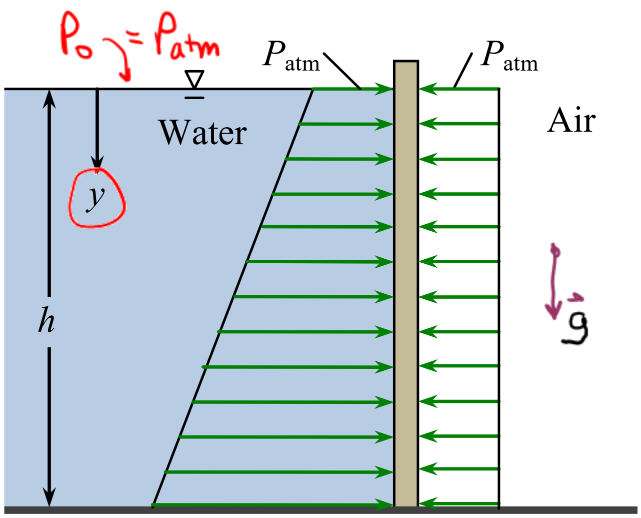
SUBMERGED VERTICAL PLATE

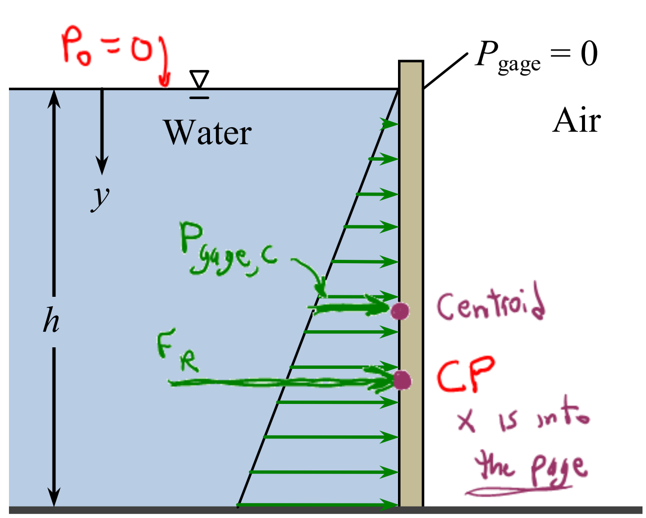
In this lesson, we will:

- Begin a discussion about hydrostatic forces on submerged surfaces
- In particular, we will examine the resultant force on a submerged vertical plate
- Do an example problem

Simplest Case: Rectangular Container with Vertical Walls



Absolute pressure distribution



Gage pressure distribution

$$\boxed{F_R = \int_A P_{gage}\, dA}$$

We can integrate for any shape plate @ any depth

CP = Center of Pressure = location on the plate where the resultant force acts

[Imagine replacing the actual force distribution by a single resultant force — this force acts at the CP]

$$\boxed{F_R = P_{gage,\,average} \cdot A}$$

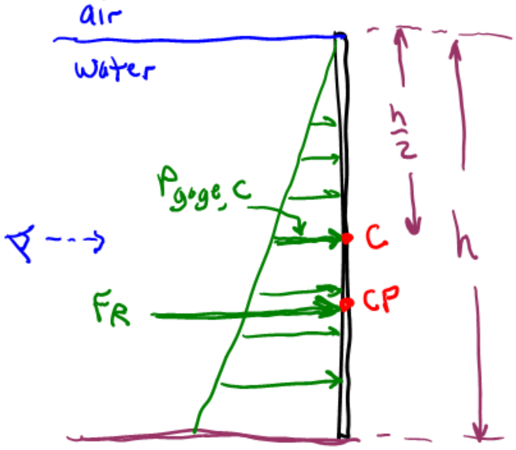
* Acts ⊥ to the plate

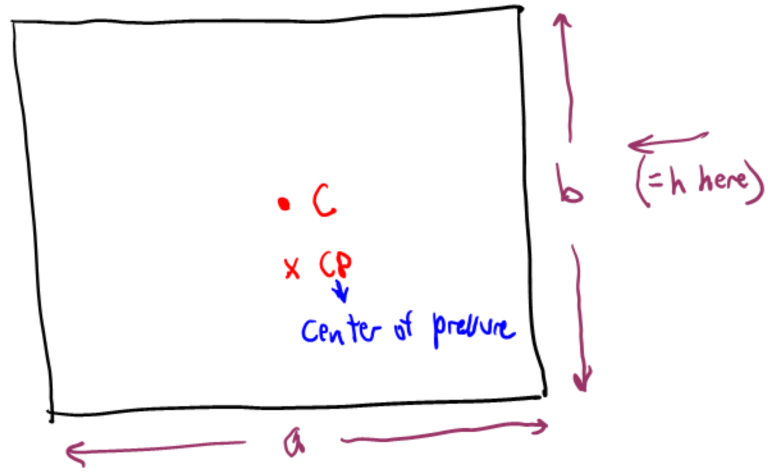
P half-way down

Area of the plate (one side)

