_L - C_P$ Pareto optimality plots, the efficiency is evaluated by using a single scalar performance measure as a function of $C_L$. For this purpose, the dimensionless PL $(C_L / C_P)$ is used as the scalar objective function. The change of PL with respect to $C_L$ is presented in figures 6(a)–(c) for $Re = 100$, $Re = 900$ and $Re = 8000$. These figures show that when the wing operates at a higher $C_L$ region, flapping wings have higher efficiency even for high $Re$. On the other hand, revolving wings are clearly more efficient at lower $C_L$ where peak efficiency is located. For instance, at $Re = 8000$ (figure 6(c)), the peak efficiency of rotary flight is almost double that of flapping flight. However, as $Re$ decreases, the relative superiority of flapping wings drops and the peak efficiency becomes almost the same for flapping and revolving wings at $Re = 100$. From figure 6, it can also be seen that the maximum efficiency is weakly correlated to the energy storage value for this specific objective function since it occurs around $C_L = 1$ where Pareto fronts with different energy storage values have almost the same $C_P$.

To identify the critical $Re$, denoted as $Re_c$, where flapping wings surpass revolving wings in efficiency, the maximum PL as a function of $Re$ is also presented in figure 6(d). $Re_c$, identified by the intersections of the revolving wing and flapping wing PL–Re curves, is slightly outside of the investigated $Re$ range, but can be approximated to be around $Re_c \approx 100$. Additionally, the efficiency, and consequently $Re_c$, are observed to be only weakly dependent on energy storage for the investigated cases. Therefore, it can be concluded that flapping wings can be more efficient than revolving wings, only when $Re$ is sufficiently small ($Re < 100$ for this performance measure), even if almost all the excess instantaneous energy is preserved in the system.

The overall shape of the optimal wing trajectories and their dependence on $Re$ are also analyzed. The trajectories corresponding to those of the maximal PL for different $Re$ are shown in figure 7 while the instantaneous lift and power coefficients for these trajectories can be found in the appendix. Additionally, the key parameters defining the trajectory characteristics are plotted with varying $Re$ (figure 8). From figure 8(a), it can also be seen that for all energy storage cases, the stroke amplitude settles to the $180^\circ$ limit ($90^\circ$ for a half stroke) when $Re > 300$. This implies that for $Re > 300$, flap-
Figure 6. Power loading graphs: (a) $C_{\lambda}$ versus $C_{\lambda}/C_p$ for $Re = 100$. (b) $C_{\lambda}$ versus $C_{\lambda}/C_p$ for $Re = 900$. (c) $C_{\lambda}$ versus $C_{\lambda}/C_p$ for $Re = 8000$. (d) $Re$ versus $\max(C_{\lambda}/C_p)$.

Figure 7. Optimized trajectories for $\max(C_{\lambda}/C_p)$ (solid line: $Re = 100$, dashed line: $Re = 900$, dotted line: $Re = 8000$) (a) with $\beta = 0$, and (b) with $\beta = 0.5$. Wing rotation delayed significantly for all cases.
ping flight tends to approximate rotary flight with larger flapping amplitudes. On the other hand, the optimal stroke amplitude converges to smaller values (84° at Re = 100 with full energy dissipation and 56° at Re = 100 with half energy dissipation) indicating the unsteady effects becoming more dominant.

The dependence of the optimal pitch angle for revolving wings on Re is shown in figure 8(e), which shows that the optimal angle of attack decreases as Re increases. The optimal angle of attack starts with a relatively high value (18°) at Re = 100 and monotonically decreases to lower values (7° at Re = 8000) similar to those observed in larger air vehicles (Leishman 2006). Furthermore, the mean pitch (figure 8(c)) remains almost at 90° as expected, which corresponds to symmetric angles of attack during downstroke and upstroke. The amplitude of pitch (figure 8(b)), on the other hand, shows a steady but slow increase with Re while the overall α_m is high (meaning that the angle of attack is small). This is expected, since low angle of attack rotary flight is preferred for this performance measure (figure 8(e)).

