RIGID-BODY ACCELERATION

LESSON 03E

In this lesson, we will:

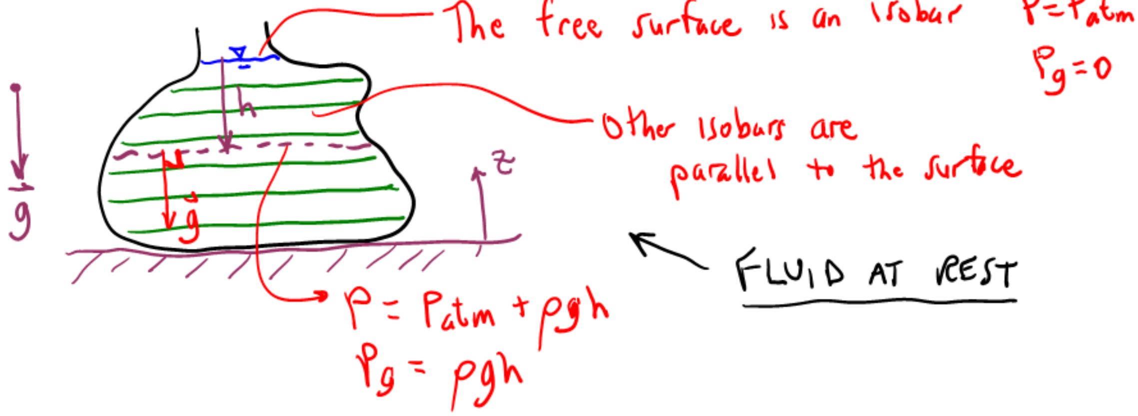
- Define **isobars** and discuss how they change when a fluid is accelerating
- Derive the equation of motion for a fluid in rigid-body linear acceleration
- Do an example problem

Isobars

"bar" = pressure

An isobar is a surface of constant pressure in a fluid.

"iso" = same

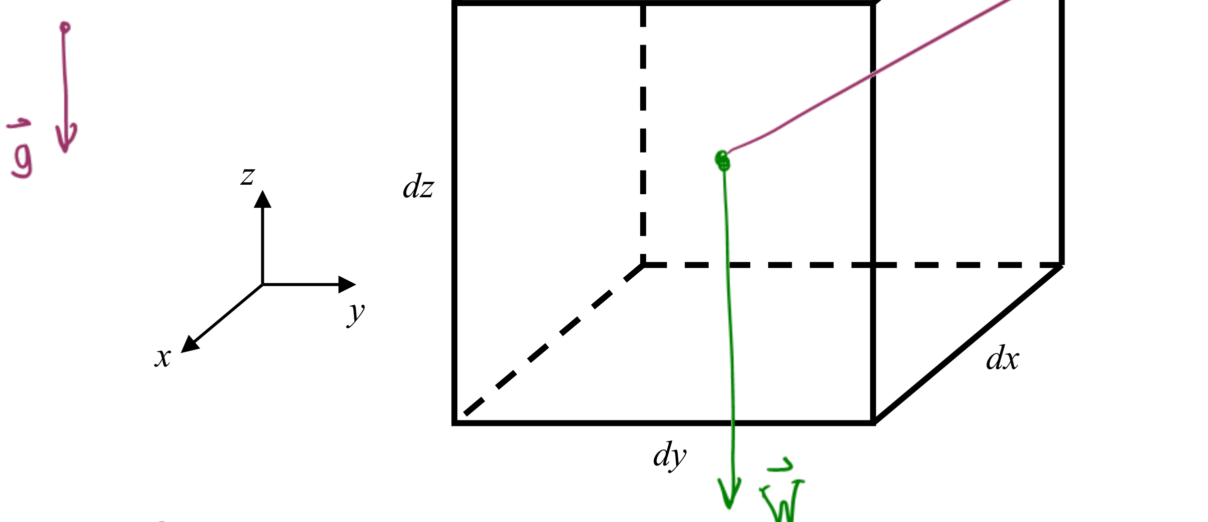


Rigid-Body Linear Acceleration of a Fluid

Recall our little element of fluid from a previous lesson.

