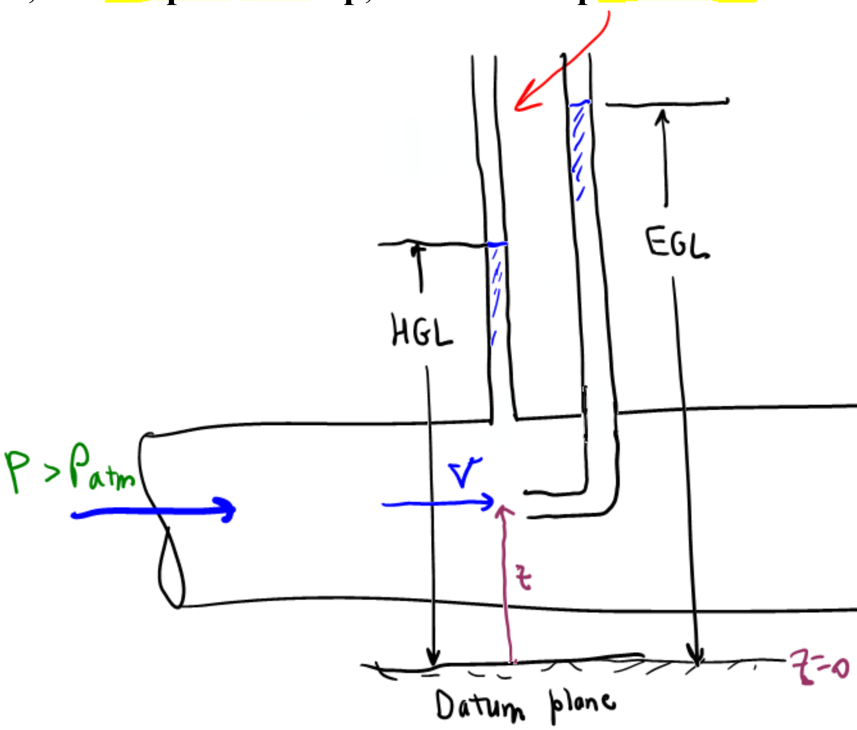
GRADE LINES

In this lesson, we will:

- Define **Grade Lines**: **Energy Grade Line** (**EGL**) and **Hydraulic Grade Line** (**HGL**)
- Discuss applications of these grade lines and their relationship to the energy equation
- Discuss how irreversible head losses affect EGL and HGL
- Do some example problems

Hydraulic Grade Line (HGL)

Definition: **HGL** is the height to which a liquid rises from a pressure tap *normal* to the flow, i.e., a **static pressure tap**, also called a **piezometer**.



Energy Grade Line (EGL)

Definition: **EGL** is the height to which a liquid rises from a total pressure probe aligned *parallel* to and facing *into* to the flow, i.e., a **total pressure probe**, also called a **stagnation pressure probe**.

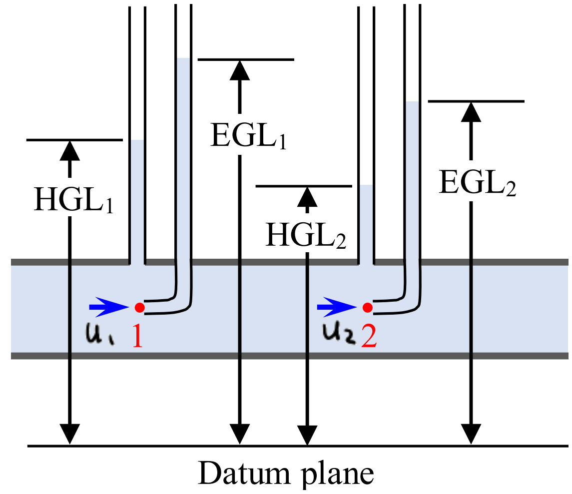
$$\boxed{HGL = \frac{P}{\rho g} + z} \qquad \boxed{EGL = \frac{P}{\rho g} + \frac{V^2}{2g} + z}$$

$$EGL - HGL = \frac{V^2}{2g}$$

$$\Downarrow$$

$$\boxed{\therefore\ V = \sqrt{2g\,(EGL - HGL)}}$$

Example: HGL and EGL



Given: Water at 20ºC flows through a pipe. Two peizometers and two stagnation presssure probes are inserted into the pipe as sketched, and their column heights are measured from an arbitrary datum plane:

- HGL_1 = 6.85 cm
- EGL_1 = 8.56 cm
- HGL_2 = 5.63 cm
- EGL_2 = 7.34 cm

To do:

(a) Calculate the water speed at locations 1 and 2 in units of m/s.

(b) Do you suspect that this flow is fully developed? Why or why not?

