MANOMETERS

In this lesson, we will:

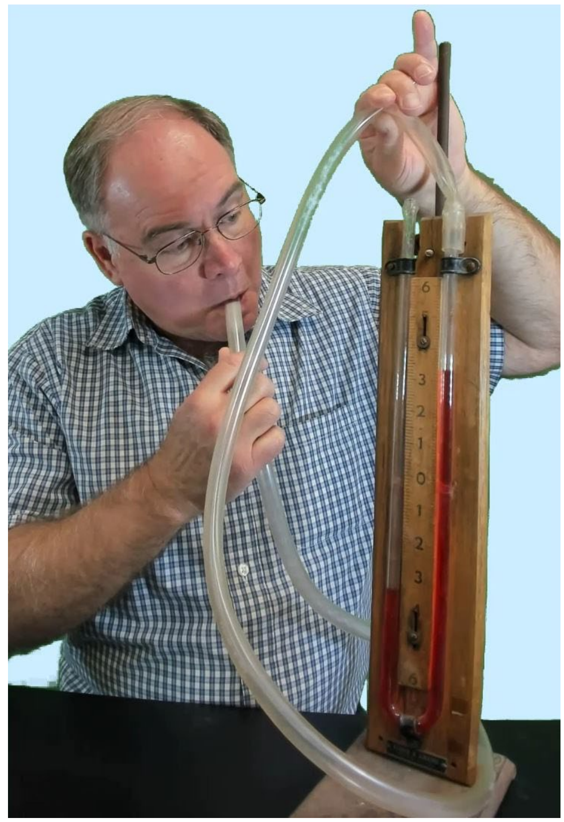
- Describe the purpose of a **manometer** and demonstrate how it works
- Discuss a simple way to analyze manometers of any shape and/or fluids
- Do some example problems

Manometer Basics: Purpose, Equation, and Demonstration

- The purpose of a manometer is to measure an unknown pressure or pressure difference.
- The only equation we need is our workhorse equation for hydrostatics,

$$\boxed{P_{\text{below}} = P_{\text{above}} + \rho g |\Delta z|}$$

- Quick demonstration of a U-tube manometer



Notes from the demonstration:

1. When Δz = 8.0 inches of water, we calculate the gage pressure in Professor Cimbala's mouth: blowing

$$\Delta P = \rho g h = \left(1000\ \frac{kg}{m^3}\right)\left(9.807\ \frac{m}{s^2}\right)(8\ in)\left(\frac{0.0254\ m}{in}\right)\left(\frac{N \cdot s^2}{kg \cdot m}\right)\left(\frac{kPa \cdot m^2}{1000\ N}\right) = 1.993\ kPa$$

$$\boxed{\Delta P = 2.0\ kPa}$$

2. When Δz = –8.0 inches of water, we calculate the gage pressure in Professor Cimbala's mouth: suction

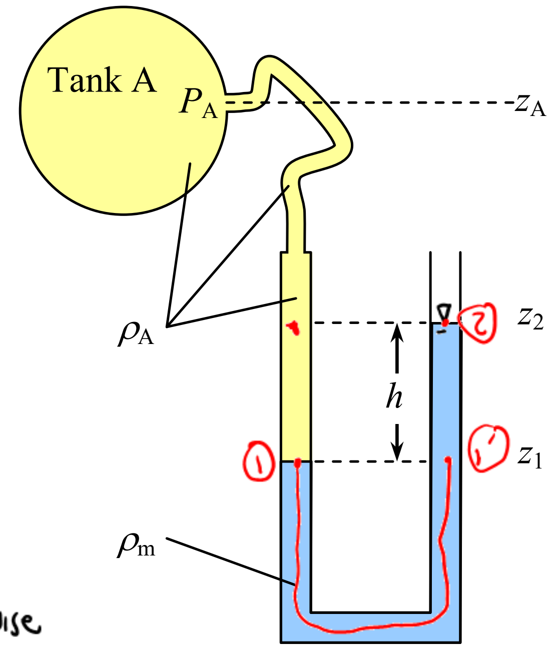
$$\boxed{\Delta P = -2.0\ kPa} \quad \text{or} \quad \boxed{\Delta P = 2.0\ kPa\ \text{vacuum}}$$

Example: Pressure measurement with a U-tube manometer

Given: A U-tube manometer is used as an instrument to measure the pressure in a tank. The right leg of the manometer is open to atmospheric pressure.

(*a*) To do: Calculate the absolute and gage pressure P_A and $P_{A,gage}$ for the general case in which ρ_A is not small compared to ρ_m.

(*b*) To do: Simplify for the case in which $\rho_A << \rho_m$ (e.g., A is air and m is mercury).