For statics, $\sum \vec{F} = 0$

with $\vec{a}$, $\boxed{\sum \vec{F} = m\vec{a}}$



$$\sum \vec{F} = \sum \vec{F}_{grav} + \sum \vec{F}_{pressure} = m\vec{a}$$

Vector eq.

$$\sum \vec{F} = \rho \vec{g}\,\cancel{dx\,dy\,dz} - \underbrace{\left(\frac{\partial P}{\partial x}\vec{i} + \frac{\partial P}{\partial y}\vec{j} + \frac{\partial P}{\partial z}\vec{k}\right)}_{\vec{\nabla} P \,=\, \text{gradient of } P}\cancel{dx\,dy\,dz} = \rho\,\cancel{dx\,dy\,dx}\,\vec{a}$$

SHOULD BE dz not dx here

$$\rho\vec{g} - \vec{\nabla}P = \rho\vec{a}$$

$$\star\ \boxed{\vec{\nabla}P = \rho(\vec{g}-\vec{a})} \quad (1)$$

Recall, for hydrostatics

$$\boxed{\vec{\nabla}P = \rho\vec{g}} \quad (2)$$

Compare (1) & (2)

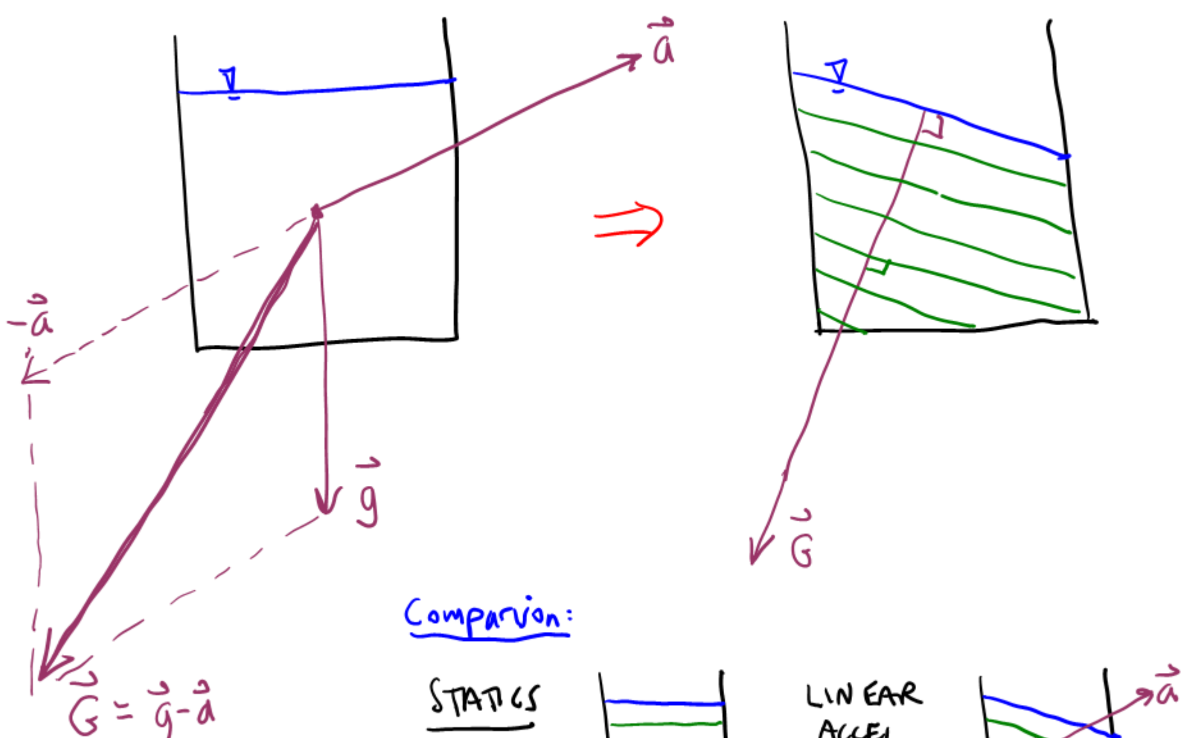
$\vec{g}$ replaced by $\vec{G}-\vec{a}$

$$\star\ \boxed{\vec{G} = \vec{g} - \vec{a}}$$

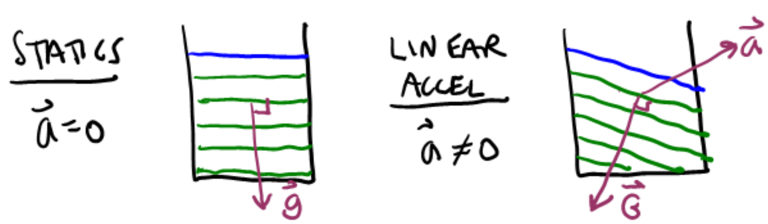