Interestingly, the phase difference between the pitch and stroke (figure 8(d)) is between −97° and −112°, which corresponds to a delayed wing rotation rather than a symmetric (≈ −90°) or advanced rotation (> −90°). Even though advanced wing rotation can increase the lift produced (Dickinson et al 1999, Sane and Dickinson 2002), it does not necessarily improve the efficiency, possibly due to the concomitant increase of drag. In fact, to the best of the authors’ knowledge, natural fliers rarely use advanced rotation, while delayed or symmetric wing rotation is observed more often (e.g. Cheng et al 2011, Muijres et al 2014).

Figure 8(f) presents the ratio of the lift generated by unsteady components to the steady lift for the optimal wing trajectories for different Re. In this figure, the relative contributions of the lift generation mechanisms do not change significantly with Re. However, it is apparent that the rotational lift causes a decrease in

![Figure 8](image-url)
the total lift resulting from the delayed wing rotation. This indicates the power reduction caused by the delay exceeds the decrease in lift generation.

### 3.2. Effects of aspect ratio (Rossby number)

In this section, the dependence of the optimal flapping and revolving wing trajectories and the corresponding efficiency on $Ro$ (or $AR$) is investigated for constant $Re = 500$. The trajectories are optimized for $AR = 2, 4, 6$ and $8$ which correspond to $Ro = 1.15, 2.31, 3.46$ and $4.62$. The Pareto fronts for $AR = 2, 4$ and $8$ are shown in figure 9.

These results reveal that flapping wings with higher $AR$ are more efficient at the $\tilde{C}_L < 0.9$ region while low $AR$ leads to higher efficiency if higher $\tilde{C}_L$ is needed. Comparison of flapping and revolving wings (figures 9(a) and (b)) shows that for both $AR$ cases, revolving wings are superior to flapping wings except at the high $\tilde{C}_L$ region. However, the crossover $\tilde{C}_L$ becomes smaller as $AR$ increases. As in the $Re$ comparison, a change in energy storage results in an improvement in efficiency for the low $\tilde{C}_L$ region while it becomes insignificant as the required $\tilde{C}_L$ increases.

In the $PL$ versus $\tilde{C}_L$ plots (figures 10(a) and (b)) it can be observed that the maximum $PL$ remains approximately unchanged with $AR$ while the values of $\tilde{C}_L$ for the maximum $PL$ decrease as $AR$ increases, indicating the maximum efficiency occurs at lower $\tilde{C}_L$ for higher $AR$. Similar results can be seen in the maximum $PL$ versus $AR$ plot (figure 10(c)), as the efficiency of revolving wings remains constant when $AR$ changes. The efficiency of flapping wings only weakly depends on $AR$ for the specific objective function and $Re$ chosen here.

The wing trajectories that maximize the $PL$ are shown in figure 11 whereas the instantaneous force and power coefficients corresponding to these trajectories are shown in the appendix. The patterns of wing stroke change drastically with $AR$. As $AR$ increases, the stroke velocity becomes more trapezoidal. Especially at $AR = 8$ (figure 11(b)), the wing rotation period is shorter while the stroke velocity and angle of attack vary less throughout the stroke compared to the trajectories of lower $AR$ wings. In other words, within each half stroke, the trajectory becomes similar to that of a revolving wing with a constant angle of attack. This might be because the inertial power associated with the wing rotation becomes higher as $AR$ decreases, enabling the optimizer to reduce rotational acceleration for wing geometries with lower $AR$. 

![Figure 9: $\tilde{C}_L$ versus $\tilde{C}_P$ Pareto fronts for various $AR$. (a) $AR = 2$, (b) $AR = 8$, (c) Comparison of Pareto fronts of flapping flight with different $AR$ and no energy storage.](image-url)
Figure 10. Power loading graphs. (a) $\bar{C}_L$ versus $\bar{C}_L/\bar{C}_P$ for $\mathcal{AR} = 2$, (b) $\bar{C}_L$ versus $\bar{C}_L/\bar{C}_P$ for $\mathcal{AR} = 8$, (c) $\mathcal{AR}$ versus max$(\bar{C}_L/\bar{C}_P)$.