Manometer

Solution:

Use $\boxed{P_{below} = P_{above} + \rho g |\Delta z|}$

(a) • $P_2 = P_{atm}$

• $P_1 = P_1'$

• Start @ P_A & work around counterclockwise

$$P_A \underbrace{+ \rho_A g (z_A - z_2) +}_{\text{going down}} \rho_A g (z_2 - z_1) - \rho_m g (z_2 - z_1) = P_2 = P_{atm}$$

going up

Simplify:
$$\boxed{P_A = P_{atm} + (\rho_m - \rho_A) g (z_2 - z_1) - \rho_A g (z_A - z_2)} \; ☆$$

neglect

(b) If $\rho_A << \rho_m$,

$$\boxed{P_A \approx P_{atm} + \rho_m g (z_2 - z_1) - \rho_A g (z_A - z_2)}$$

• It is best to keep all terms

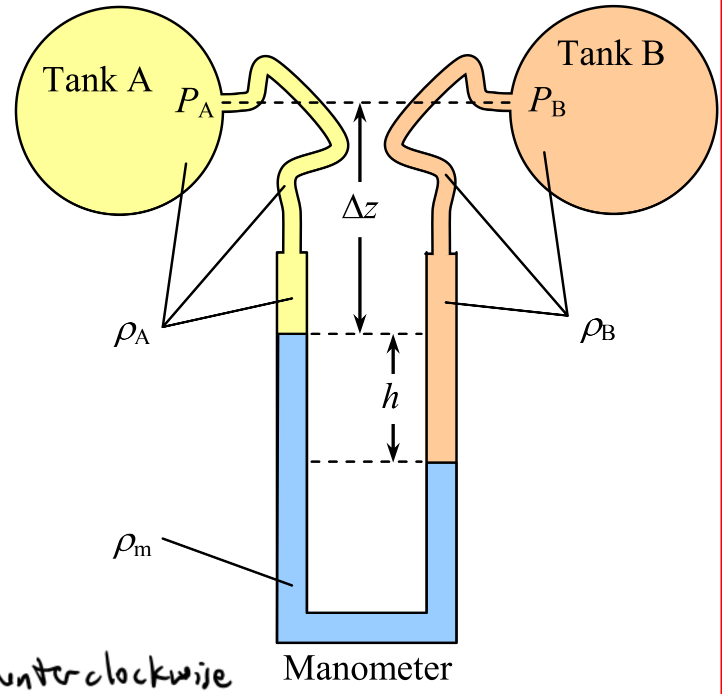
• You can plug in some numbers

Example: Pressure measurement with a U-tube manometer

Given: A U-tube manometer is used as a differential pressure measurement instrument to measure the pressure difference between two tanks. The two tanks are at the same elevation.

(*a*) To do: Calculate the pressure difference $P_B - P_A$ for the general case in which ρ_A is not the same as ρ_B (they are different fluids.

(*b*) To do: Simplify for the case in which $\rho_A = \rho_B$ (they are the same fluid).



Solution:

$$\boxed{P_{below} = P_{above} + \rho g |\Delta z|}$$

(a) Start @ A & work around counterclockwise

$$P_A \underbrace{+ \rho_A g \Delta z +}_{\text{going down}} \rho_m g h \underbrace{- \rho_B g h -}_{\text{going up}} \rho_B g \Delta z = P_B$$

$$☆ \quad \boxed{P_B - P_A = (\rho_m - \rho_B) g h + (\rho_A - \rho_B) g \Delta z} \quad ☆$$

