INCOMPLETE SIMILARITY

In this lesson, we will:

- Discuss why we sometimes encounter **Incomplete Similarity**
- Discuss how to design useful model/prototype experiments when we have incomplete similarity
- Do some example problems

$$\Pi_1 = fnc(\Pi_2, \Pi_3)$$

Definition of Incomplete Similarity

Incomplete similarity is when it is impossible (or prohibitively expensive) to match *all* the independent Π parameters between model and prototype.

$$\Pi_1 = fnc(\Pi_2)$$

Example of Incomplete Similarity: Wind Tunnel Testing

Example: Wind Tunnel Test of a Model Truck

Given: The drag on a model tractor-trailer truck is to be measured in a wind tunnel and then scaled up to predict the drag on the prototype. The model is 1:20 scale (model is 1/20th the size of the prototype). The prototype (real) truck travels at 60 mph (26.82 m/s).

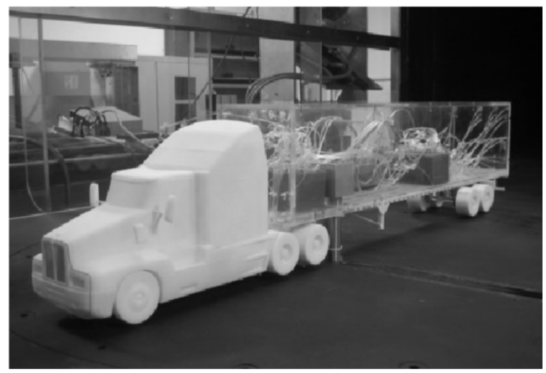


Photo from resarchgate.net.

To do: Calculate the required speed of the wind tunnel and comment about the feasibility/accuracy of the test.

Solution:

We know $C_D = fnc(Re)$

So → match $Re_m = Re_p$ to achieve complete similarity

$$Re_m = \frac{\rho_m V_m L_m}{\mu_m} = \frac{\rho_p V_p L_p}{\mu_p} = Re_p$$

Here we use air @ same P & T, model & prototype

$$V_m L_m = V_p L_p \Rightarrow V_m = V_p \frac{L_p}{L_m} = V_p (20)$$

$$V_m = \left(26.82 \frac{m}{s}\right)(20) = \boxed{536.4 \frac{m}{s} = V_m}$$

PROBLEM:
- Most wind tunnels don't go that fast
- $C = 343 \frac{m}{s}$

THIS IS SUPERSONIC !!

$$M_a = \frac{V}{c} = \frac{536.4 \text{ m/s}}{343 \text{ m/s}} = \underline{1.56} > 1$$

If we ran the wind tunnel at this speed, we would have shock waves & other compressible flow problems

WE CANNOT ACHIEVE COMPLETE SIMILARITY

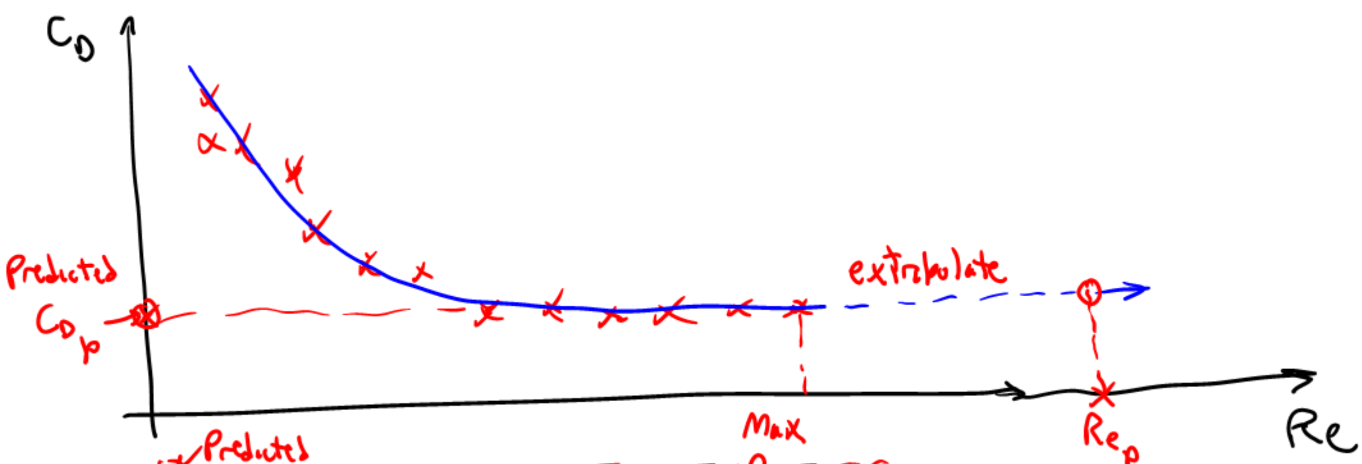
if we add c to our list of independent variables