Figure 11. Optimized trajectories for max$(\bar{C}_L/\bar{C}_P)$ (solid line: $\mathcal{AR} = 2$, dashed line: $\mathcal{AR} = 8$) (a) with $\beta = 0$, and (b) with $\beta = 0.5$. Wing rotation is delayed significantly for $\mathcal{AR} = 2$ whereas it is either symmetric or slightly advanced for $\mathcal{AR} = 8$. 
Another reason for this drastic change might be that the added mass and rotational force terms decrease more quickly with $AR$ than the translational force terms do (18)–(22). This indicates that an increase in $AR$ diminishes the unsteady forces more quickly than the steady components, enabling cases with higher $AR$ to favor trajectories mainly exploiting translational forces. The only other consistent change is the reduction in phase difference between the wing rotation and stroke as $Ro$ increases (figure 12(d)), leading to a more symmetric wing rotation. Note that, for flapping wings with $AR = 8$ the pitching amplitudes for all the cases are similar to those for revolving wings (12°). The effects of these differences are apparent in figure 12(f), in which the ratios of the cycle-averaged lift force created by unsteady forces to the translational forces are shown. As speculated above, the force components due to both the added mass and the rotational forces have a considerable contribution to force generation (or power reduction if negative) at lower $AR$. As $AR$ (or $Ro$) increases, this contribution almost completely disappears compared to that of translational force components.

4. Discussion

4.1. Importance of multi-objective dimensionless optimization

A multi-objective dimensionless optimization framework is used here for investigating the optimal
hovering flight for low Re fliers. Dimensionless optimization yields solutions for a range of problems sharing similar physical characteristics while multi-objective optimization generates Pareto fronts that can be used for performance measures based on monotonic functions of \( C_L \) and \( C_P \). As a result, given optimizations with a relatively small parameter space, e.g. different wing dimensionless geometries, \( Re, Ro \), etc., the optimized wing trajectories can be used for evaluating the performance of a wide range of physical wing trajectories. Therefore, the results of the current work, i.e. the Pareto fronts of \( C_L \) and \( C_P \) (figure 5), can have two major applications: (1) serving as a benchmark for analyzing the performance of a given wing trajectory, and (2) providing guidance for determining optimal wing trajectories in the conceptual design stage of MAVs, especially for determining whether flapping or revolving wings should be used.

First, the Pareto fronts of \( C_L \) and \( C_P \) obtained here can be effectively used for the performance analysis of known wing trajectories and dimensions. For example, if the overall \( F_L \) and \( P \) for a given trajectory are well above the physical Pareto front obtained by dimensionalizing the \( C_L - C_P \) Pareto front based on the known dimensions, it is clear that there exists a more efficient wing trajectory for hovering.

The second and probably more important application of having dimensionless Pareto fronts of \( C_L \) and \( C_P \) for various \( Re \) and \( Ro \) is the potential guidance it can provide for the conceptual design of MAVs. With a sufficient amount of dimensionless Pareto fronts generated for a wide range of \( Re \) and \( Ro \), one can first determine whether flapping or revolving wings should be used, and then the optimal wing trajectory can be designed in an iterative fashion. An example of such a design methodology is described below. Assuming the desired wing length \( R \), the maximum power available and the minimum lift required are known for a particular design of MAV, one can determine the flapping frequency \( f \) associated with each point on every Pareto front, with the same \( Re \). Note that each point on the Pareto front corresponds to a different dimensionless wing trajectory and therefore a different dimensionless wing velocity. Therefore, with the derived time scale, all the dimensionless Pareto fronts can be dimensionalized keeping the optimality for \( C_L - C_P \). Then, the dimensionless Pareto fronts which have the points corresponding to the minimum PL required (i.e. the minimum weight and highest power consumption) falling outside of them will be infeasible (note that the Pareto fronts are convex in all the cases); in other words, the trajectories on these Pareto fronts cannot meet the lowest efficiency requirements. On the other hand, if the Pareto fronts enclose the minimum or any feasible region of PL, then the most desirable optimal trajectory on the feasible Pareto fronts can be selected based on other design objectives. If none of the Pareto fronts are feasible, then the design parameters, constraints and/or objectives should be changed. Therefore, in this scenario, the \( C_L - C_P \) Pareto fronts provide guidance for determining both the vehicle design parameters and the flapping/revolving trajectories of the MAV.