Solution:

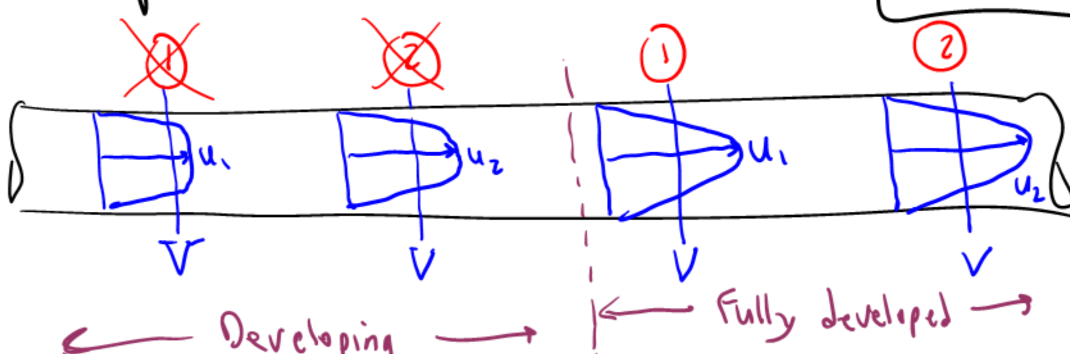
(a) $$u_1 = \sqrt{2g\,(EGL_1 - HGL_1)}$$

$$= \sqrt{2\left(9.807\ \frac{m}{s^2}\right)\left(8.56\text{ cm} - 6.85\text{ cm}\right)\left(\frac{m}{100\text{ cm}}\right)} = 0.57914\ \frac{m}{s}$$

@ 1 $$\boxed{u_1 = 0.579\ \frac{m}{s}}$$

@ 2 — repeat → $u_2 = 0.57914\ \frac{m}{s}$ → $\boxed{u_2 = 0.579\ \frac{m}{s}}$

(b)

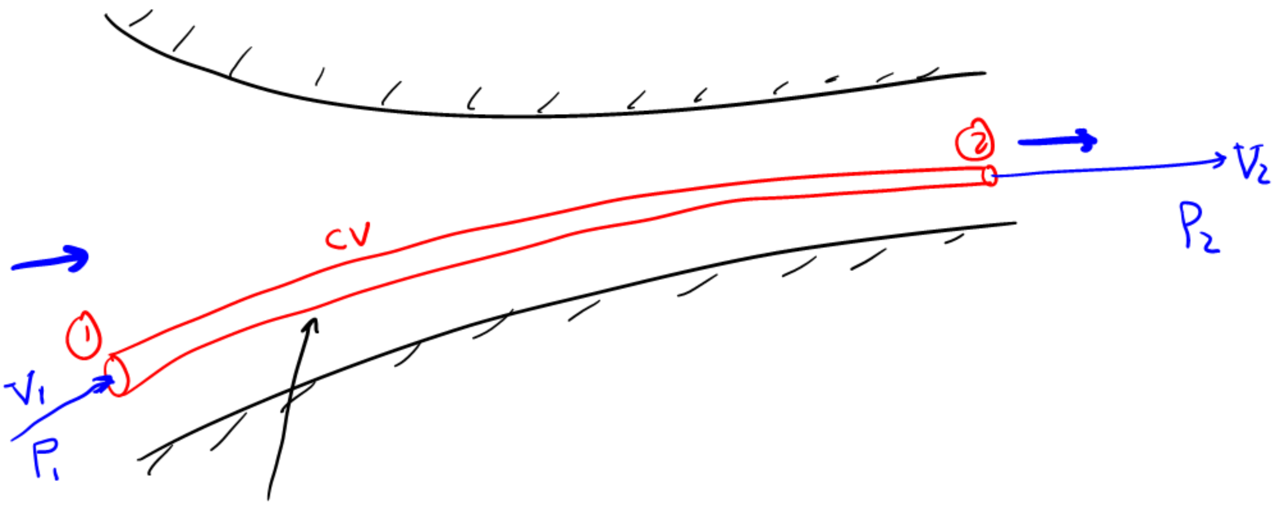


$u_1 = u_2 = u_{max}$ @ ℄

$\boxed{\text{Yes}}$

$\boxed{\text{Since } u_1 = u_2 \text{ the profile is most likely not changing} \therefore \text{The flow is fully developed}}$

Relationship of EGL to the Head Form of the Energy Equation



Streamtube is a "thick" streamline

$$\frac{P_1}{\rho g} + \alpha_1 \frac{V_1^2}{2g} + z_1 + \cancel{h_{pump,u}} = \frac{P_2}{\rho g} + \alpha_2 \frac{V_2^2}{2g} + z_2 + \cancel{h_{turbine,e}} + h_L$$

