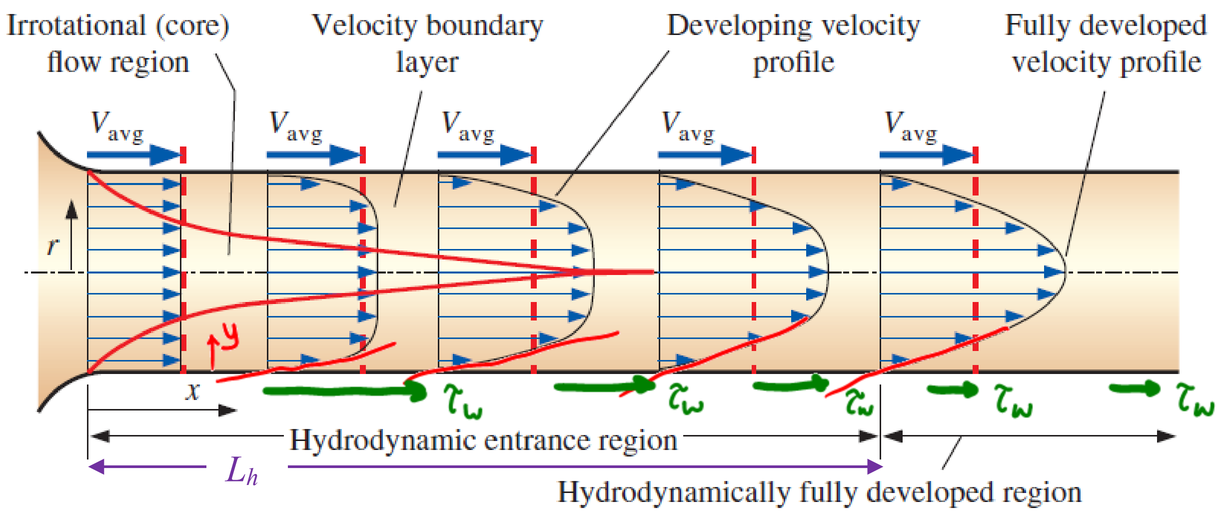
PIPE FLOW ENTRANCE REGION

In this lesson, we will:

- Define **Hydrodynamic Entrance Length** and discuss empirical equations for it
- Do an example problem

Entrance Region



From Çengel and Cimbala, Ed. 4.

- $\tau_w \downarrow$ as $x \uparrow$ in the entrance region (developing region)
- $V = V_{avg} =$ constant

$\boxed{L_h = \text{hydrodynamic entrance length}}$

Example: Hydrodynamic entrance length

Given: **Shear stress** along the inner pipe wall τ_w is a function of (ρ, μ, D, V, ε, x), where we typically drop the "avg" subscript (let $V = V_{avg}$).

To do: Use Dimensional analysis to generate the relationship.

Solution:

$n=6,\ j=3, \rightarrow k = 3\ \Pi_s$

Recall, for fully developed pipe flow, $\tau_w = \text{fnc}(\rho, \mu, D, V, \varepsilon)$

& we got $\boxed{f = \dfrac{8\tau_w}{\rho V^2} = \text{fnc}\left(Re, \dfrac{\varepsilon}{D}\right)}$

Here, we have $\tau_w = \text{fnc}(\rho, \mu, D, V, \varepsilon, x)$ $n=7,\ j=3,\ k=4\ \Pi_s$

Since $\{x\} = \{L\} \rightarrow$ same as $\{\varepsilon\} = \{L\}$

$\therefore$ For entrance region, $\boxed{f = \dfrac{8\tau_w}{\rho V^2} = \text{fnc}\left(Re, \dfrac{\varepsilon}{D}, \dfrac{x}{D}\right)}$

Example: Hydrodynamic entrance length

Given: **Hydrodynamic entrance length** L_h is a function of (ρ, μ, D, V, ε).

To do: Use Dimensional analysis to generate the relationship.

Solution:

$$\boxed{\text{Get} \quad \frac{L_h}{D} = \text{fnc}\left(Re, \frac{\varepsilon}{D}\right)} \quad \text{where} \quad \boxed{Re = \frac{\rho V D}{\mu} = \frac{VD}{\nu}}$$

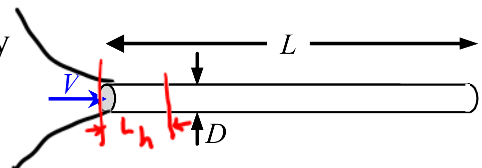
In practice, ε has little effect on L_h

LAMINAR PIPE FLOW: $\boxed{\frac{L_h}{D} \approx 0.050\, Re}$ "rules of thumb"

TURBULENT PIPE FLOW: $\boxed{\frac{L_h}{D} \approx 1.359\, Re^{\frac{1}{4}}}$

Example: Hydrodynamic entrance length

Given: Water with $\nu = 1.00 \times 10^{-6}$ m^2/s flows at a steady average speed of 5.70 m/s through a long pipe of diameter 25.4 cm. The pipe is 1.80 km long.



To do: What percent of the pipe length can be considered to be fully developed?

Solution:

$$Re = \frac{VD}{\nu} = \frac{(5.70\ \frac{m}{s})(0.254\ m)}{1.00 \times 10^{-6}\ m^2/s} = 1.4478 \times 10^6 \gg 4000$$

DEFINITELY TURBULENT

$$L_h = 1.359\, Re^{1/4} D = \underbrace{1.359(1.4478 \times 10^6)^{\frac{1}{4}}}_{47.141}(0.254\ m) = \boxed{\begin{array}{l} 11.973\ m \\ = L_h \end{array}}$$

$$\%\ \text{fully developed} = \frac{L - L_h}{L} \times 100\% = \frac{1800\ m - 11.973\ m}{1800\ m} \times 100\%$$

$$= 99.335\ \%$$

ANS $\boxed{\approx 99.3\ \%\ \text{of the pipe is fully developed}}$

- For very long, straight pipes, entrance length is negligible
- Later, we will learn how to estimate the additional head loss due to entrance effects