DIMENSIONAL ANALYSIS INTRODUCTION

In this lesson, we will:

- Define the **seven primary dimensions** and discuss how to convert any other dimension in terms of primary dimensions
- Reinforce the concept of **dimensional homogeneity** of equations
- Define **dimensionless parameters** and how to analyze them
- Do some example problems

The Seven Primary Dimensions

DIMENSION	SYMBOL	
mass	m	mass
length	L	
time	t	
temperature	T	(some authors use θ)
electric current	I	
amt. of light	C	
amt of matter	N	→ moles, $\{M\} = \left\{\frac{m}{N}\right\}$

$$\boxed{\text{ALL OTHER DIMENSIONS CAN BE FORMED BY COMBINATION OF THESE PRIMARY DIMENSIONS}}$$

E.g. Force $\quad \{\text{Force}\} = \{ma\} = \left\{m \frac{L}{t^2}\right\}$

Some authors use force instead of mass as a primary dimension

Example: Primary dimensions – shear stress, force per unit length, and power

(*a*) Given: In fluid mechanics, shear stress τ is expressed in units of N/m^2.

To do: Express the primary dimensions of τ, i.e., write an expression for $\{\tau\}$.

Solution:

$$\{\tau\} = \left\{\frac{\text{Force}}{\text{area}}\right\} = \left\{\frac{mL/t^2}{L^2}\right\} \longrightarrow \boxed{\{\tau\} = \left\{\frac{m}{Lt^2}\right\}}$$

$$\text{or } \boxed{\{\tau\} = \{m^1 L^{-1} t^{-2}\}}$$

(*b*) Given: We are conducting an experiment in which quantity *a* has dimensions of force per unit length.

To do: Express the primary dimensions of *a*, i.e., write an expression for $\{a\}$.

Solution:

$$\{a\} = \left\{\frac{F}{L}\right\} = \left\{\frac{m\cancel{L}/t^2}{\cancel{L}}\right\} = \left\{\frac{m}{t^2}\right\}$$

$$\boxed{\{a\} = \left\{\frac{m}{t^2}\right\} = \{m^1 t^{-2}\}}$$

(*c*) Given: Power $\dot{W}$ has the dimensions of energy per unit time.

To do: Write the dimensions of power in terms of primary dimensions.

Solution:

Energy = force × distance

Power = energy/time = rate of energy

$$\{\dot{W}\} = \left\{\frac{F \cdot L}{t}\right\} = \left\{\frac{mL}{t^2}\frac{L}{t}\right\} = \left\{\frac{mL^2}{t^3}\right\}$$

$$\boxed{\{\dot{W}\} = \left\{\frac{mL^2}{t^3}\right\} = \{m L^2 t^{-3}\}}$$

Dimensional Homogeneity

ALL ADDITIVE TERMS IN AN EQUATION MUST HAVE THE SAME DIMENSIONS

E.g., our workhorse linear momentum eq. for a fixed CV

$$\sum \vec{F} = \sum_{out} \beta \dot{m} \vec{V} - \sum_{in} \beta \dot{m} \vec{V}$$

$$\left\{\frac{mL}{t^2}\right\} \qquad \{1\}\left\{\frac{m}{t}\right\}\left\{\frac{L}{t}\right\} \qquad \leftarrow \text{same}$$

$$\left\{\frac{mL}{t^2}\right\} \qquad \left\{\frac{mL}{t^2}\right\} \qquad \left\{\frac{mL}{t^2}\right\}$$

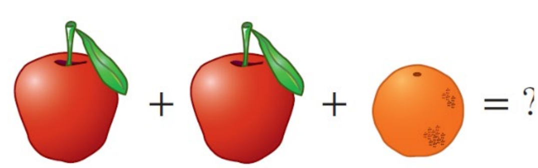
E.g., our workhorse eq. for angular momentum, fixed CV

$$\sum M = \sum_{out} r \dot{m} V - \sum_{in} r \dot{m} V$$

$$\{F \cdot L\} \qquad \left\{L \; \frac{m}{t} \; \frac{L}{t}\right\}$$

$$\left\{\frac{mL^2}{t^2}\right\} \qquad \left\{\frac{mL^2}{t^2}\right\} \qquad \left\{\frac{mL^2}{t^2}\right\}$$

If these dimensions were NOT the same, you have an error somewhere



From Çengel and Cimbala, Ed. 4.

Dimensionless Parameters

Use Π as the symbol for a dimensionless parameter
not π

$$\{\Pi\} = \{1\} = \{m^0 L^0 t^0 T^0 I^0 C^0 N^0\}$$

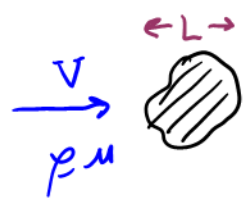
e.g., Mach number $= Ma = \frac{V}{c} \to \{Ma\} = \{1\}$

$\Pi = Ma = \frac{V}{c} \to$ Ma is a dimensionless parameter

• Another useful Π is $\boxed{\text{Reynolds number, } Re}$ ★

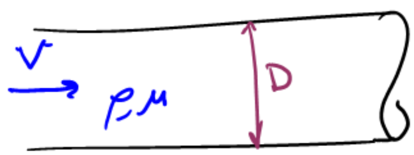
$$\boxed{Re = \frac{\rho V L}{\mu}}$$

External flow:



$\to \boxed{Re = \frac{\rho V L}{\mu}}$

Internal flow:



$\boxed{Re = \frac{\rho V D}{\mu}}$

Verify that Re is a Π:

$$\{Re\} = \frac{\left\{\frac{\cancel{m}}{\cancel{L^3}}\right\}\left\{\frac{\cancel{L}}{\cancel{t}}\right\}\{\cancel{L}\}}{\left\{\frac{\cancel{m}}{\cancel{L}\cancel{t}}\right\}} = \{1\}$$

Example: Dimensionless Parameters

Given: We have three available variables: *a*, *b*, and *c*. Here are their dimensions:

$$\{a\}=\left\{\frac{L}{t}\right\} \qquad \{b\}=\left\{\frac{L^3}{m}\right\} \qquad \{c\}=\left\{\frac{m}{t^2L}\right\}$$

We need to construct a dimensionless parameter using only these three variables and with variables *b* and *c* in the denominator. We construct our Π as follows,

$$\Pi=\frac{a^x}{bc}$$

To do: Calculate exponent *x*.

Solution:

$$\{\Pi\}=\{1\}=\{m^0L^0t^0T^0\ldots\}=\left\{\left(\frac{L}{t}\right)^x\left(\frac{L^3}{m}\right)^{-1}\left(\frac{m}{t^2L}\right)^{-1}\right\}$$

$$\{m^0L^0t^0\}=\{m^1m^{-1}L^{x-3+1}t^{-x+2}\}$$

$$\{m^{1-1}\} \Rightarrow \{m^0 \quad L^{x-2} \quad t^{2-x}\}$$

$$L:\quad x-2=0 \Rightarrow \boxed{x=2}$$

$$t:\quad 2-x=0 \Rightarrow \boxed{x=2}$$

$$\boxed{\Pi=\frac{a^2}{bc}} \quad \text{where } \boxed{x=2}$$

Verify:

$$\{\Pi\}=\left\{\frac{\cancel{L}}{\cancel{t}}\right\}^2\left\{\frac{\cancel{m}}{\cancel{L^3}}\right\}\left\{\frac{\cancel{Lt^2}}{\cancel{m}}\right\}=\{1\} \checkmark$$