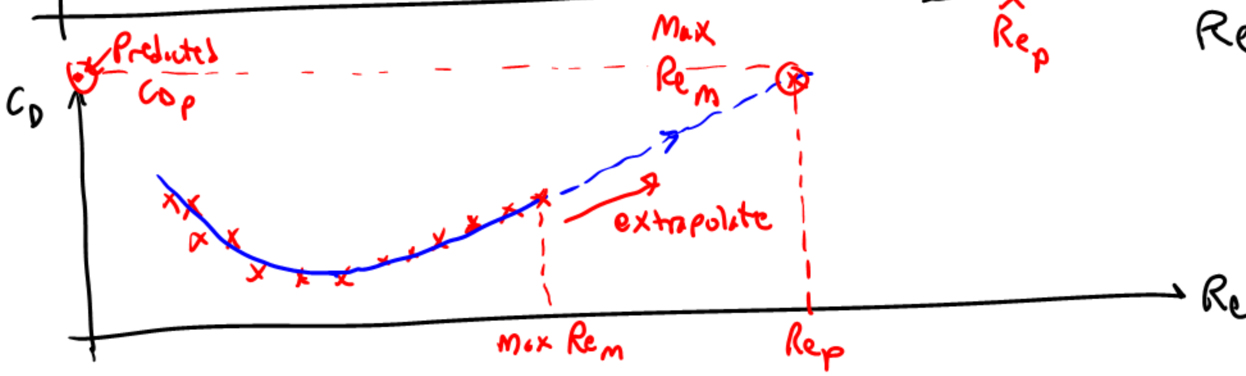
$$\boxed{C_D = fn(Re, M_a)}$$

It is difficult (or impossible) to match both Re & Ma

WHAT TO DO?

- Ma not important if $M_a \lesssim 0.3$
- C_D often becomes Reynolds number independent at high enough Re → we can extrapolate

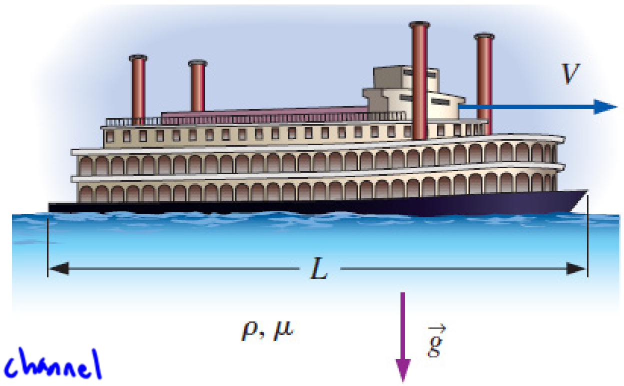




Example of Incomplete Similarity: Free Surface Effects

Example: Scale Modeling of a Ship

Given: The drag on a model ship is to be measured in a water channel and then scaled up to predict the drag on the prototype. The model is 1:100 scale (model is 1/100th the size of the prototype). The prototype (real) ship travels at 20 knots (10.29 m/s).



$$\text{Re} = \frac{\rho V L}{\mu} = \frac{VL}{\nu} \qquad \text{Fr} = \frac{V}{\sqrt{gL}}$$

From From Çengel and Cimbala, Ed. 4.

To do: Calculate the required speed of the ~~wind tunnel~~ water channel and comment about the feasibility/accuracy of the test.

Solution:

Dimensional Analysis → $\boxed{C_D = \text{fnc}(Re, Fr)}$

try to match both of these

To match Re, $\dfrac{V_m L_m}{\nu_m} = \dfrac{V_p L_p}{\nu_p}$

To match Fr, $\dfrac{V_m}{\sqrt{g L_m}} = \dfrac{V_p}{\sqrt{g L_p}}$ (since g is a fixed constant)

$$V_m = V_p \sqrt{\frac{\cancel{g} L_m}{\cancel{g} L_p}} = V_p \sqrt{\frac{L_m}{L_p}} = V_p \sqrt{\frac{1}{100}} = \frac{V_p}{10}$$

To match Fr, set $V_m = V_p/10$ → $V_m = \dfrac{10.29 \text{ m/s}}{10} = 1.03 \dfrac{m}{s}$

Now match Re → $\boxed{\nu_m = \dfrac{V_m}{V_p} \dfrac{L_m}{L_p} \nu_p}$

$\nu_p = \nu_{water} \approx 1 \times 10^{-6} \dfrac{m^2}{s}$

set $\nu_m = \left(\dfrac{1}{10}\right)\left(\dfrac{1}{100}\right)\left(1 \times 10^{-6} \dfrac{m^2}{s}\right) = \boxed{1 \times 10^{-9} \dfrac{m^2}{s} = \nu_m}$

* NO SUCH FLUID EXISTS!! → WE CANNOT ACHIEVE DYNAMIC SIMILARITY !!

WHAT TO DO?

Fortunately, Fr is way more important than Re in flows with a free surface

So, <u>match Fr</u> & test @ highest possible Re & extrapolate