Let streamtube shrink to a streamline $\rightarrow \alpha_1 = \alpha_2 = 1$

$$\boxed{\underbrace{\frac{P_1}{\rho g} + \frac{V_1^2}{2g} + z_1}_{EGL_1} = \underbrace{\frac{P_2}{\rho g} + \frac{V_2^2}{2g} + z_2}_{EGL_2} + h_L}$$

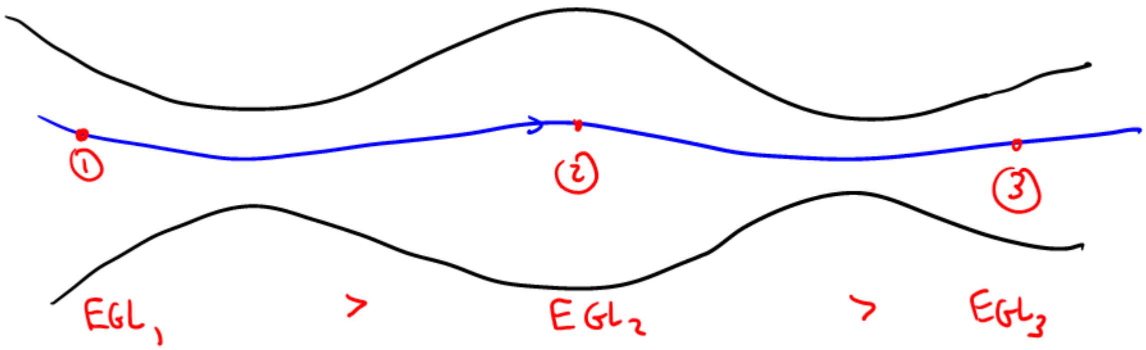
along a streamline

also called the "total head" or the "Bernoulli head"

$$\boxed{EGL_1 = EGL_2 + h_L}$$

along a streamline

Note: h_L is always > 0, EGL must continually decrease



Example: HGL and EGL (continued from above)

Given: Water at 20°C flows through a pipe. Two peizometers and two stagnation presssure probes are inserted into the pipe as sketched, and their column heights are measured from an arbitrary datum plane:

- HGL_1 = 6.85 cm
- EGL_1 = 8.56 cm
- HGL_2 = 5.63 cm
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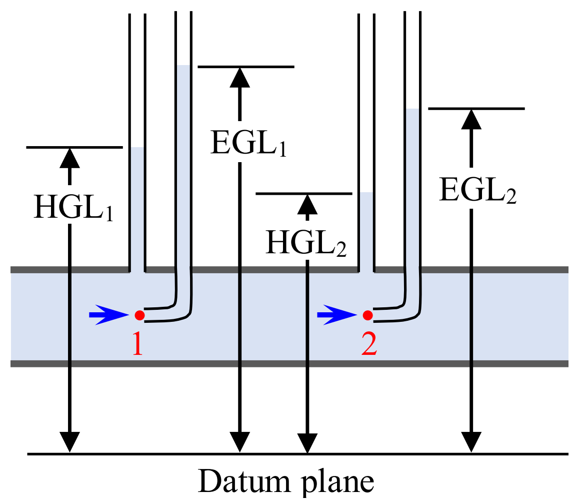


To do:

(c) Calculate the irreversible head loss in this pipe flow between locations 1 and 2 in units of m.

(d) Calculate the pressure loss in this pipe flow between locations 1 and 2 in units of Pa.

Solution:

(c) $$h_L = EGL_1 - EGL_2 = (8.56 - 7.34)\,\text{cm}\left(\frac{1\ \text{m}}{100\ \text{cm}}\right) = \boxed{0.0122\ \text{m}}$$