$$\boxed{P_{gage,\,average} = P_{gage,c}}\ *$$

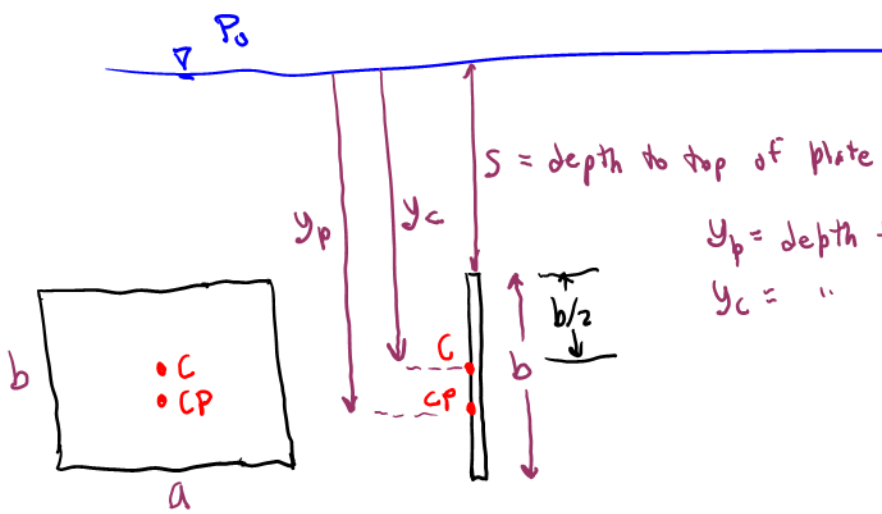
CENTROID = the mathematical center of the surface area





- How to calculate the location of the CP?

- <u>Submerged vertical plate</u>



y_p = depth to center of pressure

y_c = " " centroid of the plate

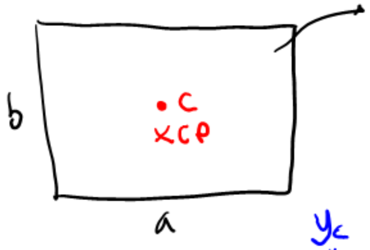
- For a rectangular plate, C is at the center, so $\boxed{y_c = s + \frac{b}{2}}$ ☆

- What about y_p? [We analyze the general case in the next lesson]

☆ $$\boxed{y_p = y_c + \frac{I_{xx,c}}{y_c A}}$$

where $I_{xx,c}$ = 2^{nd} moment of area about the axis into the page (x-axis) passing through the centroid

$I_{xx,c}$ = <u>centroidal moment of inertia</u>



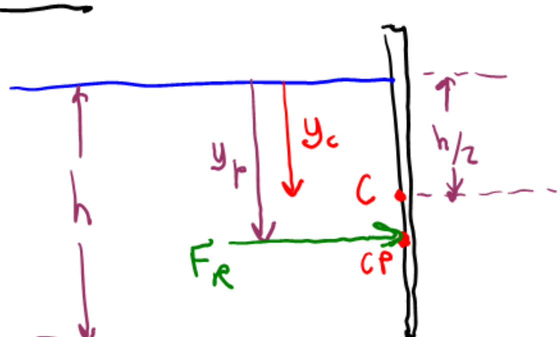
$$I_{xx,C} = \frac{ab^3}{12}$$

So $y_p = \underbrace{s + \frac{b}{2}}_{y_c} + \frac{ab^3}{12\underbrace{\left(s+\frac{b}{2}\right)}_{y_c}\underbrace{ab}_{A}}$

$$\boxed{y_p = s + \frac{b}{2} + \frac{b^2}{12\left(s+\frac{b}{2}\right)}}$$

CP depth for a Vertical submerged rectangular plate

Simplest case:



s=0 in this case

Eq's

$$\boxed{y_c = \frac{h}{2}}$$

$$F_R = P_{gage,avg} \, A = \left(\rho g \frac{h}{2}\right)(ah)$$

$P_{gage,avg} \rightarrow P_{gage,c} = \rho g \frac{h}{2}$

$A \rightarrow ah$

$$\boxed{F_R = \rho g a \frac{h^2}{2}}$$

(b=h here)

$$y_p \ (\text{or } y_{CP}) = \cancel{s} + \frac{h}{2} + \frac{h^2}{12(\cancel{s} + h/2)} = \frac{h}{2} + \frac{h^2}{6h} = \frac{2h}{3}$$

