PIPE FLOW INTRODUCTION

In this lesson, we will:

- Briefly review **Average Speed** and **Mass Conservation** for pipe flow
- Discuss differences between **Laminar** and **Turbulent** pipe flow and define **Critical Re**
- Define **Hydraulic Diameter** and discuss its application
- Do some example problems

Review: Average Speed and Conservation of Mass

Recall the definition and usefulness of average speed in a pipe:

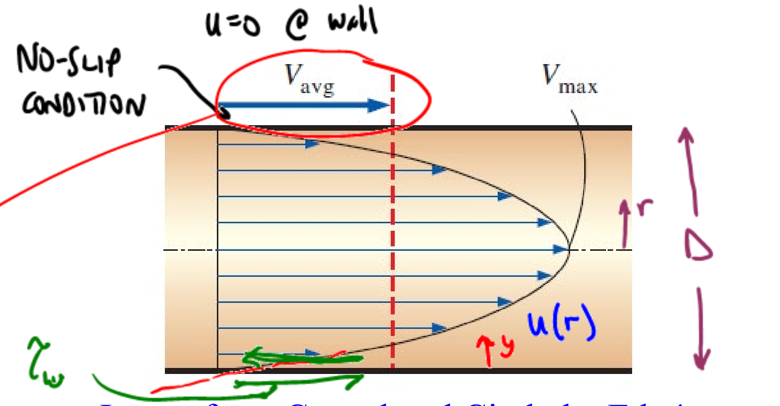


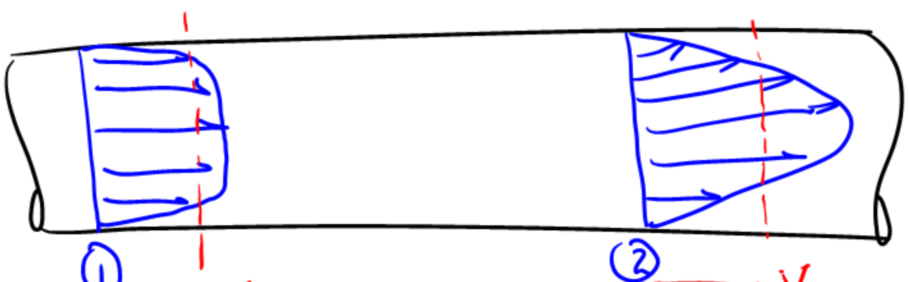
Image from Çengel and Cimbala, Ed. 4.

V_{avg} = avg. speed at a cross section

We usually drop the "avg" subscript

V = average speed

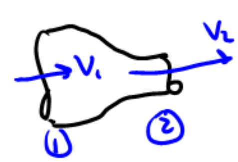
For constant diameter pipes & for incompressible flow, V = const



$\rightarrow V_1$ $\quad\rightarrow V_2$

$V_1 = V_2$

$\dot{m} = \rho V A = \text{constant}$ $\rightarrow$ here $\rho_1 V_1 A_1 = \rho_2 V_2 A_2$



Laminar vs. Turbulent Flow

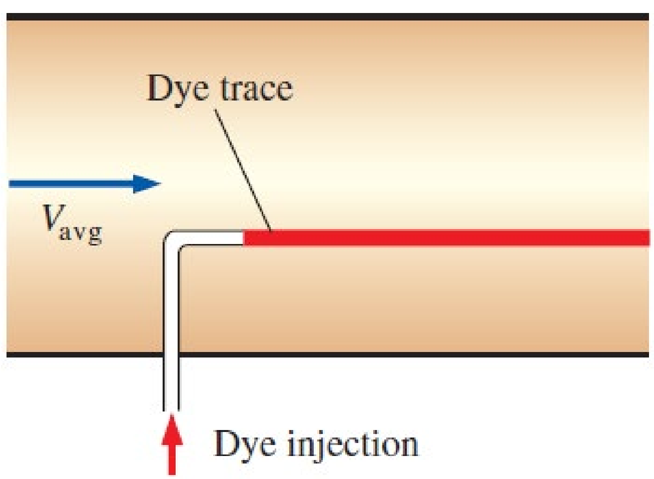
Laminar Flow

Can be steady or unsteady.

(Steady means that the flow field at any instant in time is the same as at any other instant in time.)

Can be one-, two-, or three-dimensional.