h_L

(d) recall, $\underline{\underline{\Delta P = \rho g \Delta z}}$

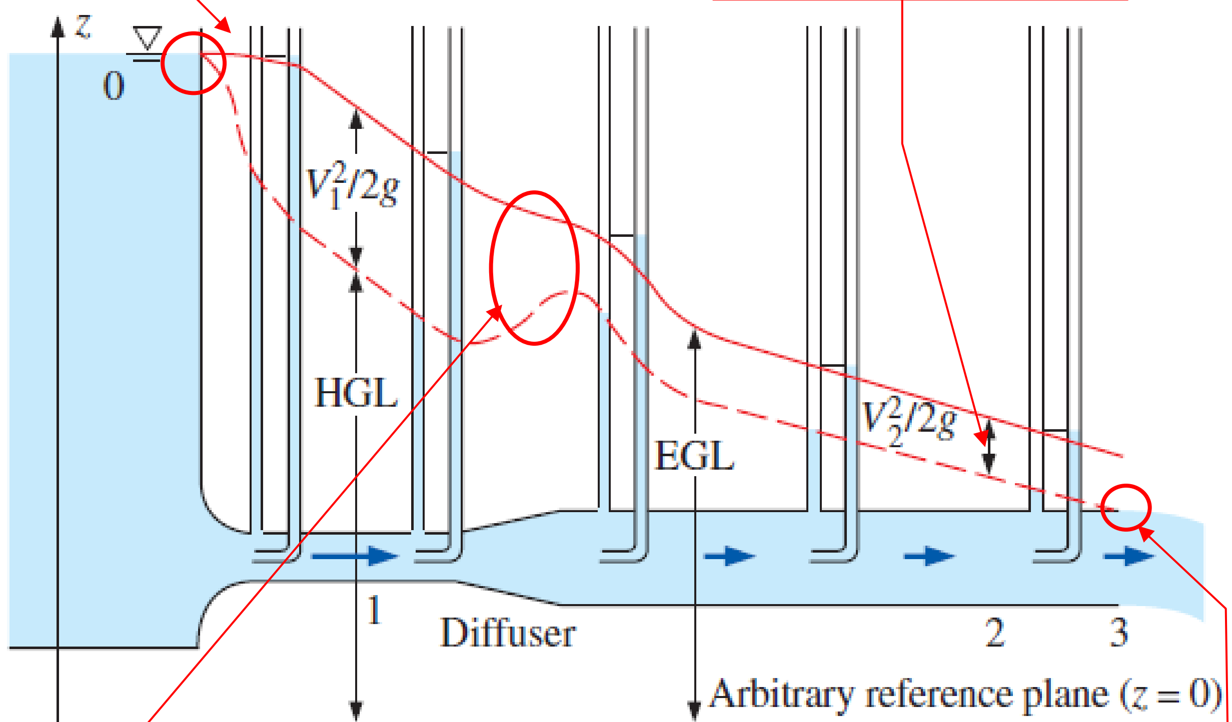
$$\therefore\ \Delta P_{loss,\,1\text{-}2} = \rho g h_L$$

$$\Delta P_{loss,\,1\text{-}2} = \left(998.0\ \frac{\text{kg}}{\text{m}^3}\right)\left(9.807\ \frac{\text{m}}{\text{s}^2}\right)(0.0122\ \text{m})\left(\frac{\text{N s}^2}{\text{kg m}}\right)\left(\frac{\text{Pa}\cdot\text{m}^2}{\text{N}}\right) = \boxed{119.\ \text{Pa}}$$

Qualitative Example of Grade Lines in a Variable-Area Pipe Flow

At point 0, HGL = EGL inside the tank since the fluid is at rest ($V = 0$). Neither EGL nor HGL can rise above this value unless work is added to the flow (e.g., with a pump).

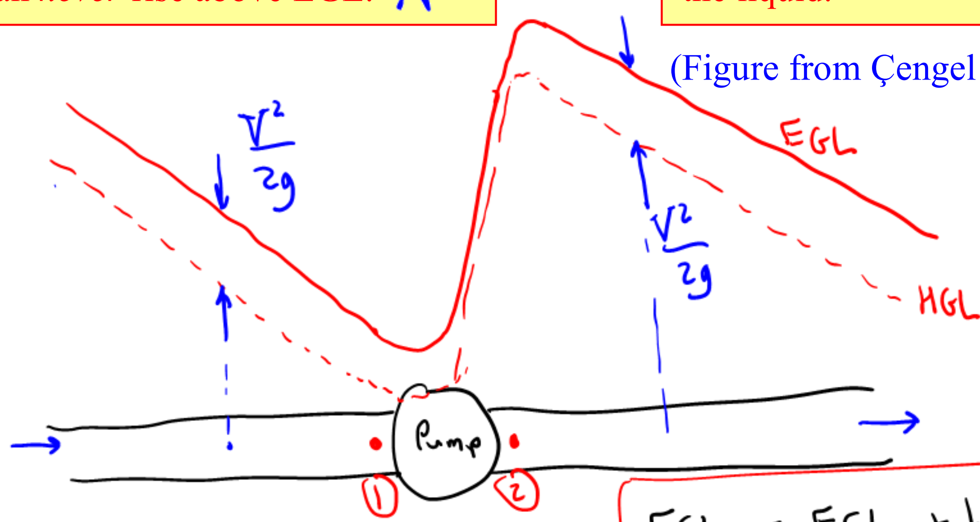
At any location in the duct, the difference between EGL and HGL is $V^2/(2g)$.



EGL continually falls due to irreversible losses, but HGL can rise or fall. Overall, however, HGL also must fall. In fact, HGL can *never* rise above EGL.

Since the jet exits at atmospheric pressure at the outlet of the pipe, $P_3 = P_{atm}$, and HGL is equal to the height of the free surface of the liquid.

(Figure from Çengel and Cimbala, Ed. 4)



$$\boxed{EGL_2 = EGL_1 + h_{pump,u} - h_L}$$