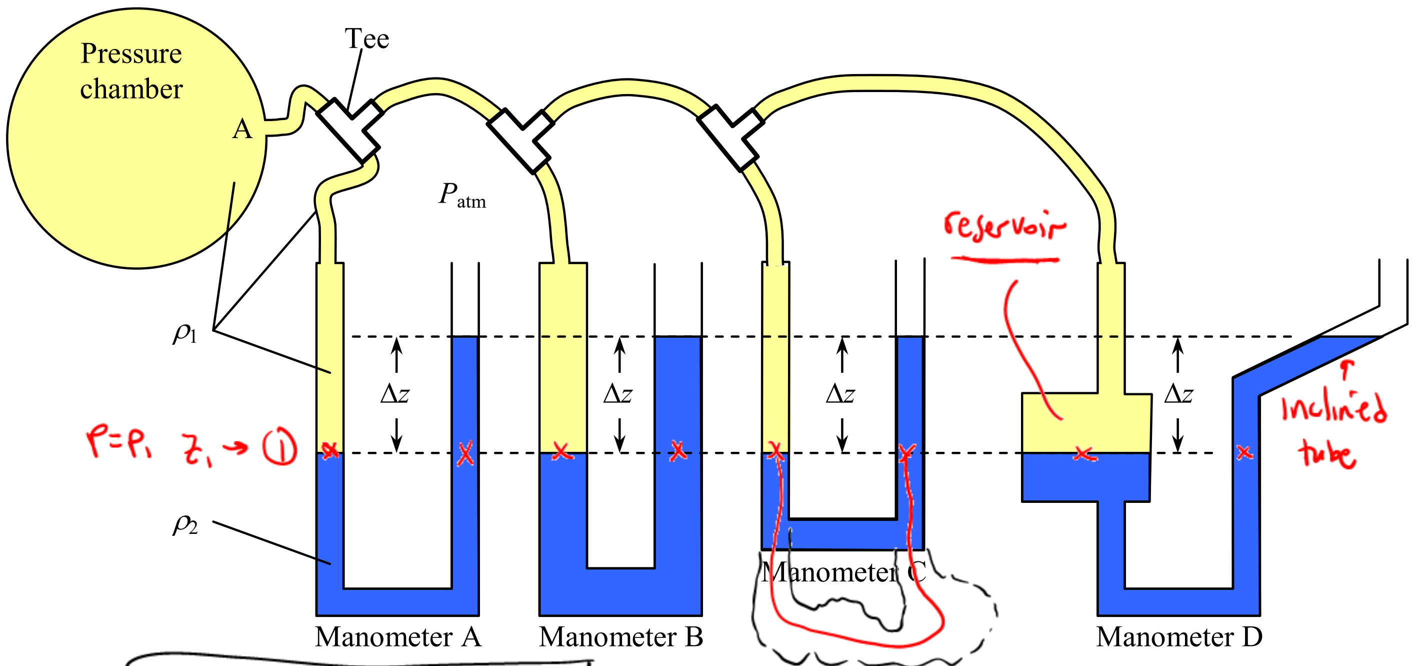
(b) If $\rho_A = \rho_B$, the last term = 0

$$\boxed{P_B - P_A \approx (\rho_m - \underset{(\text{or } \rho_A)}{\rho_B}) g h}$$

Some Notes About Manometry

The elevation difference Δz in a U-tube manometer does *not* depend on the following:

1. **U-tube diameter** (provided that the tube diameter is large enough that capillary effects are negligible). In the sketch below, for a given pressure in the tank, Δz is the same in manometers A and B, even though the tube diameter of manometer B is larger than that of manometer A. Note that the amount of manometer liquid in each of the U-tube manometers has been adjusted such that the level of the interface between fluids 1 and 2 on the left side of each manometer is at the same elevation, for direct horizontal comparison.



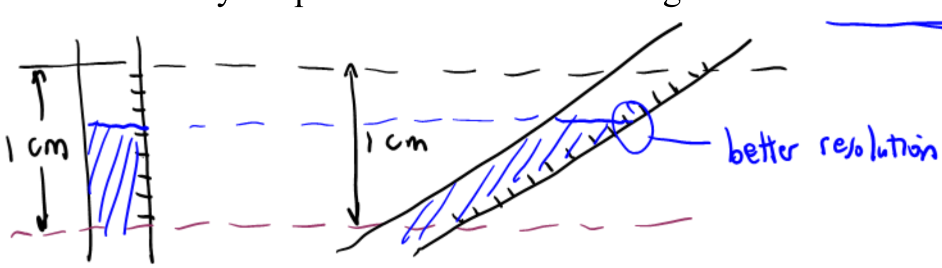
$\boxed{P_{below} = P_{above} + \rho g |\Delta z|}$ → $P \neq$ fnc of x or y, only z

∴ Diameter does not matter

2. **U-tube length** (provided that the tubes are long enough to include elevation difference Δz). In the sketch, Δz is the same in manometers A and C, even though manometer C is shorter than manometer A. Why?

