SIMILARITY, MODELS AND PROTOTYPES

In this lesson, we will:

- Discuss the **purposes** of dimensional analysis
- Define **similarity** and how to apply it between **models** and **prototypes**
- Do an example problem

The Purposes of Dimensional Analysis

- To help plan & carry out experiments
- To obtain scaling laws [e.g., design a larger pump based on an existing small pump]
- To (sometimes) predict trends [e.g., predict Power $\propto$ Dia^3]

Similarity

Consider a model & a prototype

model → a scale item, e.g. a model car used for testing

prototype → The real item

To properly scale (model to prototype) we must have complete similarity [Dynamic Similarity]

Criteria:

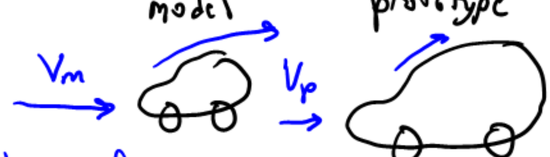
model prototype

1. Geometric Similarity

☆ Proportional geometry

model prototype

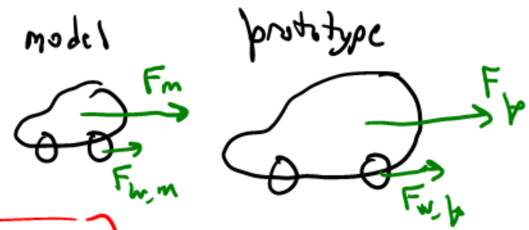
2. Kinematic Similarity



☆ Proportional velocity fields

3. Dynamic Similarity

☆ COMPLETE SIMILARITY → Proportional Forces



☆ WE MUST HAVE ALL THREE TO ACHIEVE COMPLETE SIMILARITY

→ If so, we can confidently scale up (or down) from model to prototype

$$\Pi_1 = fnc(\Pi_2, \Pi_3, \cdots \Pi_k)$$

Π_1: DEPENDENT Π

$\Pi_2, \Pi_3, \cdots \Pi_k$: INDEPENDENT Πs

[k = # of Πs]

☆ SIMILARITY BETWEEN MODEL & PROTOTYPE

If $\Pi_{2,m} = \Pi_{2,p}$ &

$\Pi_{3,m} = \Pi_{3,p}$

⋮

$\Pi_{k,m} = \Pi_{k,p}$

THEN $\Pi_{1,m} = \Pi_{1,p}$ ☆

☆ TRUE ONLY IF ALL INDEPENDENT Πs MATCH

i.e. Complete similarity

(dynamic similarity)

Example: Similarity Between Model and Prototype

Given: Engineers need to predict the vibration frequency of electrical cables subjected to wind. [This is the prototype.] Here are the prototype values:

- Air is at 25ºC → $\rho_p = 1.184\ kg/m^3$, $\mu_p = 1.849\times10^{-5}\ \frac{kg}{m\cdot s}$
- Wire diameter = 0.851 cm $= D_p$
- Wind speed = 5.32 m/s $= V_p$

A young engineer studied dimensional analysis in fluids class and figures out that for this flow situation,

St (Strouhal number) = function of Re (Reynolds number)

The engineers set up a model test in a water tunnel using a cable. They set it up with the same aspect ratio and end conditions as the prototype. Here are the model values:

- Water is at 20ºC → $\rho_m = 998.0\ kg/m^3$, $\mu_m = 1.002\times10^{-3}\ \frac{kg}{m\cdot s}$
- Wire diameter = 0.394 cm $= D_m$

To do:

(**a**) Calculate the water tunnel speed required to achieve dynamic similarity.

(**b**) When the water tunnel is run at the speed calculated in Part (a), the vibration frequency of the model cable is 38.1 Hz. Predict the vibration frequency of the prototype cable in Hz.

Solution:

(a) $St = fnc(Re) \rightarrow \Pi_1 = fnc(\Pi_2)$ $(k=2)$

(Π_1: Dependent; Π_2: Independent)

For dynamic similarity, $Re_m = Re_p \rightarrow \frac{\rho_m V_m D_m}{\mu_m} = \frac{\rho_p V_p D_p}{\mu_p}$

Solve for $V_m \rightarrow$

$$V_m = V_p\left(\frac{\rho_p}{\rho_m}\right)\left(\frac{D_p}{D_m}\right)\left(\frac{\mu_m}{\mu_p}\right) = \left(5.32\frac{m}{s}\right)\left(\frac{1.184\ \frac{kg}{m^3}}{998.0\ \frac{kg}{m^3}}\right)\left(\frac{0.851\ cm}{0.394\ cm}\right)\left(\frac{1.002\times10^{-3}\ \frac{kg}{m\cdot s}}{1.849\times10^{-5}\ \frac{kg}{m\cdot s}}\right)$$

$$V_m = 0.73875\ \frac{m}{s}$$

RUN WATER TUNNEL AT $V = 0.739$ m/s TO ACHIEVE DYNAMIC SIMILARITY

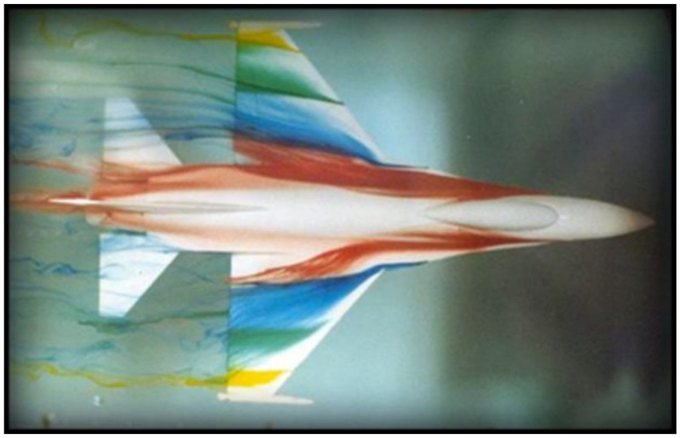
(b) Since $Re_m = Re_p$, $St_m = St_p \rightarrow St = \frac{fD}{V} \rightarrow \frac{f_m D_m}{V_m} = \frac{f_p D_p}{V_p}$

Solve for f_p: $f_p = f_m\left(\frac{D_m}{D_p}\right)\left(\frac{V_p}{V_m}\right) = \ldots$ ($f_m = 38.1$ Hz)

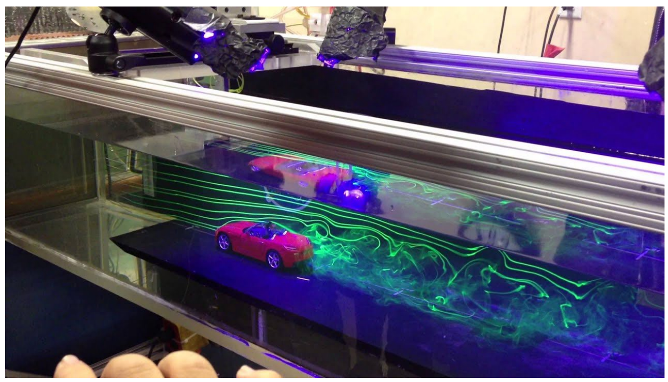
$$f_p = 127.\ Hz$$

Testing in Different Fluids

As long as there is dynamic similarity between model and prototype, the fluid used for the test does not matter.



Fighter Jet Tested in a Water Tunnel. Image from aerolab.com.



Car Tested in a Water Channel. Image from NASA Ames.

As long as there are no significant surface effects (e.g., ship waves), you can use any fluid you want for the model test.

Submarine Tested in a Wind Tunnel. Image from NASA Langley.