Has regular, *predictable* behavior



Analytical solutions are possible (see Chapter 9).

Occurs at *low* Reynolds numbers.

Turbulent Flow

Is always *unsteady*.

Why? There are always random, swirling motions (vortices or eddies) in a turbulent flow.

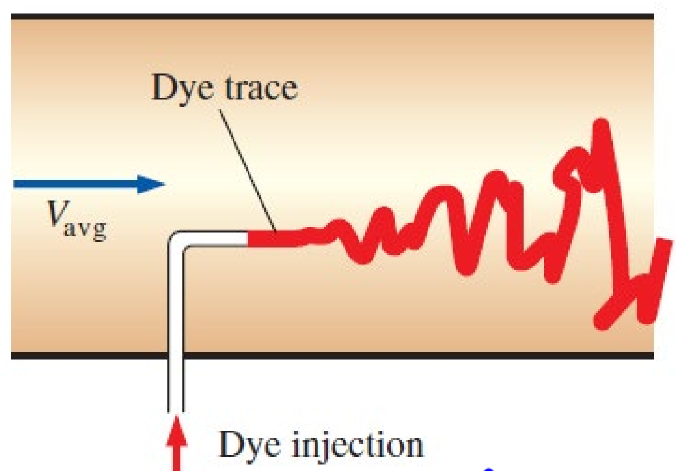
Note: However, a turbulent flow can be steady *in the mean*. We call this a *stationary turbulent flow*.

Is always *three-dimensional*.

Why? Again because of the random swirling eddies, which are in all directions.

Note: However, a turbulent flow can be 1-D or 2-D *in the mean*.

Has irregular or *chaotic* behavior (cannot predict exactly – there is some randomness associated with any turbulent flow.



Typically solve for the mean flow only

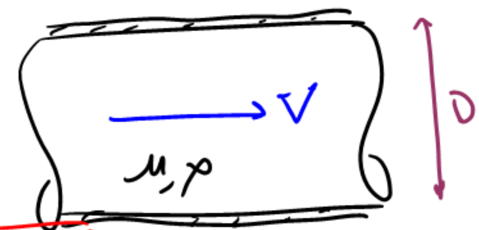
No analytical solutions exist! (It is too complicated, again because of the 3-D, unsteady, chaotic swirling eddies.)

Occurs at *high* Reynolds numbers.

Critical Reynolds Number

At what Reynolds number does pipe flow transition from laminar to turbulent?

Define Re



$$Re = \frac{\rho V D}{\mu}$$

But $\nu = \frac{\mu}{\rho}$ = kinematic viscosity

or $Re = \frac{VD}{\nu}$

For a round pipe, $Re_{critical} \approx 2300$

$Re \lessapprox 2300 \rightarrow$ laminar flow

$2300 \lessapprox Re \lessapprox 4000 \rightarrow$ transitional flow

$Re \gtrapprox 4000 \rightarrow$ turbulent

"Rules of thumb"

approximate values

vary with

- pipe roughness
- vibrations
- upstream disturbances (e.g., elbows, valves upstream)

Hydraulic Diameter

For non-round pipes, define

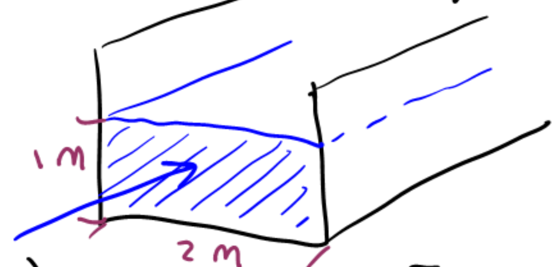
Hydraulic Diameter $$D_h = \frac{4A_c}{p}$$ ✱

where A_c = actual cross-sectional area of the pipe or duct

& p = wetted perimeter

(portion of the perimeter in contact with the fluid)

Eg, OPEN CHANNEL



$A_c = (1\ m)\times(2\ m)$

$p = 1\ m + 2\ m + 1\ m = 4\ m$

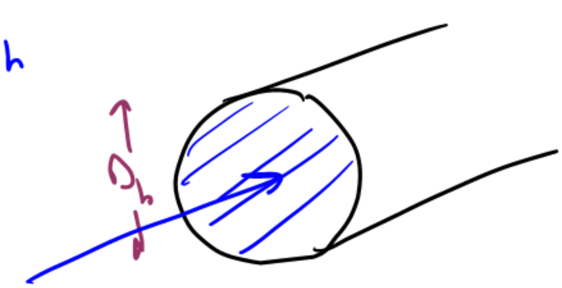
$$D_h = \frac{4A_c}{p} = \frac{4(2m^2)}{4\ m}$$

$$D_h = 2\ m$$ ✱

WHAT DOES THIS MEAN?