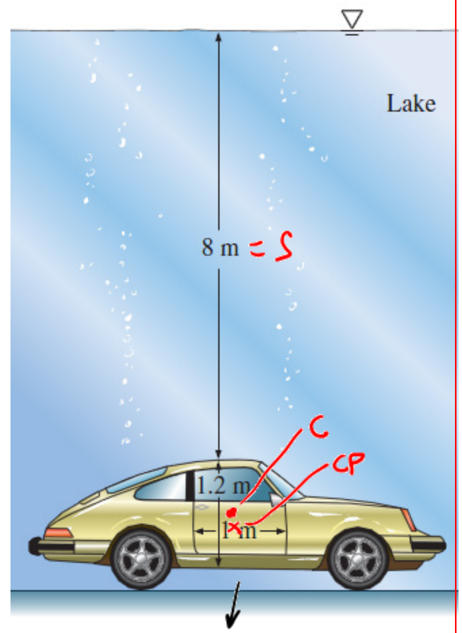
$$\boxed{y_p = \frac{2h}{3}}$$

CAUTION: THIS DOES NOT HOLD FOR CASES WHERE $s \neq 0$

Practical Example: Submerged Car (Example 3.8 of Çengel and Cimbala, Ed. 4)

EXAMPLE 3–8 Hydrostatic Force Acting on the Door of a Submerged Car

A heavy car plunges into a lake during an accident and lands at the bottom of the lake on its wheels (Fig. 3–35). The door is 1.2 m high and 1 m wide, and the top edge of the door is 8 m below the free surface of the water. Determine the hydrostatic force on the door and the location of the pressure center, and discuss if the driver can open the door.



SOLUTION A car is submerged in water. The hydrostatic force on the door is to be determined, and the likelihood of the driver opening the door is to be assessed.

Assumptions **1** The bottom surface of the lake is horizontal. **2** The passenger cabin is well-sealed so that no water leaks inside. **3** The door can be approximated as a vertical rectangular plate. **4** The pressure in the passenger cabin remains at atmospheric value since there is no water leaking in, and thus no compression of the air inside. Therefore, atmospheric pressure cancels out in the calculations since it acts on both sides of the door. **5** The weight of the car is larger than the buoyant force acting on it.

Properties We take the density of lake water to be 1000 kg/m³ throughout.

$b = 1.2\text{ m}$, $a = 1.0\text{ m}$

$$y_c = \text{depth of centroid} = \boxed{S + \frac{b}{2} = y_c} = \left(8 + \frac{1.2}{2}\right)\text{m} = \boxed{8.60\text{ m} = y_c}$$

$$\boxed{P_{gage,avg} = \rho g y_c} \quad ; \quad F_R = P_{gage,avg} \cdot A$$

$$F_R = \rho g \left(S + \frac{b}{2}\right)(a)(b) = \left(1000\frac{\text{kg}}{\text{m}^3}\right)\left(9.807\frac{\text{m}}{\text{s}^2}\right)\left(8 + \frac{1.2}{2}\right)\text{m}\,(1.0\text{ m})(1.2\text{ m})\left(\frac{\text{kN}\cdot\text{s}^2}{1000\text{ kg}\cdot\text{m}}\right)$$

$$F_R \cong 101.208\text{ kN} \qquad \boxed{F_R = 101.\text{ kN}}$$

$$CP \rightarrow @\ y_p = y_c + \frac{I_{xx,c}}{y_c A} = S + \frac{b}{2} + \frac{ab^3}{12\left(S + \frac{b}{2}\right)(ab)} = S + \frac{b}{2} + \frac{b^2}{12(S + b/2)}$$

$$y_p = \left(8 + \frac{1.2}{2}\right)\text{m} + \frac{(1.2\text{m})^2}{12(8 + 1.2/2)\text{m}} \cong \underline{8.61395\text{ m}} \qquad \boxed{y_p = 8.61\text{ m}}$$

Discussion A strong person can lift 100 kg, which is a weight of 981 N or about 1 kN. Also, the person can apply the force at a point farthest from the hinges (1 m farther) for maximum effect and generate a moment of 1 kN·m. The resultant hydrostatic force acts under the midpoint of the door, and thus a distance of 0.5 m from the hinges. This generates a moment of 50.6 kN·m, which is about 50 times the moment the driver can possibly generate. Therefore, it is impossible for the driver to open the door of the car. The driver's best bet is to let some water in (by rolling the window down a little, for example) and to keep his or her head close to the ceiling. The driver should be able to open the door shortly before the car is filled with water since at that point the pressures on both sides of the door are nearly the same and opening the door in water is almost as easy as opening it in air.

<u>You cannot open the door !!</u>