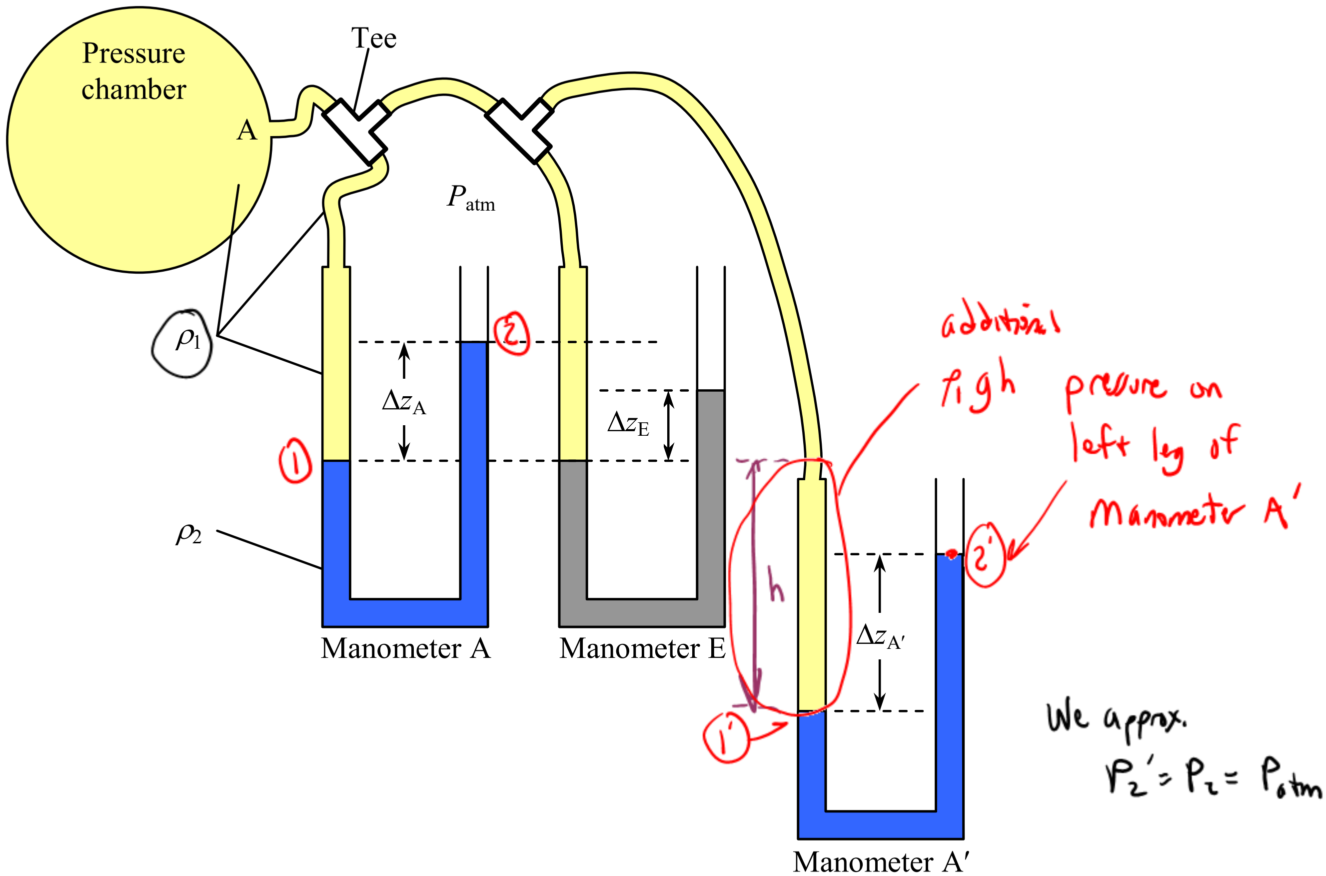
Below interface 1, nothing matters, as long as it is the same fluid

3. **U-tube shape** (again provided that capillary effects are not important and the relative elevation is the same). In the sketch, Δz is the same in manometers A and D, even though manometer D is oddly shaped. What is the advantage of an "inclined manometer?"

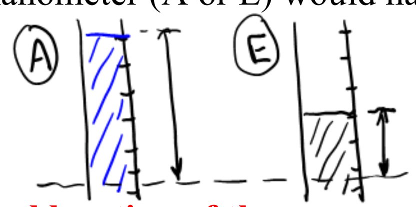


The elevation difference Δz in a U-tube manometer *does* depend on the following:

1. **Manometer fluid**. For example, if we replace the blue manometer fluid in the above sketch with a *higher density* (gray colored) fluid, as in the sketch below, Δz would *decrease*. In other words, $\Delta z_E < \Delta z_A$.



Which manometer (A or E) would have better *resolution*?



2. **Vertical location of the manometer**. For example, if we move manometer A to a lower elevation, all else being the same, and ignoring changes in atmospheric pressure (manometer A′ in the above sketch), Δz would *increase*, i.e., $\Delta z_{A'} > \Delta z_A$. Why?

- The yellow liquid has some density ρ_1
- The higher pressure in manometer A' pushes the blue manometer fluid higher on the right side

Note: if $\rho_1 << \rho_2$, then $\Delta z_{A'} \approx \Delta z_A$, regardless of the vertical location of the manometers. This is usually the case, for example, when fluid 1 is a gas, but the effect can be significant if both fluids are liquids.