$f = f(Re, \varepsilon/D)$ → correlations are based on round pipes

Equivalent to flow through a round pipe of diameter D_h



CAUTION WHEN CALCULATING $\dot{m}$ (or $\dot{\forall}$),

use the real cross-sectional area A_c

not the circular cross-sectional area $\frac{\pi D_h^2}{4}$

- $\dot{m} = \rho V A$ → use A_c & the actual V

use $$V = \frac{\dot{m}}{\rho A_c}$$

- When calculating Re to find f, use D_h & actual V

$$Re = \frac{\rho V D_h}{\mu}$$

not V for the equivalent circular pipe

In above example, suppose $\dot{\forall} = 10 \frac{m^3}{s}$. We had $A_c = 2\,m^2$

$$\therefore\ V = \frac{\dot{\forall}}{A_c} = \frac{10\ m^3/s}{2\ m^2} = 5\ \frac{m}{s}$$

Use this value in Re

"Equivalent" round pipe

D_h → $A_{round} = \frac{\pi D_h^2}{4}$

$$V = \frac{\dot{\forall}}{A_{round}} = \frac{\dot{\forall}}{\pi D_h^2/4} = 3.183\ m/s \quad X$$

* DO NOT USE THIS IN Re

Bottom Line:
- Use D_h & actual V to calc. Re
- Then use that Re to calculate f
- Use actual A_c & actual V for calculation of $\dot{m}$ or $\dot{\forall}$

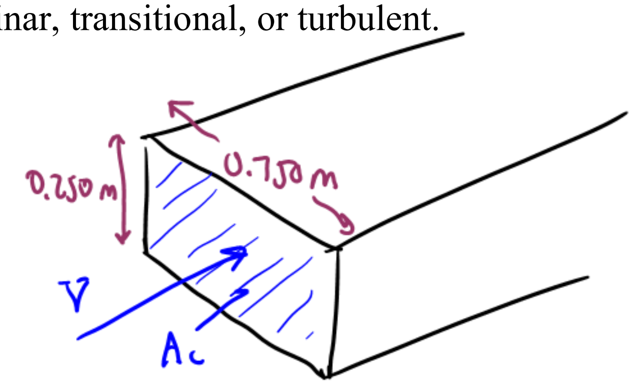
Example: Hydraulic Diameter and Critical Reynolds Number

Given: Air flows through a rectangular air conditioning duct at T = 15.0ºC. The following values are measured:

- Duct width = 0.750 m
- Duct height = 0.250 m
- Average air speed through the duct = 3.21 m/s

To do: Calculate the hydraulic diameter and the volume flow rate, then dicuss if this flow is most likely to be laminar, transitional, or turbulent.

Solution:

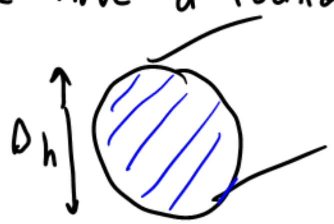


$$D_h = \frac{4A_c}{p} = \frac{4(0.750\ m)(0.250\ m)}{2(0.750\ m) + 2(0.250\ m)} = \boxed{0.375\ m = D_h}$$

$$\dot{V} = VA_c = \left(3.21\ \frac{m}{s}\right)(0.750 m)(0.250\ m) = 0.60188\ \frac{m^3}{s}$$

$$\boxed{\dot{V} = 0.602\ \frac{m^3}{s}}$$

Now "pretend" we have a round pipe of diameter D_h



@ $T = 15°C$, look up ν

$\nu = 1.470 \times 10^{-5}\ m^2/s$

$$Re = \frac{\rho V D_h}{\mu} = \frac{V D_h}{\nu}$$

$$Re = \frac{(3.21\ m/s)(0.375\ m)}{1.470 \times 10^{-5}\ m^2/s} = \boxed{81,900 = Re}$$

Since $Re >> 4000$, this is definitely turbulent