$$\therefore\ \boxed{\vec{\nabla}P = \rho\vec{G}} \quad \star(3)$$

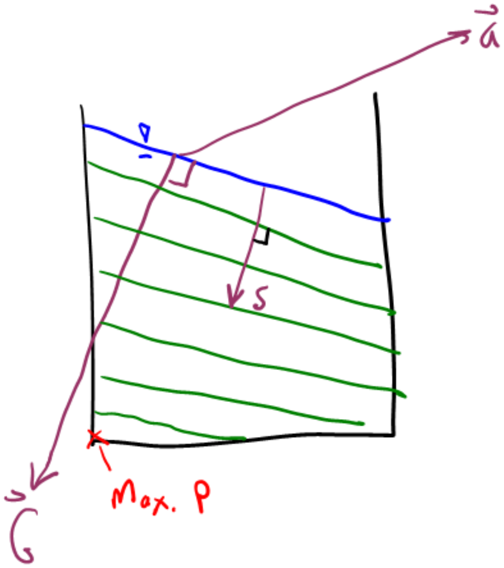
- when $\vec{a}=0$, we get our hydrostatic eq.
- when $\vec{a}\neq 0$, replace $\vec{g}$ with $\vec{G}$

let $\vec{G}$ be a <u>modified gravity vector</u>



Comparison:





S = distance from the surface parallel to $\vec{G}$.

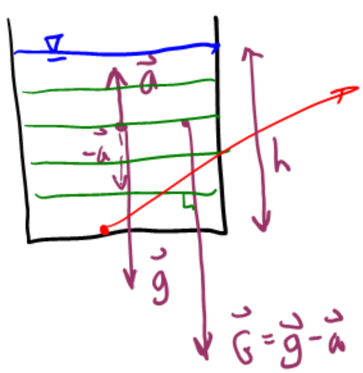
recall, hydrostatics $P_{below} = P_{above} + \rho g |\Delta z|$

here, • replace $\vec{g}$ with $\vec{G}$

• replace z with s

$$\boxed{P_{\text{"below"}} = P_{\text{"above"}} + \rho G |\Delta s|}$$

<u>Examples:</u> • Acceleration is <u>up</u>

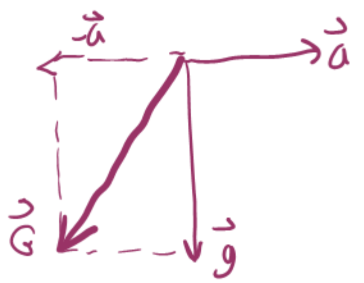


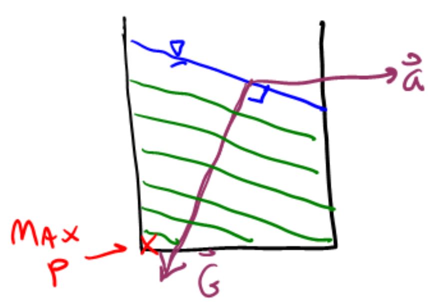
$P_{bottom} = P_{atm} + \rho G h$

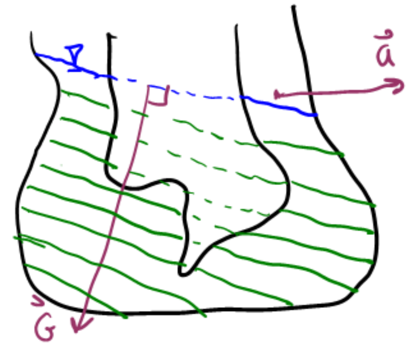
g

THE SHAPE OF THE CONTAINER DOES NOT MATTER

• Acceleration is <u>horizontal</u>







Frames from the slow-motion video:

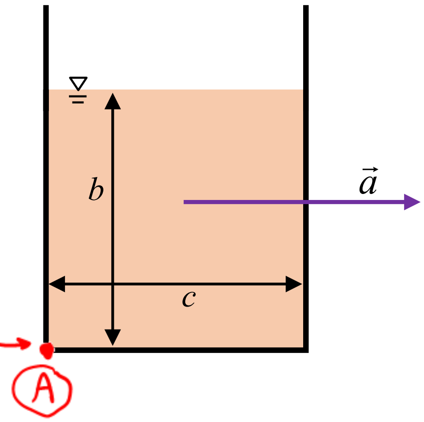
Accelerating to the left

Decelerating to the left

Example: Rigid-Body Acceleration

Given: A rectangular container of oil with SG = 0.825 is accelerated at constant acceleration to the right as sketched (not to scale). After a short adjustment time, the oil behaves as a rigid body in constant acceleration.

- Magnitude of acceleration = 3.59 m/s^2
- b = 12.5 cm
- c = 8.82 cm

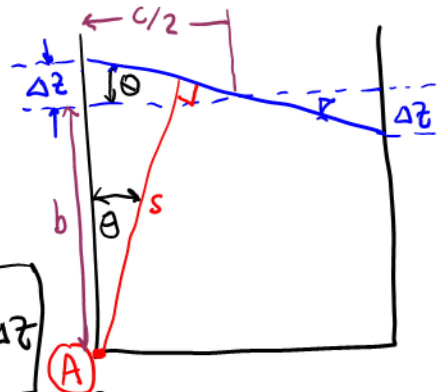


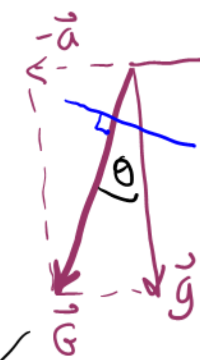
To do: Compare the gage pressure in kPa at the lower left corner of the container when the fluid is at rest and when it is accelerating.

Solution: • AT REST $P_{g,A} = \rho g b$

$$P_{g,A,rest} = \left(825\ \frac{kg}{m^3}\right)\left(9.807\ \frac{m}{s^2}\right)\left(0.125\ m\right)\left(\frac{N\cdot s^2}{kg\cdot m}\right)\underbrace{\left(\frac{kPa\cdot m^2}{1000\ N}\right)}_{\left(\frac{kPa\cdot s^2\cdot m}{1000\ kg}\right)} = \boxed{1.01\ kPa}$$

• ACCELERATING





$$\boxed{\tan\theta = \frac{\Delta z}{c/2}}$$

$$\boxed{\tan\theta = \frac{a}{g}}$$

$$\Delta z = \frac{c\tan\theta}{2} = \boxed{\frac{ca}{2g} = \Delta z}$$

$$\boxed{P_{g,A,accel} = \rho G s}$$

$$G = \sqrt{a^2 + g^2} = \boxed{10.4434\ \frac{m}{s^2} = G}$$

$$\boxed{s = (b + \Delta z)\cos\theta} \longrightarrow \cos\theta = \frac{g}{G},\quad \therefore\ s = \frac{(b+\Delta z)g}{G} = \frac{\left(b + \frac{ca}{2g}\right)g}{G}$$

$$\boxed{P_{g,A,accel} = \rho G s = \rho\left(bg + \frac{ca}{2}\right)}$$

* Answer in variable form

$$P_{g,A,accel} = \left(825\ \frac{kg}{m^3}\right)\left[\left(0.125\ m\right)\left(9.807\ \frac{m}{s^2}\right) + \frac{(0.0882\ m)\left(3.59\ \frac{m}{s^2}\right)}{2}\right]\left(\frac{kPa\cdot s^2\cdot m}{1000\ kg}\right)$$

$$\boxed{P_{g,A,accel} = 1.14\ kPa}$$

Larger than $P_{g,A,rest}$

since • $s > b$

• $G > g$