Despite the appealing applications of the optimization framework in terms of compiling a set of Pareto fronts as a catalog for low Re flights, there are several drawbacks worth noting. For instance, parameterization and the constraints imposed on the trajectory parameters limit the trajectory space leading to suboptimal solutions for flapping wings. This problem can be solved by repeating the same procedure with different sets of parameterization so that the majority of the trajectory space is covered in the optimization. Additionally, the limitations on parameterization can be relaxed to some extent resulting in a larger variable space and a closer approximation to physically achievable trajectories. However, these constraints might be difficult to remove completely because they prevent the optimizer from exploiting the non-dissipative nature of the aerodynamic model, which itself is the third source for suboptimal results. Clearly, the optimized solutions can only be as good as the aerodynamic model permits. In this regard, quasi-steady models could perform poorly in some regions (e.g. for very small stroke amplitudes and for highly unsteady forces), since they are only local approximate models of the actual fluid dynamics. The potential issues related to quasi-steady models in optimization are further discussed in section 4.3.

### 4.2. Optimality of the results in dimensional space

The objective functions for the optimization framework selected above are \( C_L \) and \( C_P \). While converting the resulting optimal trajectories and Pareto fronts to these in dimensional space, our assumption is that the resulting trajectories and the optimality are approximately the same as the ones obtained by optimizing for the average \( F_L \) and \( P \) for a specific dimensional case with fixed \( Re \). However, this only holds if \( C_L \) and \( C_P \) are linear to \( F_L \) and \( P \) respectively which is not valid. Specifically, using the definitions in (50) and (51), we can derive

\[
\int_0^1 F_L(t) \, dt \propto f^2 C_L \int_0^1 \left| \phi' \right|^2 \, dt, \tag{58}
\]

\[
\int_0^1 P(t) \, dt \propto f^3 C_P \int_0^1 \left| \phi' \right|^3 \, dt. \tag{59}
\]

Clearly, this is not a linear relationship as \( \tilde{C}_L \) and \( \tilde{C}_P \) are multiplied with different functionals of \( \phi(t) \) which is one of the optimized variables. Thus, the optimality of the proposed trajectories is not guaranteed if the overall objective is to optimize \( F_L \) and \( P \). However, we expect the proposed model’s results to be close to
the optimal ones in dimensional space since the ratio between \( f_0^1 \int_0^1 \hat{\phi}^3 \, dt \) and \( f_0^1 \int_0^1 \hat{\phi}^3 \, dt \) does not change significantly for the given feasible trajectory space and the constant \( f_0^1 \int_0^1 \hat{\phi}^3 \, dt \) that is ensured by keeping \( Re \) constant.

This problem can be fixed and dimensionless objective functions can be made linear with dimensional ones by revising the definitions of \( C_L \), \( C_p \) and \( Re \), which are shown in the following. Let the force and power coefficients be defined as

\[
C_{L/norm} = \frac{\int_0^1 F_L(\hat{t}) |F_L(\hat{t})|^{1/2} \, d\hat{t}}{f^3 \int_0^1 \hat{\phi}^3 \, d\hat{t}}, \quad (60)
\]

\[
C_p = \frac{\int_0^1 \bar{\gamma}(\hat{t}) \, d\hat{t}}{f^3 \int_0^1 \hat{\phi}^3 \, d\hat{t}}, \quad (61)
\]

where \( \gamma_{C_L} \) and \( \gamma_{C_p} \) denote the constant values obtained from the fluid medium and wing geometry (e.g. \( \rho, f_2 \), etc.). Then, if we define another \( Re \) denoted as \( Re_{norm} \), which is kept constant in the optimization framework and defined as

\[
Re_{norm} = \gamma_{Re} \left( f^3 \int_0^1 \hat{\phi}^3 \, d\hat{t} \right)^{1/5}, \quad (62)
\]

where \( \gamma_{Re} \) is a constant value that is the combination of the fluid and wing geometries, \( C_L \) and \( C_p \) can be written as

\[
C_{L/norm} = \frac{\int_0^1 \bar{\gamma}(\hat{t}) |F_L(\hat{t})|^{1/2} \, d\hat{t}}{\frac{Re_{norm}^3}{\gamma_{Re}}}, \quad (63)
\]

\[
C_p = \frac{\int_0^1 \bar{\gamma}(\hat{t}) \, d\hat{t}}{\frac{Re_{norm}}{\gamma_{Re}}}. \quad (64)
\]

With \( Re_{norm} \), this set of force coefficients, if used as the dimensionless objective functions, will always be linear to the dimensional ones if the dimensional ones selected are \( \int_0^1 F_L(\hat{t}) |F_L(\hat{t})|^{1/2} \, d\hat{t} \) and \( \int_0^1 \bar{\gamma}(\hat{t}) \, d\hat{t} \). Therefore, the resulting trajectories for dimensionless optimization are also the solutions for the ones with dimensional optimization. Additionally, Pareto fronts with the same \( Re_{norm} \) retain their relative values during dimensionalization; thus, the comparison of results, such as \( Re_{c} \), in dimensionless space will also be valid for dimensional comparisons. Note that the \( Re \) which is used in the aerodynamic model is different from \( Re_{norm} \). These two can be converted to each other with the following relation:

\[
Re = Re_{norm} \frac{\int_0^1 \hat{\phi} \, d\hat{t}}{\left( \int_0^1 \hat{\phi}^3 \, d\hat{t} \right)^{1/3}}, \quad (65)
\]

Although this normalization method is more straightforward for navigating between dimensionless and dimensional spaces, in this work a more conventional method is used to nondimensionalize lift force and power (section 2.5). The major reason behind this selection is our aim to stay as consistent as possible with the definitions in the literature. However, regardless of our selection here, \( \int_0^1 F_L(\hat{t}) |F_L(\hat{t})|^{1/2} \, d\hat{t} \) and \( \int_0^1 \bar{\gamma}(\hat{t}) \, d\hat{t} \) can be suitable measures of lift and power respectively. Thus, this normalization method can be superior to what is used in this paper. For this reason, we will test this normalization method in future work.

4.3. Issues and limitations of quasi-steady model in the optimization framework

The most critical issue of the quasi-steady model in optimization that needs to be addressed is probably the non-dissipative nature of the rotational lift term \( (20) \). Although current models of rotational lift in the literature work well for approximating experimental or numerical results generated based on a specific range of wing trajectories, they appear to be non-dissipative, i.e. they can generate lift while generating power, which is obviously unphysical. Furthermore, this unphysical behavior of rotational lift stands out more in an optimization, since the optimizer, if not designed or constrained properly, can exploit such loopholes and generate unrealistic trajectories. In this work, this problem is overcome by taking advantage of the energy dissipation term in the form of the energy storage ratio \( \beta \), which in fact has physical significance since it represents the loss of energy related to the mechanical properties of the actuation mechanism. Nonetheless, to achieve more reliable optimal wing trajectories with the proposed framework, the rotational lift term in the current model should be improved so that it becomes dissipative.

In addition, the quasi-steady model used in this work has limited accuracy for the estimation of lift and power. Many unsteady aerodynamic lift generation mechanisms that are observed in insect flight, such as wing–wake interaction (Sane and Dickinson 2002), the Wagner effect (Wagner 1925), clap and fling (Weis-Fogh 1973) etc., are ignored in the current aerodynamic model. Therefore, it is desirable to include these unsteady mechanisms in the optimization framework using a more complex aerodynamic model or using dynamically scaled robotic experiments (Motamed and Yan 2007), to identify and compare the optimal solutions of flapping and revolving wings more accurately. Since the unsteadiness ignored here can only be exploited by flapping wings, the Pareto fronts for flapping wings can change if a more accurate model is used. However, we are unable to assess the extent and direction of this change only based on the quasi-steady...
model, since the trade-off between additional benefits of the unsteadiness and the corresponding power consumption is unknown. In this regard, optimization using dynamically scaled wing experiments can lead to more accurate results (see section 5).

Regardless of these problems, after manipulations preventing the exploitation of unphysical mechanisms, solutions of the optimization problem are observed to be classical flapping wing trajectories, for which quasi-steady models are locally fitted and give accurate estimates. Specifically, the quasi-steady model used in this work is validated by Lee et al (2016) for such trajectories and force and torque estimations observed in validation to be a good match with the CFD results. Therefore, we expect the \( C_L \) and \( C_P \) values reported in this study to be a decent representation of the actual aerodynamics. Furthermore, the scope of this work is to present the general trend of the effects of \( Re \) and \( Ro \) on optimality rather than to provide accurate efficiency values for every single point on the Pareto fronts. Thus, the general accuracy of the quasi-steady model is acceptable, considering the resulting trajectories.

5. Conclusions and future work

In this study, we present a dimensionless optimization framework that is developed for low \( Re \) hovering flight. Since the optimization is performed in dimensionless space, the optimal solutions possess a generality lacking in dimensional optimization. Additionally, the proposed framework utilizes a multi-objective optimization method which reveals explicitly the trade-off between the lift generation and power consumption at low \( Re \) flight.

The proposed optimization method is used to identify the optimal wing trajectories for a rectangular wing. The maximal efficiencies of revolving and flapping trajectories are compared for which \( Re \) and \( Ro \) act as control variables. Our findings indicate that rotary flight is more efficient if high \( C_L \) (\(<1 \) for \( Re = 100 \) and \(<1.6 \) for \( Re = 8000 \)) is not required. However, the flapping trajectories potentially perform comparably or better than revolving wings for lower \( Re \) cases, regardless of the energy storage provided. On the other hand, flapping wings are more efficient for tasks demanding high \( C_L \). Using the dimensionless PL \( (C_L/C_P) \) as the performance measure, rotary flight is observed to be dominant over flapping flight since the optimal solutions are located in low \( C_L \) regions. It is also observed that delayed wing rotations are preferred to advanced and symmetric rotations for the solved problem. Moreover, flapping wings with high \( Ro \) (or \( AR \)) are observed to be more efficient at low \( C_L \) regions while wings with lower \( Ro \) have higher efficiency for \( C_L > 0.9 \). For the selected case and the single-objective performance measure \( C_L/C_P \), the effects of \( Ro \) on efficiency are relatively negligible. Due to the constraints on the kinematic space and the aerodynamic model used, this work sets the lower boundary for the efficiency of flapping wings. Furthermore, it should be noted that the conclusions made in this study are only valid for the specific case described in section 2.2. For instance, wings with different or morphing cross-sections can yield a higher aerodynamic efficiency for both rotary and flapping wings.

Our results clearly indicate that the Pareto fronts, whether with flapping or revolving wings, move to a considerably more efficient region as \( Re \) increases. Therefore, from the design perspective, to increase efficiency, the size of the MAV should be increased since the increase of \( Re \) is approximately proportional to the lift or the weight. However, one should also note that smaller vehicle sizes are more advantageous for maneuverability.

This work is our first step in comparing flapping and revolving wings systematically in which we lay the framework that can be used for further studies. As a future work, this framework will be applied to various cases with different wing geometries and trajectory parameterizations where the normalization method proposed in section 4.2 will also be tested. Additionally, since the quasi-steady model used in the current study could limit the optimality of the solution, future efforts should be made to include unsteady mechanisms in the optimization framework to identify and compare the optimal solutions of flapping and revolving wings more accurately. Toward this goal, we are currently investigating the real-time optimization of wing trajectories by combining experimental fluid dynamics and reinforcement learning in a hardware-in-the-loop configuration (Bayiz et al 2018).

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Appendix. Force and power coefficients for solutions

In this section, the instantaneous force and power coefficients are presented for the trajectories in figures 8 and 12.
Figure A1. The instantaneous force and power coefficients for the trajectories in figure 8. (a) and (b) The coefficients for $\beta = 0$. (c) and (d) The coefficients for $\beta = 0.5$. Note that the $\bar{C}_P$ presented here represents the total power consumption, including the inertial power.

Figure A2. The instantaneous force and power coefficients for the trajectories in figure 12. (a) and (b) The coefficients for $\beta = 0$. (c) and (d) The coefficients for $\beta = 0.5$. Note that the $\bar{C}_P$ presented here represents the total power consumption, including the inertial power.
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